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THEORETICAL PREDICTION OF ACOUSTIC-GRAVITY PRESSURE WAVEFORMS GENERATED BY LARGE EXPLOSIONS IN THE ATMOSPHERE

by

Allan D. Pierce and Joe W. Posey

Department of Mechanical Engineering
Massachusetts Institute of Technology

Contract No. F19628-67-C-0217

Project No. 7639

Task No. 763910

Work Unit No. 76391001

FINAL REPORT

Period Covered: February 1, 1967 through January 31, 1970

30 April, 1970

Contract Monitor: Elisabeth F. Iliff
Terrestrial Sciences Laboratory

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ABSTRACT

A computer program is described which enables one to compute the pressure waveform at a distant point following the detonation of a nuclear explosion in the atmosphere. The theoretical basis of the program and the numerical methods used in its formulation are explained; a deck listing and instructions for the program's operation are included. The primary limitation on the program's applicability to realistic situations is that the atmosphere is assumed to be perfectly stratified. However, the temperature and wind profiles may be arbitrarily specified. Numerical studies carried out by the program show some discrepancies with previous computations by Harkrider for the case of an atmosphere without winds. These discrepancies are analyzed and shown to be due to different formulations of the source model for a nuclear explosion. Other numerical studies explore the effects of various atmospheric parameters on the waveforms. In the remainder of the report, two alternate theoretical formulations of the program are described. The first of these is based on the neglect of the vertical acceleration term in the equations of hydrodynamics and allows a solution by Cagniard's integral transform technique. The second is based on the hypothesis of propagation in a single guided mode and permits a study of the effects of departures from stratification on the waveforms.

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Chapter I

INTRODUCTION

1.1 SCOPE OF THE REPORT

The present report summarizes investigations carried out by the authors during the years 1966-1968 on the propagation of low frequency pressure disturbances under Air Force Contract No. F19628-67-C-0217 with the Air Force Cambridge Research Laboratories, Bedford, Massachusetts. The study performed was theoretical in nature.

The principal problem which the authors chose to study was that of the theoretical prediction of the pressure waveform (acoustic pressure versus time) which would be detected near the ground at a large horizontal distance (between 500 and 20,000 km) from a nuclear explosion in the lower atmosphere. This problem was selected for several reasons.

Nuclear explosions, particularly those in the megaton range, excite waves whose dominant periods lie in the so-called acoustic-gravity range (roughly between 1 and 20 minute periods). These acoustic-gravity waves are relatively little understood at present and exhibit many interesting properties which invite serious inquiry.

Of all the known sources which may excite acoustic-gravity waves capable of being detected at large distances, the nuclear explosions correspond most closely to point sources. This allows a considerable simplification in the analysis. Furthermore, the nuclear explosions are better understood and may be described in a more detailed quantitative fashion than would be natural sources such as volcano eruptions, weather disturbances, etc.

A considerable amount of data on pressure waveforms recorded following nuclear explosions exists and is published in the scientific literature. Since the explanation of data should be a principal reason for any theoretical development, it is natural to begin with the study of phenomena for which a large bulk of systematically obtained data exists.

It would appear that the possible application of a theory of waves generated by nuclear explosions to the interpretation of experimental data would be of some practical importance, either as an aid in a possible acoustic detection system of nuclear

explosions, or as a means of inferring some of the as yet imperfectly known properties of the earth's upper atmosphere. In this respect, the first step would clearly be the development of a deterministic theory which, given certain properties of the explosion and of the atmosphere, allows one to predict the waveforms.

It should be mentioned at the outset that the problem under consideration has been of considerable interest to a large segment of the scientific community for some time and that the problem has an illustrious background. The present report merely reports a continuation of one facet of a lengthy pattern of research which has been carried on by a large number of investigators in the past. In a somewhat restricted sense, the work reported here is a continuation of work done by one of the authors (A. Pierce) under Air Force Contract No. AF 19 (628) - 3891 with Avco Corporation during 1964-1966.

A major part of the present report is concerned with the explanation, presentation, and description of a computer program (which we refer to by the name INFRASONIC WAVEFORMS) which was developed during the course of the contract. This program is based on a theory which assumes the atmosphere to be perfectly stratified and to have somewhat arbitrary temperature and wind velocity profiles. This theory, described in some detail in Chapter II, is based on a number of approximations which restrict its application to waveforms recorded near the ground at large distances from low to moderate altitude nuclear explosions. In addition, the computational method restricts the application of the program to the earliest portion of the dominant signal, which travels with a speed roughly equal to the speed of sound at the ground.

Chapter III gives a user's manual for the program, with instructions for preparing input, description of the possible output of the program, and sample input and output. A deck listing of the program is given in Appendix B.

In Chapter IV, some numerical studies made using the program are reported. These studies include the explanation of some discrepancies with previous computations by Harkrider for the case of atmospheres without winds; the discrepancies being due to differences of methods of incorporating a source model into the formulation. A discussion, with numerical examples, is also given of the effects of variations in parameters describing the atmosphere on the waveforms. There we conclude that the physical significance of the individual guided modes predicted for a given atmosphere model is extremely limited and that the total waveform

is relatively insensitive to variations in the parameters characterizing the atmosphere. An extensive comparison with data remains to be carried out. The only example presented in this report is for the case recorded at Berkley, California, on 30 October, 1962 following an explosion 13.60° N. 172.22° W. near Johnson Island. We chose this record as it appeared to be the least affected by ambient noise of the records exhibited by Donn and Shaw. Although the choice may therefore appear somewhat fortuitous, the agreement of the theoretically obtained waveform with this record would appear, from a subjective standpoint, to be extremely good.

The following two chapters, V and VI, present two alternate theoretical formulations of the problem of predicting waveforms. The first of these, described in Chapter V, represents an application of various mathematical techniques generally known as Cagniard's method to the idealized case when the vertical acceleration term in the equations of hydrodynamics is neglected at the outset. The second, described in Chapter VI, is based on the assumption that the propagation is predominantly in a single quasi-mode which resembles Lamb's mode for an isothermal atmosphere. This theory represents an extension of some ideas recently expressed by Bretherton (1969) and by Garrett (1969), and shows considerable promise in that it is not restricted to a stratified atmosphere or to linear equations. The quantitative implications of these theories are not explored, but are discussed in the present report with the hope that they may be of interest to other researchers concerned with acoustic-gravity wave propagation. At the time of this writing, we are especially optimistic about the single mode theory and hope to have some quantitative assessment of its applicability in the very near future.

Chapter II

THEORETICAL BASIS OF INFRASONIC WAVEFORMS

2.1 SUMMARY OF THE THEORETICAL MODEL

The mathematical model on which the computer program INFRASONIC WAVEFORMS is based is briefly summarized here. The geometry adopted (Fig. 2-1) is that of a stratified atmosphere above a rigid flat earth. The ambient atmosphere is described by a sound speed profile $c(z)$ and a wind velocity profile $v(z)$, where z is height above the ground. Both of these profiles are assumed independent of horizontal coordinates x and y and of time t . Moreover, the ambient winds are assumed to be horizontal.

The air comprising the atmosphere is taken as an ideal gas of constant specific heat ratio $\gamma = 1.4$ and of constant mean molecular weight. Thus the ambient pressure p_0 and density ρ_0 satisfy the hydrostatic relation and the ideal gas law

$$p_0(z) = p_0(0) \exp \left[- \int_0^z (\gamma g / c^2) dz \right] \quad (2.1.1)$$

$$\rho_0 = \gamma p_0 / c^2 \quad (2.1.2)$$

where $p_0(0)$ (taken as 10^6 dynes/cm²) is the ambient pressure at the ground. Since the propagation is considered to be predominantly in the lower atmosphere, the acceleration of gravity g is taken to be constant with height and equal to the typical earth surface value of .0098 km/sec².

The neglect of earth curvature is in accordance with the results of previous studies by Weston (1961) and by Harkrider (1964) which indicate that the curvature of the earth can approximately be accounted for by multiplying the flat earth waveform by the factor

$$[(r/r_e)/\sin(r/r_e)]^{1/2} \quad (2.1.3)$$

where r is the great circle propagation distance and r_e is the radius of the earth. This result holds in particular for waves which have traveled somewhat less than one-fourth the circumference of the earth. Since the intended application of the program is for the interpretation of data recorded at distances less

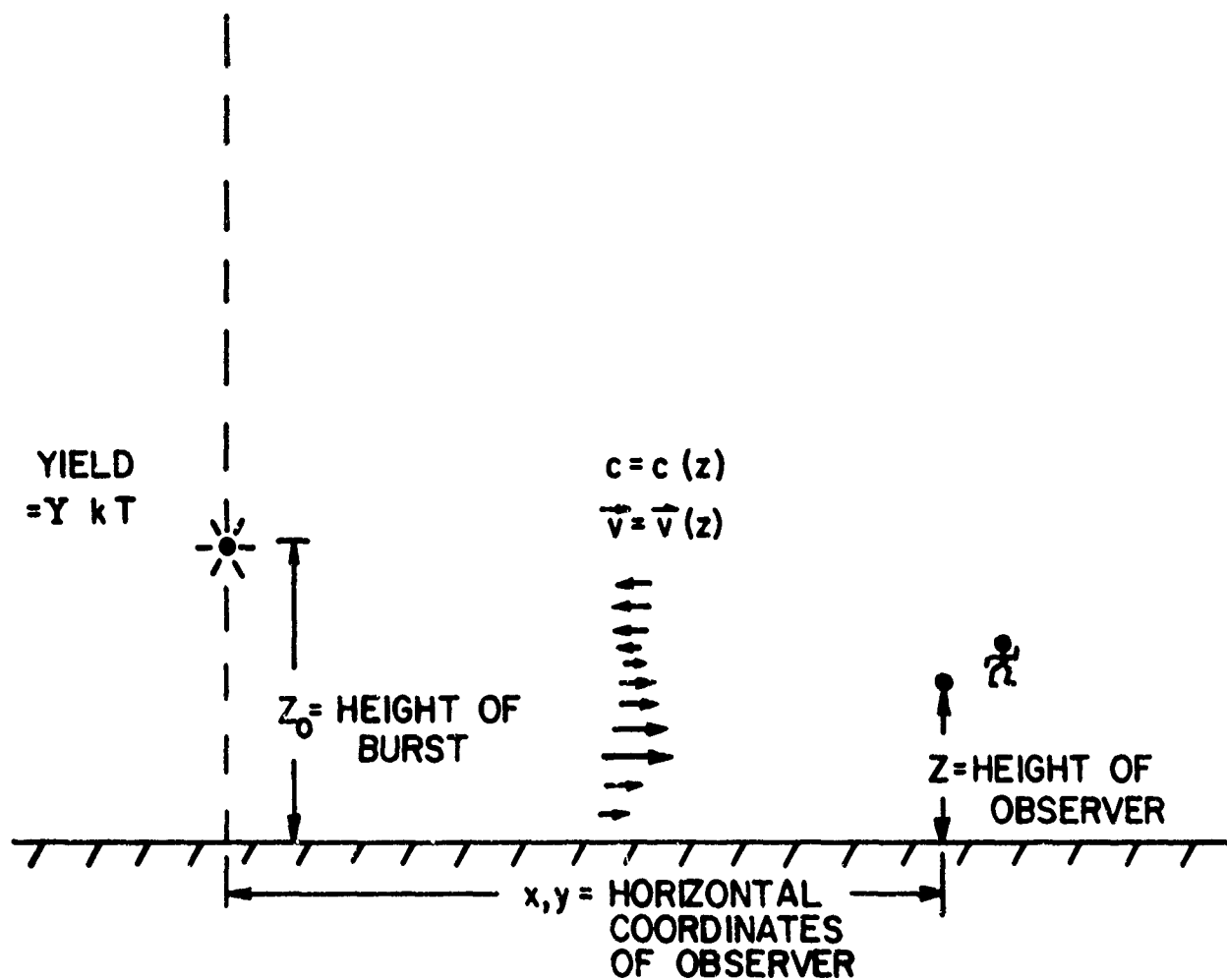


Figure 2-1. Sketch showing the general model adopted in the theoretical formulation. An explosion of yield Y is at height z_0 above a flat rigid ground in an atmosphere with stratified sound speed c and horizontal wind velocity v . The wave disturbance is detected by an observer with coordinates (x, y, z) .

than this and since the factor above does not vary appreciably from 1 for such a range of distances, this correction is neglected. In general, we consider such a correction to be minor compared with the inevitable uncertainties in the source model and the ambient atmosphere.

In order that the model be amenable to computation, we consider the propagation to be governed by the linearized equations of hydrodynamics. This would appear to be a fair approximation at large distances from the explosion, although it is clearly not applicable close-in. It is therefore implicitly assumed that any near field nonlinear effects may be taken into account by a judiciously chosen source model.

The source model adopted here (whose rationale is discussed at some length in the next section) is that where the presence of the source and the near field nonlinear effects are represented by a time-varying point energy source term added to the right hand side of the linearized equation which corresponds to energy conservation. Thus the governing equations (which are to be satisfied everywhere above the ground) are taken to be of the form

$$\rho_0 \left[D_t \vec{u} + (\vec{u} \cdot \nabla) \vec{u} \right] = - \nabla p - g \rho \vec{e}_z \quad (2.1.4a)$$

$$D_t \rho + \nabla \cdot (\rho_0 \vec{u}) = 0 \quad (2.1.4b)$$

$$\begin{aligned} \left(D_t p + \vec{u} \cdot \nabla p_0 \right) - c^2 \left(D_t \rho + \vec{u} \cdot \nabla \rho_0 \right) \\ = 4\pi c^2 f_E(t) \delta(\vec{r} - \vec{r}_0) \end{aligned} \quad (2.1.4c)$$

where

$$D_t = (\partial/\partial t) + \vec{v} \cdot \nabla$$

is the time derivative corresponding to an observer moving with the ambient wind. In the above, p , ρ , and \vec{u} represent the deviations of pressure, density, and fluid velocity from their ambient values. The quantity \vec{e}_z is the unit vector in the vertical direction, while \vec{r}_0 denotes the source location.

The time-dependent quantity $f_E(t)$ represents a function characteristic of the source. For convenience of referral, we state here our choice of this function prior to a discussion of

the rationale behind such a choice. We take

$$f_E(t) = \int_0^t f'_E(t) dt \quad (2.1.5a)$$

where

$$f'_E(t) = Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_s f_{1KT}(t/\lambda_o Y_{KT}^{1/3}) \quad (2.1.5b)$$

Here Y_{KT} is the explosion yield in KT, $p_o(z_o)$ is the ambient pressure at the height of burst, and λ_o is a scaling factor given by

$$\lambda_o = [c(0)/c(z_o)][p_o(0)/p_o(z_o)]^{1/3} \quad (2.1.6)$$

The quantity L_s is a length identically equal to 1 km, which we include in the theory for dimensional "purity". The function $f_{1KT}(t)$ is given by

$$f_{1KT}(t) = (P_s)(1 - t/t_s)e^{-t/t_s} U(t) \quad (2.1.7)$$

where

$$P_s = 1.61 \times 34.45 \times 10^3 \text{ dynes/cm}^2$$

$$t_s = 0.48 \text{ sec.}$$

Here $U(t)$ is the Heaviside unit step function and z_o denotes the height of burst.

In the actual computation, the source location \vec{r}_o is taken to be a fixed point in space. An alternative assumption which might seem more plausible is to take the source as moving with the ambient wind at the height z_o of burst. However, the results of the computation should be insensitive as to just which assumption is made. This follows since we limit our analysis to disturbances which travel with speeds near the speed of sound near the ground and since the wind speeds are invariably much less than the sound speed. Any phenomenon analogous to a doppler shift would undoubtedly be smaller than could feasibly be detected by experiment.

Boundary conditions on Eqs. (2.1.4) are that $u_z = 0$ at the ground $z = 0$ and that of causality (no disturbance z in the far

field before $f_E(t)$ first becomes nonzero). The time origin here is not considered of too much relevance. With the definition given for $f_E(t)$, it is approximately (the discrepancy being due to nonlinear effects) equal to the time of detonation. We also take the source location to be on the z axis ($x_0 = y_0 = 0$). Generally, we consider the $+x$ axis to point eastward, and the $+y$ axis to point northward.

2.2 THE SOURCE MODEL

Here we summarize the rationale behind the choice of the source model given in the preceding section. The discussion partly follows the development previously given by Pierce (1968).

We consider a basic nonlinear hydrodynamics model of a nuclear explosion consisting of an initially isothermal sphere of vanishingly small radius in an unbounded homogeneous atmosphere with negligible gravity. The initial isothermal sphere has ambient density and fluid velocity, but is assumed to have very high temperature and pressure. The total thermal energy (the specific heat of air is assumed independent of temperature) inside the sphere is denoted by E_{hy} , which represents the total hydrodynamic energy released by the explosion. This, according to what is given in Glasstone's text (1962) should be roughly 1/2 of the total energy of the explosion.

The pressure waveform of the explosion in this idealized model can easily be shown to correspond to hydrodynamic scaling, i.e.

$$p = p_0 F(R/\lambda, ct/\lambda) \quad (2.2.1)$$

where p_0 and c are ambient pressure and sound speed, F is a universal function, and λ is a characteristic length given by

$$\lambda = (E_{hy}/p_0)^{1/3} \quad (2.2.2)$$

Experiment and numerical computations show that at moderate distances (between 3 and 10λ from a nuclear explosion) the time dependence of the blast overpressure is approximately of the form (t relative to time of shock arrival)

$$p = (P)(1 - t/\tau)e^{-t/\tau}U(t) \quad (2.2.3)$$

where P and τ are functions of distance. According to Eq.

(2.2.1) above, we may take

$$\tau = (\lambda/c)A(R/\lambda) \quad (2.2.4a)$$

$$P = p_0 B(R/\lambda) \quad (2.2.4b)$$

where A and B are universal functions of (R/λ) . It is clear, since the far field propagation should be governed by linear acoustics, that A should at large R be a relatively slowly varying function of R/λ and that, at large R, B should be approximately (spherical spreading)

$$B \sim B_0 \lambda/R \quad (2.2.5)$$

where B_0 is a very slowly varying function of R/λ .

The basic idea in our source term selection is that $f_E(t)$ in Eq. (2.1.4c) should be such that the solution of the linearized equations of hydrodynamics with the neglect of gravity and winds and for the same uniform ambient atmosphere should agree with Eq. (2.2.1) at moderate distances. The ambient atmosphere for this matching process is taken as that characteristic of the burst location. The solution of the linearized equations under the circumstances outlined above gives

$$p = R^{-1} f'_E(t - R/c) \quad (2.2.6)$$

Thus, we would choose f'_E to be

$$f'_E(t) = p_0 B_0 \lambda (1 - t/\tau) e^{-t/\tau} U(t) \quad (2.2.7)$$

The value of R/λ chosen for the matching is that corresponding to 1 km from a 1 KT explosion at sea level. According to Glasstone's text(1962) the value of P at one mile from such an explosion is 34.45×10^3 dynes/cm² while the value of τ is 0.48 sec. Since we expect the shock overpressure to fall off nearly inversely with R between 1 km and 1 mile we may take P to be $34.45 \times 10^{-3} \times 1.61$ dyne/cm² at one km. (Here we use the fact that 1 mile is 1.61 km.) Thus

$$p_0 B_0 \lambda = P_s L_s Y_{KT}^{1/3} [p_0(z_0)/p_0(0)]^{2/3} \quad (2.2.8a)$$

$$\tau = Y_{KT}^{1/3} [p_0(0)/p_0(z_0)]^{1/3} [c(0)/c(z_0)] t_s \quad (2.2.8b)$$

where P_s , L_s , and t_s are as defined in the preceding section and Y_{KT} is the energy yield in KT. Equations (2.2.7), (2.2.8a,b) agree with the definition of f'_E given in the preceding section.

It now remains to examine the limitations of this source model. The basic assumption we have made is that gravity and atmospheric gradients have relatively little effect on the early development of the blast wave. Another important assumption is that the initial fireball radius (before the shock detaches from the fireball) is sufficiently small that the radius of the initial sphere may be idealized as zero. This radius is conjectured by Pierce (1968) to be about $.05 \lambda$. The two approximations would clearly be inappropriate if the initial sphere radius were of the order of a scale height H_s . Thus one should certainly require $\lambda \ll 20 H_s$.

The establishment of a more realistic upper bound appears to be a somewhat complicated problem. Our best guess to date is that the positive phase duration by the time the blast overpressure is down to 10% ambient should be smaller than 1/10 the period (about 5 minutes) corresponding to Brunt's frequency ω_B or Hines' ω_A . This would insure that there be negligible acoustic-gravity wave dispersion in the waveform while non-linear effects are appreciable. This requirement gives roughly

$$Y_{MT} < 200 P_{ATMOS} \quad (2.2.9)$$

where Y_{MT} is the yield in MT and P_{ATMOS} is the ambient pressure at the height of burst in atmospheres [see Fig. 2-2]. We are not sure if this requirement is too conservative or too generous at present, but we offer it as a rough guideline to workers who might wish to use the program.

It should be noted that, in our source model, we have taken a point energy source rather than a point mass source. In actual practice, for the computation of ground level waveforms, it makes relatively little difference whether one adopts an energy source or a mass source. However, when one considers the fact that a nuclear explosion adds much energy but relatively little mass to the atmosphere, it is clear that the energy source model is a priori the more realistic. One of the authors (Pierce, 1968) has examined the relative merits of the two types of sources. He found the linear initial value problem of waves generated by the release of an initially isothermal sphere of ambient density was better represented by the point energy source. [It should be noted that the use of a point energy source rather than a point mass source is a relatively new concept in the theory of acoustic-gravity wave propagation. In particular, all of the

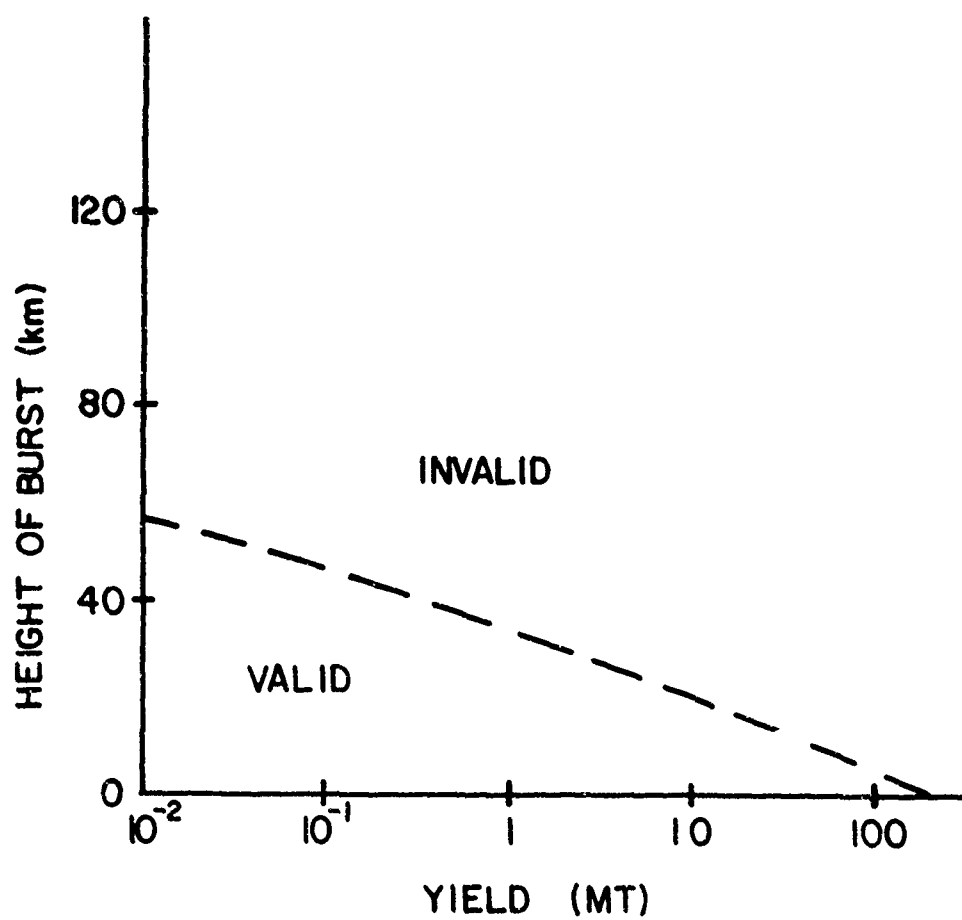


Figure 2-2. Estimated range of yields and source altitudes for which the effective point source model is valid.

authors' work prior to 1968 was based on the use of a point mass source model.]

One inherent defect in the model is that we have no mechanism for taking into account the far field nonlinear effects. In the related problem of sonic boom propagation, these are known not to be negligible. While they do not change waveforms appreciably over shorter distances, their accumulative effects can cause large distortions over very large distances. While these far field nonlinear effects should be examined in some detail in the future, we suspect that they are not as important in the infrasonic nuclear explosion problem as they are for sonic booms. The basis for this belief is that the inherent dispersive nature of the atmosphere as a waveguide tends to filter out the higher frequencies from the lowest part of the atmosphere and causes the lower frequencies to tend to arrive first. The nonlinear effects should be of lesser importance for the dominant lower frequency arrivals since the time for a wave peak to overtake a node is correspondingly longer for lower frequencies.

2.3 THE SOLUTION IN TERMS OF FOURIER TRANSFORMS

Since the linearized equations of hydrodynamics do not depend explicitly on time and, except at the source location, do not depend explicitly on horizontal position x (due to the assumed stratification of the ambient atmosphere), one may express their solution as a triple Fourier transform over frequency ω and horizontal wave number vector components, k_x and k_y . Thus one may write the acoustic pressure p , for example, as

$$p = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{ik \cdot \vec{x}} \left\{ \int_{-\infty+i\epsilon}^{\infty+i\epsilon} \hat{f}_E(\omega) \hat{p}(\omega, \vec{k}, z, z_0) e^{-i\omega t} d\omega \right\} dk_x dk_y \quad (2.3.1)$$

Here $\hat{f}_E(\omega)$ is the Fourier transform of $f_E(t)$, i.e.

$$\hat{f}_E(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} f_E(t) e^{i\omega t} dt \quad (2.3.2)$$

The quantity ϵ is chosen sufficiently large that the integral over ω vanishes identically at times t before the source is first excited, if k_x and k_y are real. Thus the line ϵ above the real axis should pass above all poles and branch lines of the inte-

grand. The function $\hat{p}(\omega, \vec{k}, z, z_0)$ must be defined such that this can be accomplished. Fourier transforms \hat{u} and \hat{p} are defined analogously.

A direct substitution of (2.3.1) and its counterparts for ρ and \vec{u} into the linearized equations gives the following ordinary differential equations for \hat{p} , $\hat{\rho}$, and \hat{u} :

$$-\rho_0 i\Omega \hat{u} = -d\hat{p}/dz - g\hat{\rho} \quad (2.3.3a)$$

$$-\rho_0 i\Omega \hat{u}_H + \rho_0 \hat{u} d\vec{v}/dz = -ik\hat{p} \quad (2.3.3b)$$

$$-i\Omega \hat{p} + \rho_0 \vec{k} \cdot \hat{u}_H + d(\rho_0 \hat{u})/dz = 0 \quad (2.3.3c)$$

$$-i\Omega(\hat{p} - c^2 \hat{\rho}) + \rho_0 \hat{u}[(\gamma - 1)g + dc^2/dz] = (c^2/\pi) \hat{f}_E \delta(z - z_0) \quad (2.3.3d)$$

Here we have abbreviated

$$\Omega = \omega - \vec{k} \cdot \vec{v}$$

for the Doppler shifted angular frequency and also have abbreviated \hat{u} and \hat{u}_H for the vertical and horizontal components of \hat{u} .

It turns out [Pierce, 1965] that the above set of ordinary differential equations may be reduced to two differential equations for

$$Z = \hat{p}/\rho_0^{1/2} \quad (2.3.4a)$$

$$Y = i\rho_0^{1/2} \hat{u}/\Omega \quad (2.3.4b)$$

These equations are

$$\left(\frac{d}{dz} + S\right)Z - S_{12}Y = \left(\frac{-g}{i\Omega\rho_0^{1/2}\pi}\right)\delta(z - z_0) \quad (2.3.5a)$$

$$\left(\frac{d}{dz} - S\right)Y - S_{21}Z = \left(\frac{-1}{i\Omega\rho_0^{1/2}\pi}\right)\delta(z - z_0) \quad (2.3.5b)$$

where

$$S = (1-\gamma/2)g/c^2 - c^{-1} dc/dz \quad (2.3.6a)$$

$$S_{12} = \Omega^2 - (\gamma - 1)g^2/c^2 - (g/c^2)(dc^2/dz) \quad (2.3.6b)$$

$$S_{21} = (k^2/\Omega^2) - c^{-2} \quad (2.3.6c)$$

The remaining quantities of interest are given by

$$\hat{\rho} = \rho_0^{1/2} Z/c^2 + \rho_0^{1/2} [(\gamma - 1)g + dc^2/dz] Y/c^2 + [1/(\pi i \Omega)] \delta(z - z_0) \quad (2.3.7a)$$

$$\hat{u}_H = (\vec{k}/\Omega) \rho_0^{-1/2} Z - \rho_0^{-1/2} Y \quad (2.3.7b)$$

The solution to Eqs. (2.3.5) is easily worked out [Pierce, 1967] in terms of solutions of the homogeneous equations

$$\frac{d}{dz} \begin{bmatrix} Z \\ Y \end{bmatrix}_{u,l} = \begin{bmatrix} -S & S_{12} \\ S_{21} & S \end{bmatrix} \begin{bmatrix} Z \\ Y \end{bmatrix}_{u,l} \quad (2.3.8)$$

Let Z_u and Y_u be nonzero solutions which satisfy these equations for $z > z_0$ and which are analytic functions of ω for given real k_x and k_y and for all $\text{Im } \omega$ greater than some finite value. Similarly, let Z_l , Y_l be a nonzero set of solutions for $z < z_0$ which satisfy the boundary condition $Y_l = 0$ at $z = 0$. Then the solutions of the inhomogeneous equations are given by

$$\begin{bmatrix} Z \\ Y \end{bmatrix} = \alpha_l(z_0) \begin{bmatrix} Z_u(z) \\ Y_u(z) \end{bmatrix} \quad (z > z_0) \quad (2.3.9a)$$

$$\begin{bmatrix} Z \\ Y \end{bmatrix} = \alpha_u(z_0) \begin{bmatrix} Z_l(z) \\ Y_l(z) \end{bmatrix} \quad (z < z_0) \quad (2.3.9b)$$

where

$$\alpha_{l,u} = \frac{-1}{i\Omega \rho_0^{1/2} \pi W} [Z_{l,u} - g Y_{l,u}] \quad (2.3.10)$$

with the Wronskian W given by

$$W = (Y_u Z_l - Z_u Y_l)_{\text{any } z} = Y_u(0) Z_l(0) \quad (2.3.11)$$

That W is independent of height follows directly from the differential equations (2.3.8).

It now follows from the preceding analysis that the integrand in Eq. (2.3.1) is given by

$$p = \left\{ \frac{\rho_o(z)}{\rho_o(z_o)} \right\}^{1/2} \frac{1}{\pi[\omega - \vec{k} \cdot \vec{v}(z_o)]} \left\{ \frac{\Psi(z, z_o)}{Z_\ell(0)Y_u(0)} \right\} \quad (2.3.12)$$

where

$$\Psi(z, z_o) = [Z_u(z_o) - gY_u(z_o)]Z_\ell(z) \quad z_o > z \quad (2.3.13a)$$

$$= [Z_\ell(z_o) - gY_\ell(z_o)]Z_u(z) \quad z_o < z \quad (2.3.13b)$$

It should be noted that a previous statement of the above result has a misprint, $[\rho_o(z)/\rho_o(z)]$ instead of $[\rho_o(z)/\rho_o(z_o)]$ in the paper by Pierce (1968).

2.4 THE GUIDED MODE APPROXIMATION

The Fourier transform solution given in the previous section is too complicated as it stands for direct integration. Thus it is appropriate to take advantage of any approximations which may be appropriate to the cases of primary interest (namely, waves detected near the ground at large distances which arrive at times roughly corresponding to the lower atmosphere sound speed). The primary approximation we make in this respect is the guided mode approximation. The mathematical manipulations leading to this approximation are described in many texts for waves in stationary media and were first discussed by Pridmore-Brown (1962) and later modified by Pierce (1965) for the case of waves from a point source in a windy stratified atmosphere.

Before introducing the guided mode approximation, we first consider the symmetry properties of the factors \hat{f}_E , \hat{p} , and of their product

$$I(\omega, \vec{k}) = \hat{f}_E(\omega) \hat{p}(\omega, \vec{k}, z, z_o) \quad (2.4.1)$$

which appears in the integrand of Eq. (2.3.1). We set $\vec{k} = \vec{k}_R + i\vec{k}_I$ where \vec{k}_R and \vec{k}_I are real vectors and similarly set

$\omega = \omega_R + i\omega_I$. Since p is real, we have (or at least can choose)

$$I(\omega, \vec{k})^* = I(-\omega_R + i\omega_I, -\vec{k}_R + i\vec{k}_I) \quad (2.4.2)$$

Thus, taking the complex conjugate of I is equivalent to changing the signs of the real parts of ω and \vec{k} . Since $f_E(\omega)$ must a priori have this property [$f_E(t)$ is real], it follows that \hat{p} must also have this property.

A final symmetry property which can be insured by appropriate choice of branch lines is that, for real \vec{k} and complex ω ,

$$\hat{p}(-\omega, -\vec{k}, z, z_0) = -\hat{p}(\omega, \vec{k}, z, z_0) \quad (2.4.3)$$

To prove this, one should first note that the differential equations (2.3.5) are invariant if $\omega \rightarrow -\omega$, $\vec{k} \rightarrow -\vec{k}$, $Z \rightarrow -Z$, $Y \rightarrow -Y$. Also, the lower boundary condition is unchanged. This suggests that we may be able to take

$$Z(\omega, \vec{k}) = -Z(-\omega, -\vec{k}) \quad (2.4.4)$$

While it would appear that this neglects the consideration of an upper boundary condition, this is actually not the case, since any posed upper boundary condition would of necessity have to be equivalent to the requirement that $Z(\omega, \vec{k})$ be analytic for $\omega_I > \epsilon$. Thus Eq. (2.4.4) merely states that we are free to define $Z(\omega, \vec{k})$ for values of ω in the lower half of the complex ω plane in such a manner that Eq. (2.4.4) is satisfied. In what follows we consider that we have made such a definition with an appropriate selection of branch lines. Equation (2.4.3) then follows from the above and (2.3.4a). Since \hat{p} is almost everywhere analytic in \vec{k} , it would seem appropriate to consider (2.4.3) to hold also when the components of \vec{k} are complex.

Returning now to the central task of deriving the guided mode approximation, let us first interchange the order of ω and \vec{k} integrations in Eq. (2.3.1). It follows that we can do this if we can find an ϵ which does not depend on \vec{k} . The analysis by Friedland and Pierce (1969) would apparently indicate that such an ϵ can be found providing the atmosphere is inherently stable. Let us assume this is the case. Then the integration over k_x and k_y is replaced by one over k and θ_k (polar coordinates) where

$$k_x = k \cos \theta_k ; \quad k_y = k \sin \theta_k \quad (2.4.5)$$

Since the integrand is clearly periodic in θ_k and since increasing the value of θ_k by π is equivalent to changing the sign of k , we obtain

$$p = \int_{-\infty+i\epsilon}^{\infty+i\epsilon} e^{-i\omega t} \hat{f}_E(\omega) \int_{\theta-\pi/2}^{\theta+\pi/2} Q D\theta_k \quad (2.4.6)$$

where

$$Q = \int_0^\infty \hat{p}(\omega, \vec{k}) e^{i\vec{k} \cdot \vec{x}} k dk - \int_{-\infty}^0 \hat{p}(\omega, \vec{k}) e^{i\vec{k} \cdot \vec{x}} k dk \quad (2.4.7)$$

Here θ is the angle (reckoned counterclockwise) which the horizontal vector \vec{x} makes with the x axis. It should be noted that the factor $\vec{k} \cdot \vec{x}$ in the exponent is $kR \cos(\theta - \theta_k)$ where R is the net horizontal distance from source to receiver.

Using Cauchy's theorem, we may show for $\cos(\theta - \theta_k) > 0$ that

$$Q = Q_R - Q_L + 2Q_I \quad (2.4.8)$$

where Q_R , Q_L and Q_I are contour integrals of the form

$$\int_C \hat{p}(\omega, \vec{k}) e^{ikR \cos(\theta - \theta_k)} k dk \quad (2.4.9)$$

where the contour C is taken as follows:

Q_R : C circles the upper right quadrant of the k plane in the counter-clockwise sense.

Q_L : C circles the upper left quadrant of the k plane in the counter-clockwise sense.

Q_I : C goes straight up the imaginary axis from 0 to ∞ .

The integral Q_I is readily seen to be relatively small for large R compared to Q_R and to Q_L and is accordingly neglected at the outset. The contour integrals Q_R and Q_L are then evaluated by Cauchy's method of residues. It is anticipated that contributions from any branch lines encircled in the shrinking process are of minor importance at large R . Thus we obtain the

approximate result

$$Q = 2\pi i \{ (\sum \text{Res})_R - (\sum \text{Res})_L \}$$

where the quantities $(\sum \text{Res})_R$ and $(\sum \text{Res})_L$ are the sums of the residues of

$$k \hat{p}(\omega, \vec{k}) e^{ikR \cos(\theta - \theta_k)}$$

at the poles in the right upper and left upper quadrants, respectively.

The integrals over θ_k are performed by the saddle point approximation [see, for example, Morse and Feshbach (1954)] under the assumption that the poles $k_n(\omega, \theta_k)$ are much more slowly varying functions of θ_k than is $\cos(\theta - \theta_k)$. Physically, this assumption is equivalent to the neglect of crosswinds. Thus we have a typical integral evaluated in the manner

$$\begin{aligned} & \int_{\theta - \pi/2}^{\theta + \pi/2} \phi_n(\omega, \theta_k) e^{-ik_n R \cos(\theta - \theta_k)} d\theta_k \\ & \sim e^{ik_n(\omega, \theta)R} \phi_n(\omega, \theta) \int_{-\infty}^{\infty} e^{ik_n R (\theta - \theta_k)^2 / 2} d\theta_k \\ & \sim \left(\frac{2\pi}{|k_n R|} \right)^{1/2} e^{i(k_n R - \pi/4)} e^{i\text{Ph}(k_n)/2} \quad (2.4.10) \end{aligned}$$

where the pole location is taken at $\theta_k = \theta$. Here $\text{Ph}(k_n)$ is the phase of k_n (between 0 and π).

Thus Eq. (2.4.6) becomes

$$p = 2\pi i \int_{-\infty + i\epsilon}^{\infty + i\epsilon} \hat{f}_E(\omega) \sum_n \left(\frac{2\pi}{|k_n| R} \right)^{1/2} k_n S_n e^{i[k_n R - \omega t - \pi/4]} e^{i\text{Ph}(k_n)/2} \phi_n d\omega \quad (2.4.11)$$

where ϕ_n is the residue of \hat{p} at $k = k_n(\omega, \theta)$ and S_n is 1 if k_n corresponds to a pole of \hat{p} in the upper right quadrant; -1 for the upper left quadrant. It is assumed throughout the preceding analysis that ω has a nonzero imaginary part. The k_n in general will be complex numbers with positive imaginary parts.

We may let $\epsilon \rightarrow 0$ as long as the $k_n(\omega_R + i\omega_I, \theta)$ have non-negative imaginary parts as $\omega_I \rightarrow 0$ from above. We assume this is the case, since otherwise the ambient atmosphere would be unstable. It is necessary to consider the possibility (since it is a certainty in the absence of dissipation) that some of the k_n may be real when ω is real. However, we wish to avoid spurious terms which correspond to poles lying below the real k axis when $\omega_I > 0$. This can be accomplished by simply requiring that, for real ω and real k_n , only those terms be included for which

$$\partial k_n / \partial \omega \geq 0 \quad (2.4.12)$$

(which is equivalent to the requirement that the group velocity be positive.)

At this point we make use of the symmetry properties of \hat{p} . The integral over ω may be separated into one from $-\infty$ to 0 and one from 0 to ∞ . The former is then subjected to a change of variable $\omega \rightarrow -\omega$. One can readily show from (2.3.2) that the former must be just the complex conjugate of the latter. Thus

$$p = \text{Re} \int_0^{\infty} \hat{f}_L(\omega) \sum_n A_n(\omega, \theta) e^{i[k_n R - \omega t - \pi/4]} d\omega \quad (2.4.13)$$

where

$$A_n = 4\pi i \left[\frac{2\pi}{|k_n| R} \right]^{1/2} S_n e^{i\text{Ph}(k_n)/2} \phi_n k_n \quad (2.4.14)$$

The pole locations are assumed to be piecewise continuous functions $k_n(\omega, \theta)$ of ω . Thus we can interchange the sum and integral in Eq. (2.4.13), obtaining

$$p = \sum_n p_n \quad (2.4.15)$$

where p_n is the contribution from the n^{th} guided mode, given by

$$p_n = \text{Re} \int \hat{f}_E(\omega) A_n(\omega, \theta) e^{i[k_n R - \omega t - \pi/4]} d\omega \quad (2.4.16)$$

The integration limits extend over a range of positive ω for which $k_n(\omega, \theta)$ is defined. It should be noted that the k_n are in general

complex. Their imaginary parts must be positive, but (at least formally) their real parts could be either positive or negative. In the terminology used by Friedman (1967), those modes with a non-zero imaginary part of k_n are called leaky modes. It is possible that a given mode may be leaky over a given range of ω and then be non-leaky (real k_n) over another range of ω .

It is tempting to discard all leaky modes or leaky portions of modes at the outset with the glib statement that at sufficiently large R they are negligible. However, just whether or not they are negligible depends on the magnitude of $\text{Im}(k_n)$. Since we are primarily interested in propagation to distances of the order of 10,000 km, the values of $\text{Im}(k_n)$ should be greater than, say, 10^{-3} km^{-1} if we are to consider a leaky mode to be negligible. We might term modes where $\text{Im}(k_n)$ is less than this value as slowly leaking modes.

Just when slowly leaking modes are important in waveform synthesis is intimately related to the nature of the topmost region of the assumed model atmosphere. If the top of the atmosphere is adjacent to a rigid surface or is bounded by a free surface, then there is nowhere for energy to leak and there are no leaking modes. On the other hand, as was originally observed by Press and Harkrider (1962), if the uppermost region of the atmosphere is taken as an isothermal half-space, then there are certain regions of the k vs. ω (for fixed θ) plane (with k and ω real) in which the dispersion curves for non-leaking modes cannot penetrate. If a mode's dispersion curve apparently terminates at the edge of such a region, then it would seem that the extension of the mode into such a region would be a leaky mode. None of these three types of models is too fair a representation of the upper atmosphere, but one may argue that, if the major portion of the energy is channeled near the ground, then the variations in the model atmosphere above 150 km should have relatively little effect on the actual waveform. Numerical studies such as described elsewhere in this report would seem to support this conclusion.

The discussion given above would suggest that we may avoid the consideration of leaky modes by adopting a suitable model of the uppermost portion of the atmosphere. Just what model is adopted should probably be a compromise between what is known about the upper atmosphere and the desire to minimize the contribution from the slowly leaking modes. In what follows, we assume such a model has been selected and accordingly consider only real non-leaking modes.

Since we are interested primarily in interpreting data on

waves arriving at times corresponding to group velocities roughly equal to the sound speed in the lower atmosphere, it is clear that we may discard at the outset any modes or portions of modes which give negligible contributions at such times. Since the contribution to a mode at time t comes primarily from frequencies near that at which ω satisfies

$$\frac{t}{R} = \frac{\partial k_n}{\partial \omega} \quad (2.4.17)$$

it would seem appropriate to consider only those modes where $\partial k_n / \partial \omega$ is of the order of $1/c$ where c is a representative sound speed. To this purpose, the following theorem derived by Pierce (1965) for non-leaking modes may be of assistance.

$$\frac{\partial k}{\partial \omega} = \frac{\int_0^{\infty} \{ \Omega Y^2 + (k^2 / \Omega^3) Z^2 \} dz}{\int_0^{\infty} \{ \Omega (\vec{k} \cdot \vec{v} / k) Y^2 + k \omega \Omega^{-3} Z^2 \} dz} \quad (2.4.18)$$

The fact that wind speeds are small compared to the sound speed suggests that we may estimate the magnitude of $\partial k / \partial \omega$ for cases of interest with the neglect of wind velocity. In this limit, the above expression becomes

$$\frac{\partial k}{\partial \omega} \sim \frac{k}{\omega} + \frac{\omega^2}{k} \alpha^2 \quad (2.4.19)$$

where α^2 is positive. Thus $\partial k / \partial \omega$ is positive only if $k > 0$ (given $\omega > 0$). Furthermore, the group velocity $(\partial k / \partial \omega)^{-1}$, if positive, should always be less than the phase velocity. Thus, we may restrict our analysis to modes where $k > 0$ and where ω/k is greater than, say, half the sound speed at the ground. It would certainly seem appropriate to discard all modes where k is negative or where ω/k is less than the maximum wind speed, given that the maximum wind speed is small. (The reasoning here may be somewhat circular since we initially neglected the winds to arrive at this deduction. However, a more detailed examination seems unwarranted at the present stage.)

Let us next examine the quantity $A_n(\omega, 0)$ in Eq. (2.4.14) under the assumption that k_n is positive, real, and greater than ω times the maximum wind speed. The poles of \hat{p} which satisfy these requirements must, by Eq. (2.3.12), correspond to zeros of

$Y_u(0)$, when considered as a function of k . The residue ϕ_n of \hat{p} at such a pole is given by (2.3.12), only with $Y_u(0)$ replaced by $\partial Y_u(0)/\partial k$. The latter derivative has been shown by Pierce (1965) to be given by

$$\partial Y_u(0)/\partial k = \frac{-2 \int_0^\infty \{\Omega(\vec{k} \cdot \vec{v}/k) Y^2 + k \omega \Omega^{-3} Z^2\} dz}{Z_u(0)} \quad (2.4.20)$$

Furthermore, since $Y_u(0)$ is 0 at a pole, the upper and lower boundary conditions are both satisfied when $k = k_n$ and we may discard the subscripts l and u . Thus we have

$$\phi_n = - \left[\frac{\rho_o(z)}{\rho_o(z_o)} \right]^{1/2} \frac{1}{2\pi\Omega(z_o)} \frac{[Z(z_o) - gY(z_o)]Z(z)}{\int_0^\infty \{\Omega(\vec{k} \cdot \vec{v}/k) Y^2 + k \omega \Omega^{-3} Z^2\} dz} \quad (2.4.21)$$

where the direction of \vec{k} is θ and its magnitude is $k_n(\omega, \theta)$.

If Eq. (2.4.21) is substituted into (2.4.14), we find

$$\Lambda_n = \left[\frac{\rho_o(z)}{\rho_o(z_o)} \right]^{1/2} \left[\frac{8\pi k_n}{R} \right]^{1/2} \frac{1}{\Omega(z_o)} \frac{[Z(z_o) - gY(z_o)]Z(z)}{\int_0^\infty \{\Omega(\vec{k} \cdot \vec{v}/k) Y^2 + k \omega \Omega^{-3} Z^2\} dz} \quad (2.4.22)$$

It should be recalled that the expression for a guided mode is given by Eq. (2.4.16). The quantities $k(\omega, \theta)$ are zeros of $Y_u(0)$ and it is assumed that we need only consider contributions from mode segments where Ω is positive for all z and where $k_n > 0$.

The only remaining piece of analysis to complete our formal derivation is the derivation of an explicit expression for $f_E(\omega)$. If we insert Eqs. (2.1.5) into Eq. (2.3.2), we find

$$\hat{f}_E(\omega) = (2\pi)^{-1} Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_s \int_0^\infty e^{i\omega t} \int_0^t f_{1KT}(\tau'/[\lambda_o Y_{KT}^{1/3}]) d\tau' dt$$

or, after an integration by parts,

$$\hat{f}_E(\omega) = -(2\pi)^{-1} Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_s \left(\frac{1}{i\omega} \right) \int_0^\infty e^{i\omega t} f_{1KT}(t/[\lambda_o Y_{KT}^{1/3}]) dt$$

From Eq. (2.1.7), we find

$$\int_0^\infty e^{i\omega t} f_{1KT}(t/[\lambda_o Y_{KT}^{1/3}]) dt = \frac{P_s i\omega}{[\omega + i(\lambda_o Y_{KT}^{1/3} t_s)^{-1}]^2}$$

Thus

$$\hat{f}_E(\omega) = -(2\pi)^{-1} Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_s P_s / [\omega + i(\lambda_o Y_{KT}^{1/3} t_s)^{-1}]^2 \quad (2.4.23)$$

The symbols in this expression are as defined in Sec. 2.1.

2.5 SUMMARY OF THE GUIDED MODE SOLUTION

For convenience of referral, the solution derived in the previous sections is summarized. First, we have the wave as a sum of guided modes, the acoustic overpressure p being given by

$$p = \sum_n p_n(R, \theta, z_o, z, t) \quad (2.5.1)$$

where the arguments of p_n are

R = horizontal distance from source

θ = azimuth angle of observer (counter-clockwise in horizontal plane)

z_o = height of burst

z = height of observation location

t = time of observation

In addition p_n depends on the sound speed and wind velocity profiles, $c(z)$ and $v(z)$, and on the yield Y_{KT} of the explosion in KT.

A particular guided mode wave is given as an integral of

the form

$$p_n = \int D_n \cos [\omega t - k_n R + e] d\omega \quad (2.5.2)$$

where the integration extends over all positive frequencies ω for which a real $k_n(\omega, \theta)$ is defined. The quantities D_n and e are real functions of ω . We may define D_n as $\pm |f_n A_n|$ and take e as $\pi/4$ minus the phase of $\pm f_n A_n$. The choice of sign depends on just which real factors are incorporated into D_n .

The particular forms which we may take for D_n and e_n are

$$D_n = \left[\frac{\rho_o(z)}{\rho_o(z_o)} \right]^{1/2} \left[\frac{2k}{\pi R} \right]^{1/2} \frac{Y_{KT}^{1/3} [p_o(z_o)/p_o(0)]^{2/3} L_s P_s}{[\omega^2 + (\lambda_o Y_{KT}^{1/3} t_s)^{-2}]}$$

$$\times \frac{1}{\Omega(z_o)} \frac{[Z(z_o) - gY(z_o)]Z(z)}{\int_0^\infty \{\Omega \vec{k} \cdot \vec{v}/k\} Y^2 + k \omega \Omega^{-3} Z^2\} dz} \quad (2.5.3)$$

$$e = 5\pi/4 + 2 \times \text{phase} [\omega + i(\lambda_o Y_{KT}^{1/3} t_s)^{-1}] \quad (2.5.3)$$

where

$$\begin{aligned} P_s &= 1.61 \times 34.45 \times 10^3 \text{ dynes/cm}^2 \\ L_s &= 1 \text{ km} \\ t_s &= 0.48 \text{ sec} \\ \lambda_o &= [c(0)/c(z_o)] [p_o(0)/p_o(z_o)]^{1/3} \\ \vec{k} &= k_n(\omega, \theta) [\vec{e}_x \cos \theta + \vec{e}_y \sin \theta] \end{aligned}$$

The functions Y and Z are eigenfunctions of two coupled homogeneous ordinary differential equations and $k_n(\omega, \theta)$ may be considered the corresponding eigenvalue.

To introduce the nomenclature used in the discussion of the computer program, we set

$$D_n = (1/R^{1/2}) (\text{AMPLTD}) \quad (2.5.4a)$$

$$\text{AMPLTD} = (\text{FACT}) (Y_{KT}^{2/3} k^{1/2} / \omega) |S(\omega \lambda_o Y_{KT}^{1/3})| (\text{AMP}) \quad (2.5.4b)$$

$$\text{FACT} = [4/2\pi]^{1/2} c(0) [\rho_o(z)/\rho_o(z_o)]^{1/2} [p_o(z_o)/p_o(0)]^{1/3} \quad (2.5.4c)$$

$$\text{AMP} = - \frac{[Z(z_o) + gY(z_o)]Z(z)}{\int_0^\infty \{ \Omega(\vec{k} \cdot \vec{v}/k) Y^2 + \omega k \Omega^{-3} Z^2 \} dz} \quad (2.5.4d)$$

$$S(\omega) = \frac{i\omega P_s L_s}{[\omega + i t_s^{-1}]^2} \quad (2.5.4e)$$

$$\text{PHASQ} = e = 3\pi/4 - \text{Phase} \{S(\omega \lambda_o Y_{KT}^{1/3})\} \quad (2.5.4f)$$

The subscript n on various quantities is omitted for brevity.

In terms of the quantities introduced above, the contribution p_n from the n-th guided mode becomes

$$p_n = R^{-1/2} \int \text{AMPLTD} \cos [\omega(t - R/v_p) + \text{PHASQ}] d\omega \quad (2.5.5)$$

where

$$v_p = \omega/k \quad (2.5.6)$$

is the phase velocity (varying with ω) of the guided mode.

2.6 THE MULTILAYER METHOD

In order to compute the dispersion curves $k_n(\omega, \theta)$ [or, equivalently, phase velocity v_p vs. ω] for the n guided modes and the functions Y and Z , it is convenient to formally replace the actual atmosphere by a multilayer model, in which the model is comprised of a discrete number of layers, each having constant temperature and wind velocity. Such a technique is fairly common in the numerical solution of wave propagation problems and dates at least as far back as Haskell (1950). The multi-

layer method has some shortcomings and has been criticized by various authors. It is important that the reader realize that the method is only a numerical integration technique. We do not approximate the atmosphere by a multilayer model at the outset but only use this as a device to evaluate the quantities needed for numerical evaluation of the solution summarized in the previous section. In actual fact, any given multilayer atmosphere would most likely be unstable for disturbances of sufficiently short wave length. However, for any given k and ω , we may always pick a model (by simply including enough layers) that the numerical solution of the homogeneous residual equations (2.3.8) is in good agreement with the result which might be obtained by using a given continuous atmosphere. This has been demonstrated previously by one of the authors [Pierce, 1966]. It may be argued that the multilayer method is not the most efficient numerical method, but the authors believe that, from the standpoint of coding the problem for numerical computation, the multilayer method is generally to be preferred.

For the purpose of making the organization of the computation scheme as simple as possible, it is assumed that one has picked a multilayer model of sufficient detail that it suffices for all numerical computations necessary to evaluate a given waveform. The same model will then be used throughout the computation. Guidelines for selecting such a model have been discussed by Pierce (1967) and by Vincent (1969). The user, if he so wishes, may establish his own guidelines by numerical experiment.

In multilayer computations, it is convenient to deal with quantities ϕ_1 and ϕ_2 rather than Z and Y since the latter are not in general continuous at layer boundaries. The functions ϕ_1 and ϕ_2 are defined as

$$\phi_1 = cY \quad (2.6.1a)$$

$$\phi_2 = gY/c - Z/c \quad (2.6.1b)$$

These can be shown from Eq. (2.3.8) to satisfy the residual equations

$$\frac{d}{dz} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \quad (2.6.2)$$

where the elements of the matrix $[A]$ are given by

$$A_{11} = gk^2/\Omega^2 - \gamma g/(2c^2) \quad (2.6.3a)$$

$$A_{12} = 1 - c^2 k^2/\Omega^2 \quad (2.6.3b)$$

$$A_{21} = g^2 k^2/\Omega^2 c^2 - \Omega^2/c^2 \quad (2.6.3c)$$

$$A_{22} = -A_{11} \quad (2.6.3d)$$

It follows from the form of these coefficients that ϕ_1 and ϕ_2 must be continuous with z even when c and v are discontinuous. In any given layer, the matrix $[A]$ is constant.

One restriction we place on the multilayer atmosphere is that the top-most layer (bounded below by $z = z_T$) be an isothermal halfspace with constant winds. The only solution of Eq. (2.6.2) which, for real k , is analytic in ω for $z > z_T$ for all ω for which $\text{Im}(\omega) > 0$, and which vanishes as $\text{Im}(\omega) \rightarrow \infty$ (these conditions are equivalent to the causality condition) is of the form

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = D \begin{bmatrix} -A_{12} \\ A_{11} + G \end{bmatrix} e^{-G(z - z_T)} \quad (z > z_T) \quad (2.6.4)$$

where

$$G^2 = A_{11}^2 + A_{12}A_{21} \quad (2.6.5)$$

with the coefficients A_{ij} appropriate to the upper half space. The phase of G is chosen such that G is analytic in the upper half of the ω plane and such that the phase of G approaches 0 as $\text{Im}(\omega) \rightarrow \infty$. The quantity D in the above expression is any convenient constant. A necessary consequence of this is that the phase of G must be 0 when ω is real on all regions of the real ω axis where $G^2 > 0$. If $G^2 < 0$, the phase of G could be either $\pi/2$ or $-\pi/2$, depending on which choice is compatible with the requirement that G is analytic for $\text{Im}(\omega) > 0$. It should be noted that G has branch points on the real axis.

The values of ϕ_1 and ϕ_2 at lower values of z are found by integrating Eqs. (2.6.2) down from $z = z_T$ with (2.6.4) as starting conditions. Since the equations are linear, we can determine a transmission matrix $[R]$ such that

$$\begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}_{z=0} = [R] \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix}_{z_T} \quad (2.6.6)$$

where $[R]$ is independent of the values of ϕ_1 and ϕ_2 at $z = z_T$. The condition that $\phi_1(0) = 0$ is therefore

$$F(\omega, \vec{k}) = A_{12}R_{11} - (G + A_{11})R_{12} = 0 \quad (2.6.7)$$

where G , A_{11} , and A_{12} are quantities appropriate to the upper halfspace. The function $F(\omega, \vec{k})$ for general ω and \vec{k} is called the normal mode dispersion function. It is defined here for all values of ω in the upper half plane and for all real \vec{k} .

If, as implied previously, we restrict our attention to modes where ω and \vec{k} are real, then the matrix $[R]$ will be real and G must be real if Eq. (2.6.7) is to be satisfied. This limits the region in the ω, \vec{k} space where one need search for roots of the normal mode dispersion function. Since we need only consider points such that G is real, we can simply say that the normal mode dispersion function does not exist if this condition is not satisfied. We can also say that, if ϕ_1 and ϕ_2 describe a non-leaking guided mode (which is the only type we consider), then ϕ_1 and ϕ_2 must satisfy an upper boundary condition of decaying exponentially with increasing z in the upper half space.

One of the chief advantages of the multilayer approximation is that one can formulate a straightforward algorithm which computes the normal mode dispersion function $F(\omega, \vec{k})$ for given real ω and \vec{k} . Thus the computer can formally consider $F(\omega, \vec{k})$ as a known function in obtaining dispersion curves. The details of the computation of $F(\omega, \vec{k})$ are discussed in the description of the program's subroutines. [See Appendix B.]

2.7 TABULATION OF DISPERSION CURVES

One of the principal difficulties in coding the numerical synthesis of waveforms was that of obtaining a feasible method for tabulating the dispersion curves of the guided mode. By dispersion curves, we here mean the graphs of phase velocity $v_p(\omega, \theta)$ versus ω for fixed θ , where v_p satisfies

$$F(\omega, \vec{k}) = 0 \quad (2.7.1)$$

with

$$\vec{k} = (\omega/v_p)[\vec{e}_x \cos \theta + \vec{e}_y \sin \theta] \quad (2.7.2)$$

We denote the value of $F(\omega, \vec{k})$ when \vec{k} is given by Eq. (2.7.2) as $F_\theta(\omega, v_p)$. Thus we wish to tabulate curves in the v_p, ω plane along which

$$F_\theta(\omega, v_p) = 0 \quad (2.7.3)$$

given a computational scheme which either computes F_θ for given ω and v_p or else tells us that a real $F_\theta(\omega, v_p)$ does not exist for such values.

Such a tabulation of dispersion curves is complicated by the fact that it requires some care to insure that we do not mix modes. For example, if (ω_1, v_1) and (ω_2, v_2) are two points at which $F_\theta = 0$, it is difficult to determine whether or not these points both lie on the same curve or on different curves. An obvious goal is to eliminate the need for human intervention in answering such questions. The manner in which we accomplished this may be of some intrinsic interest as analogous problems occur in many other contexts.

We specify a rectangular region of the v_p versus ω plane and consider a dense rectangular array of points P in this region. Each point lying on the same row corresponds to the same value of v_p and each point lying on the same column corresponds to the same value of ω . For each such point we compute the sign, + or - (or X if $F_\theta(\omega, v_p)$ does not exist), of the normal mode dispersion function. One can visualize such a computation as being presented in the form of a picture (which we term a table) analogous to what one sees on a television screen. (See Fig. 2-3.) In such a picture one may readily perceive (providing the density of points is sufficiently great) clear-cut regions of the v_p versus ω plane where the sign of F_θ is +, regions where F_θ is negative, and regions where F_θ does not exist (all X's). The dispersion curves would then correspond to the more or less sharply defined lines which separate regions of all +'s from regions of all -'s. The technique used by the authors was (1) to systematically and selectively increase the density of considered points to such an extent that all dispersion curves in the rectangular region could be clearly perceived and (2) to use the picture array as a guide for systematically tabulating the dispersion curves for each given mode and to define starting brackets for homing in on particular points on the curves.

PHASE NORMAL MODE DISPERSION FUNCTION SIGN

0.40000	+++++---++---++---++---++
0.58621	+++++---++---++---++---++
0.57241	---++---++---++---++---++
0.55862	---++---++---++---++---++
0.54483	X---++---++---++---++---++
0.53103	X---++---++---++---++---++
0.51724	Y---++---++---++---++---++
0.50345	X---++---++---++---++---++
0.48966	Y---++---++---++---++---++
0.47586	Y---++---++---++---++---++
0.46207	XY---++---++---++---++---++
0.44828	XY---++---++---++---++---++
0.43448	XY---++---++---++---++---++
0.42069	XY---++---++---++---++---++
0.40690	XY---++---++---++---++---++
0.39310	XY---++---++---++---++---++
0.37931	XY---++---++---++---++---++
0.36552	XY---++---++---++---++---++
0.35172	XY---++---++---++---++---++
0.33793	XY---++---++---++---++---++
0.32414	XY+++---++---++---++---++
0.31034	XY+++---++---++---++---++
0.29655	XY+++---++---++---++---++
0.28276	XY+++---++---++---++---++
0.26897	XY+++---++---++---++---++
0.25517	XY---++---++---++---++---++
0.24138	XY---++---++---++---++---++
0.22759	XY---++---++---++---++---++
0.21379	XY+++---++---++---++---++
0.20000	XY+++---++---++---++---++

OMEGA 123456789012345678901234567890
 PHASE VELOCITY DIRECTION IS 0.0 DEGREES

OMEGA =				
0.50000E-02	0.02750E-02	0.11552E-01	0.14928E-01	0.18103E-01
0.21379E-01	0.24455E-01	0.27931E-01	0.31207E-01	0.34483E-01
0.37759E-01	0.41034E-01	0.44310E-01	0.47586E-01	0.50862E-01
0.54139E-01	0.57414E-01	0.60690E-01	0.63965E-01	0.67241E-01
0.70517E-01	0.73793E-01	0.77069E-01	0.80345E-01	0.83621E-01
0.86896E-01	0.90172E-01	0.93448E-01	0.96724E-01	0.10000E 00

Figure 2-3. Sample table of normal mode dispersion function signs. An "X" means that the function does not exist at that point in the phase velocity-frequency plane.

Although the above discussion may seem to be expressed in humanistic terms, the actual computation is carried out by the machine without human intervention.

One subsidiary problem which had to be solved was that of determining the criteria for when the density of points in the "picture" is sufficiently great that all of the dispersion curves in the rectangular region may be clearly defined. For this purpose the following theoretical conjectures were of considerable utility:

1. No two dispersion curves may intersect each other.
2. As long as v_p is greater than the maximum wind speed, $dv_p/d\omega \leq 0$ for any dispersion curve.

The first conjecture follows from the fact that if two curves cross at a point (ω, v_p) then one must have $F_0 = 0, \partial F_0/\partial \omega = 0, \partial F_0/\partial v_p = 0$ all simultaneously satisfied at this point. To locate such a point, we would have to solve three equations for the two unknowns ω and v_p . Since we have more equations than unknowns, it would be highly unlikely that such a point could be found. To date, we have not found any case of this happening, although the separation between adjacent curves can be very small.

The second conjecture follows from Eq. (2.4.18). Using the fact that $v_p = \omega/k$, we find, after some algebra, that

$$\frac{dv_p}{d\omega} = \frac{-1}{k^2} \frac{\int_0^\infty \Omega^2 Y^2 dz}{\int_0^\infty [k\omega\Omega^{-3}z^2 + \Omega(\vec{k} \cdot \vec{v}/k)Y^2] dz} \quad (2.7.4)$$

This is clearly negative as long as the denominator is positive. However, if the denominator is negative, then Eq. (2.4.18) would require the group velocity to be negative. If the group velocity is negative, then the discussion associated with Eq. (2.4.12) implies that the mode should not contribute to the waveform. In any event, the denominator in Eq. (2.7.4) must be positive for no winds and would most likely be positive if the winds are sufficiently weak. (Throughout this discussion we consider $\Omega > 0$ for all z .) Thus, while we have not succeeded in giving a truly rigorous proof of either of these conjectures, they seem likely to always hold in all cases of interest.

With the acceptance of the two conjectures discussed above, we may regard a pictorial array as being sufficient to resolve the modes if it indicates no apparent violations of the two conjectures. If it does indicate an anomaly, we simply add more points to the array (i.e., increase the density of points) until the anomaly disappears. The method of adding points should be formulated in such a manner that one does not go overboard, as the signs of the normal mode dispersion function will in general occupy a large amount of storage space in the machine. The method utilized in the program seems to be fairly foolproof and yet reasonably conservative in the number of points added to correct apparent anomalies. Further details may be found in the discussion of the program's subroutines.

As an example of how the table expansion process works in practice, suppose that the region of the ω, v plane considered is that where $.2 < v < .6$ km/sec, $.005 < \omega^p < .1$ rad/sec. The first tabulation^p is made with 900 points, corresponding to 30 equally spaced values of ω and 30 equally spaced values of v . For a particular θ and a particular model atmosphere, the result (which may at the user's discretion be printed out by the machine) is shown in Fig. 2-3. We arbitrarily number the modes starting in the lower left corner of the table and going up towards the upper right corner. Note that modes 3 and 4 almost touch near (0.0214, 0.324), modes 4 and 5 seem to disappear near (0.0443, 0.320), and other modes seem to vanish as well. In order to get rid of these anomalies, the computer judiciously adds new rows and columns. The table in Fig. 2-4 is the result. Note that the ω coordinates of the rows and the phase velocity coordinates of the columns are not equally spaced in the expanded table.

2.8 OTHER NUMERICAL TECHNIQUES

The result of the dispersion curve computation is a tabulation (stored in the machine) of points (ω, v) which lie on the n -th mode's dispersion curve and which describe that portion of the curve which lies in some prespecified rectangular region of the (ω, v) plane. Some specified number of modes are tabulated. The size assumed for the rectangular region is an inherent limitation on the computation and largely determines the limits of integration used in the evaluation of (2.5.5). Since these limits are not the true limits of the mode, an additional approximation is implied by this technique. There is some degree of "art" involved in the selection of this rectangular region and in the interpretation of waveforms computed with such a truncation.

Let $v(\omega)$ be the phase velocity as a function of frequency for a given mode which is tabulated at $\omega = \omega_1, \omega_2, \dots, \omega_N$; the corresponding values of v being denoted by v_1, \dots, v_N . For each of the values ω_i , the wavenumber $k_i = \omega_i/v_i$ is computed and then the quantities AMPLTD and PHASQ in Eqs. (2.5.4) are computed. The values of AMPLTD, k , and PHASQ at values of ω between neighboring ω_i are approximated by linear interpolation, following a technique introduced by Aki (1960) for numerical integration over oscillatory integrands. This defines the integrand at all values of ω between ω_1 and ω_N . The resulting integral may then be expressed as a sum of $N - 1$ terms, each term involving elementary functions, with no further approximations. The evaluation of this sum then leads to an approximate value of p_n for given t and R .

The Aki technique described above for numerical integration, although approximate, would appear to be a considerable improvement over the method of stationary phase, commonly used in wave propagation computations. It would appear that the stationary phase approximation would probably give grossly erroneous results, in view of the fact that some of the modes are very weakly dispersive. We should point out that the technique used here was suggested to the authors by Harkrider's paper (1964).

A shortcoming of the computation scheme is that the resulting solutions formally violate the causality requirement. Although causality is guaranteed by the Fourier transform solution, the guided mode solution, being an approximation, may not preserve this property. Furthermore, the truncation of integration limits will tend to amplify the non-physical waveform predicted at times before the true wave should actually arrive. However, at moderate and large distances, the noncausal portion of the wave should have relatively small amplitudes. This is borne out by the numerical computations. The authors believe that, with proper care in the selection of input parameters, the scheme described here should give a fair representation of the dominant portion of the waveform for low altitude observation of waves from low altitude explosions - providing, of course, that the stratified model for the atmosphere is adequate.

Chapter III

USER'S GUIDE TO INFRASONIC WAVEFORMS

3.1 INTRODUCTION

INFRASONIC WAVEFORMS is a digital computer program written in FORTRAN IV for the IBM 360 system at M.I.T. A slightly modified version for the IBM 7094 is currently in operation at the Air Force Cambridge Research Laboratories in Bedford, Massachusetts. The purpose of the program is to give a theoretical prediction of the acoustic pressure waveform which would be recorded at large horizontal distances (500 km to 10,000 km) from a low to moderate altitude thermonuclear explosion in the atmosphere. The program represents a substantial extension to an earlier program, INFRASONIC MODES, written by A. Pierce (1966).

In the program, the atmosphere is assumed to consist of a number (possibly as large as 100) of horizontal layers, each having constant temperature and wind velocity. The temperatures, wind-velocity magnitudes, and wind-velocity directions are not necessarily assumed to be the same in each layer. Such a multi-layer atmosphere, if judiciously chosen, may be expected to give a reasonable approximation to any continuously stratified atmosphere in so far as the calculations of waveforms are concerned.

It is the authors' intent that the program be written in such a form as to facilitate use by anyone having access to a large digital computer. It is written in a manner such that it should not be too difficult to modify for application to similar problems or for use in other computer installations. The fact that we were able to modify the M.I.T. version for the AFCRL computer with only a moderate expenditure of time attests to the latter.

The key to insuring that any program be amenable to wide-spread usage is documentation. This report represents one such attempt to provide such documentation. In addition, the program is written with a predominance of COMMENT statements. A listing of the program is given in Appendix B. The comments at the beginning of each subroutine attempt to explain the function of each, and its purpose in relation to the main task of the program. Definitions of all variables, input and output, as well as of those presumed in COMMON storage are given. This rather elaborate procedure was suggested to the authors by a recent book on computer analysis of time series by Simpson. (Frankly, we believe that this is the manner in which all computer programs should be written.)

The present program has been continuously tested and used for well over a year now and we are reasonably certain that it is free of major coding errors. However, the sheer length of the program prohibits us from certifying this with certainty.

The theory on which the program is based is summarized in the preceding chapter. Here, we attempt to describe the program from the viewpoint of its operation - i.e., to give a user's manual for the program. To a certain extent, this duplicates the statements given in the deck listing of the program. However, in a matter such as this, only a slight amount of confusion can cause undue grief and expenditure of time and money. Thus, we feel that it is vastly preferable to give an overdetailed account of the program than to run the risk of dissuading someone from use of the program. The comments given here apply mainly to the operation of the M.I.T. version - it is to be hoped that users at other installations will be able to quickly ascertain just what modifications in the program and in its rules of usage are necessary.

3.2 GENERAL DISCUSSION OF PROGRAM USAGE

To obtain a synthesized waveform and/or other auxiliary information, the user must decide in advance on the values of various input parameters which control the operation of the program. These input parameters may be considered as falling within one of six general categories:

- 1) Parameters specifying the nature of the model atmosphere to be used.
- 2) Parameters specifying the nature of the explosion; namely, its yield and height of burst.
- 3) Parameters specifying the location of the observation point with respect to the explosion.
- 4) Parameters controlling the nature of the theoretical and numerical approximations made in the computation.
- 5) Parameters controlling the extent, detail, and type of the output.

It is important that the user realize that not all input parameters need be specified. The program is written so as to allow considerable flexibility in input and output.

Possible outputs of the computation include the following:

- 1) Punched cards containing intermediate results in a format suitable for input to future runs of the program.
- 2) A tabulation on the printout of the assumed model atmosphere's properties in a standard format. (Fig. 3-1)
- 3) Printout of all input quantities as they are read in (Fig. 3-2).
- 4) Pictorial displays of the phase velocity curves of the guided modes as being lines separating regions where a + sign is printed at every point from regions where a - (minus) sign is printed at every point of a rectangular array of points in the phase velocity versus angular frequency plane. These displays may later be used to check on whether all desired modes were included and on whether or not the computation process was successful in resolving the modes. (Figures 3-3 and 3-4)
- 5) A listing of the tabulation of phase velocity versus frequency for each guided mode (Fig. 3-5).
- 6) A second listing giving the same as in 5 plus parameters of the ϕ_1 and ϕ_2 profiles (defined in Eq. 2.6.1) for each point in the tabulation. (Fig. 3-6)
- 7) A third listing giving the same as in 5 plus an amplitude factor independent of yield. (Fig. 3-7)
- 8) A fourth listing giving the same as in 5 plus the yield dependent amplitude and phase which appear in the integrand of the integral over frequency which represents the contribution to the waveform from a given guided mode. (Fig. 3-8)
- 9) Tabulations of acoustic pressure versus time for selected guided mode waveforms. (Fig. 3-9)
- 10) Tabulation of acoustic pressure versus time for the total waveform. (Fig. 3-9)
- 11) A plot of acoustic pressure versus time for selected guided mode waveforms on the CALCOMP plotter. (Fig. 3-10)
- 12) A plot of acoustic pressure versus time for the total waveform on the CALCOMP plotter. (Fig. 3-10)

MODEL ATMOSPHERE OF 34 LAYERS

LAYER	ZB	ZT	H	C	VX	VY
34	225.00	INFINITE	INFINITE	0.8014	0.0	0.0
33	205.00	225.00	20.00	0.7655	0.0	0.0
32	195.00	205.00	10.00	0.7469	0.0	0.0
31	185.00	195.00	10.00	0.7279	0.0	0.0
30	175.00	185.00	10.00	0.7097	0.0	0.0
29	165.00	175.00	10.00	0.6882	0.0	0.0
28	155.00	165.00	10.00	0.6584	0.0	0.0
27	145.00	155.00	10.00	0.6093	0.0	0.0
26	135.00	145.00	10.00	0.5413	0.0	0.0
25	125.00	135.00	10.00	0.4783	0.0103	0.0
24	115.00	125.00	10.00	0.4007	0.0236	0.0
23	105.00	115.00	10.00	0.3168	0.0309	0.0
22	95.00	105.00	10.00	0.2833	0.0103	0.0
21	85.00	95.00	10.00	0.2718	-0.0051	0.0000
20	75.00	85.00	10.00	0.2725	0.0077	0.0
19	65.00	75.00	10.00	0.2869	0.0206	0.0
18	55.00	65.00	10.00	0.3104	0.0216	0.0
17	45.00	55.00	10.00	0.3230	0.0216	0.0
16	40.00	45.00	5.00	0.3261	0.0175	0.0
15	35.00	40.00	5.00	0.3161	0.0082	0.0
14	30.00	35.00	5.00	0.3084	0.0021	0.0
13	25.00	30.00	5.00	0.3019	-0.0021	0.0000
12	20.00	25.00	5.00	0.2938	-0.0072	0.0000
11	18.00	20.00	2.00	0.2869	-0.0058	-0.0021
10	16.00	18.00	2.00	0.2819	0.0055	0.0055
9	14.00	16.00	2.00	0.2869	0.0100	0.0040
8	12.00	14.00	2.00	0.2938	0.0139	0.0
7	10.00	12.00	2.00	0.3005	0.0154	0.0
6	8.00	10.00	2.00	0.3078	0.0129	0.0
5	6.00	8.00	2.00	0.3161	0.0098	0.0
4	4.00	6.00	2.00	0.3230	0.0046	0.0
3	2.00	4.00	2.00	0.3292	0.0046	0.0
2	1.00	2.00	1.00	0.3400	0.0011	-0.0011
1	0.0	1.00	1.00	0.3424	0.0011	-0.0011

ZB=HEIGHT OF LAYER BOTTOM IN KM
 ZT=HEIGHT OF LAYER TOP IN KM
 H=WIDTH OF LAYER IN KM
 C=SOUND SPEED IN KM/SEC
 VX=X COMP. OF WIND VEL. IN KM/SEC
 VY=Y COMP. OF WIND VEL. IN KM/SEC

Figure 3-1. Printout of model atmospheric properties for 30° N. 140° W. in October.

```
NAME HAS JUST BEEN READ IN
NAME
NSTART=          I,NPRINT=
CEND
```

4AM2 HAS JUST BEEN READ IN
2AM2

Figure 3-2. Sample of NAMELIST printout.

VPHSE ACRMAL MODE DISPERSION FUNCTION SIGN

0.60000	X---+---+---+---+---+---+---+---+---+---
0.53793	X---+---+---+---+---+---+---+---+---+---
0.57586	X---+---+---+---+---+---+---+---+---+---
0.56379	X---+---+---+---+---+---+---+---+---+---
0.55172	X---+---+---+---+---+---+---+---+---+---
0.53966	X---+---+---+---+---+---+---+---+---+---
0.52759	X---+---+---+---+---+---+---+---+---+---
0.51552	X---+---+---+---+---+---+---+---+---+---
0.50345	X---+---+---+---+---+---+---+---+---+---
0.49138	X---+---+---+---+---+---+---+---+---+---
0.47931	X---+---+---+---+---+---+---+---+---+---
0.46724	X---+---+---+---+---+---+---+---+---+---
0.45517	X---+---+---+---+---+---+---+---+---+---
0.44310	X---+---+---+---+---+---+---+---+---+---
0.43103	X---+---+---+---+---+---+---+---+---+---
0.41897	X---+---+---+---+---+---+---+---+---+---
0.40690	X---+---+---+---+---+---+---+---+---+---
0.39483	X---+---+---+---+---+---+---+---+---+---
0.38276	X---+---+---+---+---+---+---+---+---+---
0.37069	X---+---+---+---+---+---+---+---+---+---
0.35862	X---+---+---+---+---+---+---+---+---+---
0.34655	X---+---+---+---+---+---+---+---+---+---
0.33448	X---+---+---+---+---+---+---+---+---+---
0.32241	X---+---+---+---+---+---+---+---+---+---
0.31034	X---+---+---+---+---+---+---+---+---+---
0.29828	X---+---+---+---+---+---+---+---+---+---
0.28621	X---+---+---+---+---+---+---+---+---+---
0.27414	X---+---+---+---+---+---+---+---+---+---
0.26207	X---+---+---+---+---+---+---+---+---+---
0.25000	X---+---+---+---+---+---+---+---+---+---

OMEGA 123456789012345678901234567890
 PHASE VELOCITY DIRECTION IS 35.000DEGREES

OMEGA =

0.50000E-02	0.82759E-02	0.11552E-01	0.14828E-01	0.18103E-01
0.21379E-01	0.24655E-01	0.27931E-01	0.31207E-01	0.34483E-01
0.37759E-01	0.41034E-01	0.44310E-01	0.47586E-01	0.50862E-01
0.54138E-01	0.57414E-01	0.60690E-01	0.63965E-01	0.67241E-01
0.70517E-01	0.73793E-01	0.77069E-01	0.80345E-01	0.83621E-01
0.86896E-01	0.90172E-01	0.93448E-01	0.96724E-01	0.10000E-00

Figure 3-3. Tabulation of the normal mode dispersion function signs for the atmosphere of Figure 3-1 and a direction of propagation of 35° north of east.

TABULATION OF FIRST 8 MODES

MODE 1

OMEGA (RAD/SEC)	VPHSE (KM/SEC)
0.008276	0.257854
0.008685	0.256762
0.008890	0.256238
0.009095	0.255711
0.009914	0.253487
0.011013	0.249999

MODE 2

OMEGA (RAD/SEC)	VPHSE (KM/SEC)
0.008276	0.317475
0.008644	0.317133
0.008685	0.316778
0.008712	0.316379
0.008785	0.314870
0.008852	0.313362
0.008890	0.312499
0.008987	0.310344
0.009095	0.308013
0.009590	0.298275
0.009914	0.292628
0.010319	0.286206
0.011192	0.274137
0.011552	0.269688
0.012215	0.262068
0.013410	0.249999

MODE 3

OMEGA (RAD/SEC)	VPHSE (KM/SEC)
0.008276	0.328731
0.008500	0.322413
0.008621	0.319396
0.008685	0.318234
0.008727	0.317887
0.008890	0.317615
0.009095	0.317559
0.009914	0.317503
0.011552	0.317446
0.014828	0.317323
0.018103	0.317161
0.018571	0.317133

Figure 3-5. A portion of the normal mode dispersion curve tabulation printed by INFRASONIC WAVEFORMS as determined using the table in Figure 3-4.

PHI1 AND PHI2 PROFILE DATA

IAP1MX = NO. OF LAYER FOR WHICH ABS(PHI1(IAP1MX)) IS A MAXIMUM
 IAP2MX = NO. OF LAYER FOR WHICH ABS(PHI2(IAP2MX)) IS A MAXIMUM
 R1 = PHI1(IAP1MX) / ABS(PHI2(IAP2MX))
 R2 = PHI2(IAP2MX) / ABS(PHI2(IAP2MX))
 R3 = PHI2(1) / ABS(PHI2(IAP2MX))
 NZC1 = NO. OF TIMES PHI1 CHANGES SIGN
 NZC2 = NO. OF TIMES PHI2 CHANGES SIGN

MODE 1

OMEGA	VPHSE	IAP1MX	R1	NZC1	IAP2MX
0.00826	C.25785	34	7.09325	2	16
0.00869	0.25676	15	6.55537	2	16
0.00889	0.25624	15	6.57698	2	16
0.00909	C.25571	15	6.59924	2	16
0.00991	C.25349	15	6.69513	2	16
0.01101	C.25000	15	6.84304	2	16

MODE 2

OMEGA	VPHSE	IAP1MX	R1	NZC1	IAP2MX
0.00828	0.31748	33	2.07396	3	1
0.00864	C.31713	32	14.98704	3	1
0.00869	0.31678	32	28.47873	3	1
0.00871	0.31638	32	39.86269	3	24
0.00876	C.31487	32	39.20818	1	24
0.00885	0.31336	32	38.63559	1	24
0.00889	C.31250	32	38.31291	1	24
0.00899	0.31034	32	37.49858	1	24
0.00909	0.30801	32	36.60118	1	24
0.00959	C.29878	31	33.53206	1	23
0.00991	0.29 3	30	31.02254	1	23
0.01032	0.28621	29	28.49094	1	23
0.01119	0.27414	28	25.01674	1	23
0.01155	0.26969	28	24.25139	1	23
0.01222	0.26207	28	22.75203	1	23
0.01341	0.25000	27	20.41314	1	23

Figure 3-6. Sample printout of ϕ_1 and ϕ_2 profile data

TABULATION OF SOURCE FREE AMPLITUDES FROM SUBROUTINE PAMPDE

HEIGHT OF BURST = 3.000 KM
 HEIGHT OF OBSERVER = 0.0 KM
 FACT = 0.558 KM/SEC
 ALAM = 1.173

MODE 1

OMEGA	VPHSE	AMP
0.00828	0.25785	-0.00455954
0.00869	0.25676	-0.00459953
0.00889	0.25624	-0.00459171
0.00909	0.25571	-0.00457545
0.00991	0.25349	-0.00446189
0.01101	0.25000	-0.00421712

MODE 2

OMEGA	VPHSE	AMP
0.00828	0.31748	-0.02964770
0.00864	0.31713	-0.02382846
0.00869	0.31678	-0.01535209
0.00871	0.31638	-0.00928055
0.00878	0.31487	-0.00245403
0.00885	0.31336	-0.00113502
0.00889	0.31250	-0.00082699
0.00899	0.31034	-0.00046585
0.00909	0.30801	-0.00030775
0.00959	0.29828	-0.00014295
0.00991	0.29263	-0.00012778
0.01037	0.28621	-0.00013161
0.01119	0.27414	-0.00019093
0.01155	0.26969	-0.00023866
0.01222	0.26207	-0.00038533
0.01341	0.25000	-0.00098984

MODE 3

OMEGA	VPHSE	AMP
0.00828	0.32873	-0.00068337
0.00850	0.32241	-0.00049140
0.00862	0.31940	-0.00349182
0.00869	0.31823	-0.01440191
0.00873	0.31789	-0.02297221
0.00889	0.31761	-0.02886438
0.00909	0.31756	-0.02937463
0.00991	0.31750	-0.02952643

Figure 3-7. Tabulation of modes including the amplitude factor AMP which is independent of the source strength.

MODE TABULATION FOR Y= 10000.00 KILOTONS

MODE 1

OMEGA	VPHSE	AMPLTD	PHASE
C.C0828	0.25785	-67670.	3.72680
C.00869	0.25676	-70010.	3.71697
C.00889	0.25624	-70745.	3.71206
C.00909	0.25571	-71337.	3.70715
C.C0991	0.25349	-72784.	3.68753
0.01101	0.25000	-72766.	3.66127

MODE 2

OMEGA	VPHSE	AMPLTD	PHASE
C.C0828	0.31748	-3.96551E 05	3.72680
C.00864	0.31713	-3.25613E 05	3.71796
C.C0869	0.31678	-2.10378E 05	3.71697
C.C0871	0.31638	-1.27441E 05	3.71634
C.00878	0.31487	-33915.	3.71458
C.C0885	0.31336	-15781.	3.71297
C.00889	0.31250	-11538.	3.71206
C.00839	0.31034	-6555.5	3.70974
C.C0909	0.30801	-4371.9	3.70715
C.C0959	0.29828	-2116.2	3.69529
C.C0991	0.29263	-1939.9	3.68753
C.01032	0.28621	-2058.8	3.67785
C.01119	0.27414	-3169.8	3.65701
0.01155	0.26969	-4053.6	3.64844
C.01222	0.26207	-6811.7	3.63267
0.01341	0.25000	-18688.	3.60436

MODE 3

OMEGA	VPHSE	AMPLTD	PHASE
0.00828	0.32873	-898.67	3.72680
C.00850	0.32241	-6606.2	3.72143
C.00862	0.31940	-47486.	3.71851
C.C0869	0.31823	-1.96905E 05	3.71697
C.00873	0.31789	-3.14976E 05	3.71596
C.C0889	0.31761	-3.94443E 05	3.71206
C.00909	0.31756	-4.10974E 05	3.70715
C.C0991	0.31750	-4.30359E 05	3.68753
C.01155	0.31745	-4.60767E 05	3.64844
C.01483	0.31732	-5.10123E 05	3.57093

Figure 3-8. Tabulation of modes including the source-dependent amplitude and phase, AMPLTD and PHASE, respectively.

TABULATION OF RESPONSES

	TIME	TOTAL	MODE 1	MODE 2
1	16000.0	-3.21	0.04	1.20
2	16015.0	-2.57	0.01	1.38
3	16030.0	1.87	-0.01	1.54
4	16045.0	3.34	-0.04	1.63
5	16060.0	-0.06	-0.06	1.74
6	16075.0	-1.70	-0.09	1.87
7	16090.0	1.47	-0.11	1.92
8	16105.0	3.41	-0.13	1.94
9	16120.0	0.42	-0.15	1.93
10	16135.0	-1.72	-0.17	1.84
11	16150.0	1.68	-0.18	1.81
12	16165.0	5.46	-0.19	1.70
13	16180.0	3.97	-0.20	1.57
14	16195.0	1.21	-0.20	1.41
15	16210.0	2.95	-0.20	1.23
16	16225.0	5.67	-0.19	1.03
17	16240.0	3.21	-0.18	0.81
18	16255.0	-0.95	-0.16	0.58
19	16270.0	0.80	-0.15	0.34
20	16285.0	6.37	-0.12	0.09
21	16300.0	7.21	-0.10	-0.16
22	16315.0	3.09	-0.07	-0.41
23	16330.0	1.78	-0.04	-0.65
24	16345.0	4.58	-0.01	-0.88
25	16360.0	4.56	0.02	-1.09
26	16375.0	0.16	0.05	-1.29
27	16390.0	-1.55	0.08	-1.47
28	16405.0	1.82	0.10	-1.62
29	16420.0	3.29	0.13	-1.75
30	16435.0	-0.45	0.16	-1.85
31	16450.0	-2.80	0.18	-1.92
32	16465.0	0.15	0.19	-1.96
33	16480.0	1.79	0.21	-1.96
34	16495.0	-2.78	0.21	-1.93
35	16510.0	-6.93	0.22	-1.88
36	16525.0	-4.05	0.22	-1.79
37	16540.0	0.15	0.21	-1.67
38	16555.0	-2.20	0.20	-1.53
39	16570.0	-6.80	0.19	-1.36
40	16585.0	-5.11	0.17	-1.17
41	16600.0	-0.42	0.15	-0.96
42	16615.0	-2.15	0.13	-0.73
43	16630.0	-8.42	0.10	-0.50
44	16645.0	-8.92	0.07	-0.25

Figure 3-9. Sample printout of the total and modal pressure histories. The time is given in seconds after the blast, and pressure in dynes/cm².

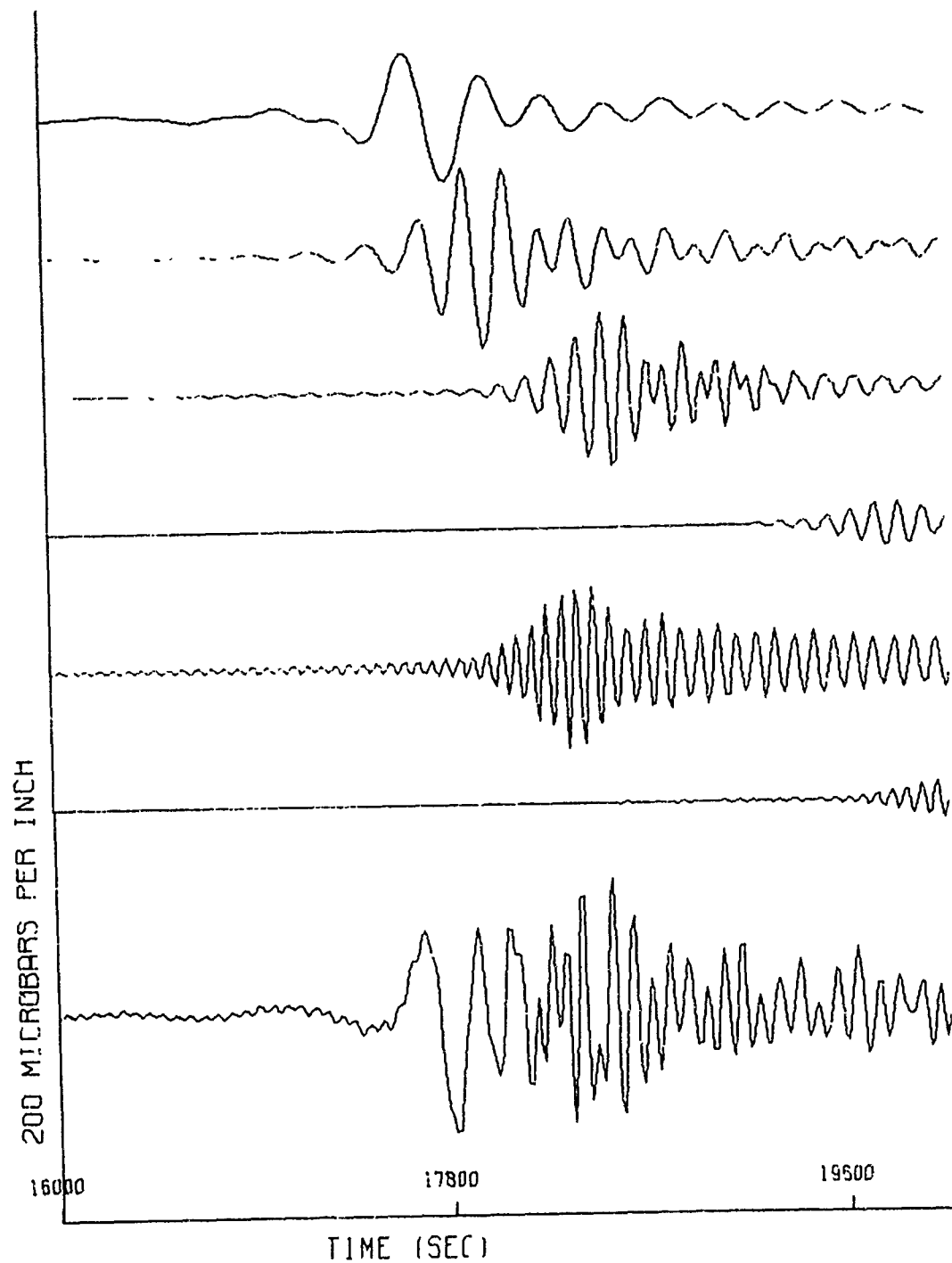


Figure 3-10. CALCOMP plot of modal and total waveforms on a common time axis and with a common pressure scale. (Reduced to 250 μ bars per inch.) See Figure 3-12 for a complete listing of the input data.

It is at the user's discretion as to just what output is actually realized in a given run of the program. A fuller discussion of input and output variables is given in subsequent sections.

3.3 INPUT PARAMETERS CHARACTERIZING THE ATMOSPHERE, THE SOURCE, AND THE OBSERVER LOCATION

The atmosphere model is characterized by three possible sets of parameters. These are listed below:

<u>Set 1</u>	<u>Set 2</u>	<u>Set 3</u>
IMAX	IMAX	IMAX
T	T	CI
VKNTX	WINDY	VXI
VKNTY	WANGLE	VYI
LANGLE	LANGLE	
ZI	ZI	HI

The variables which appear in these three lists are defined below:

1) IMAX is the number of layers of finite thickness in the multilayer atmosphere. It is an integer and may take any value between 2 and 99, inclusive.

2) T is the absolute temperature in degrees Kelvin. It is a subscripted real variable; T(1) is the temperature in the lowest layer; T(IMAX + 1) is the temperature in the upper half space. (The layers are numbered from the bottom.) Exactly IMAX + 1 values T(I) should be supplied.

3) CI is a subscripted real variable representing sound speed in km/sec. CI(I) is the sound speed in the I-th layer. Exactly IMAX + 1 values should be supplied.

4) VKNTX and VKNTY are subscripted (IMAX + 1 values) representing x and y components of wind speed in knots of the IMAX + 1 layers (including the upper half space).

5) WINDY is a subscripted variable (IMAX + 1 values) representing the wind velocity magnitude in knots of the IMAX + 1 layers.

6) WANGLE is a subscripted variable (IMAX + 1 values) representing the wind velocity direction in degrees, reckoned counter-clockwise from the x axis.

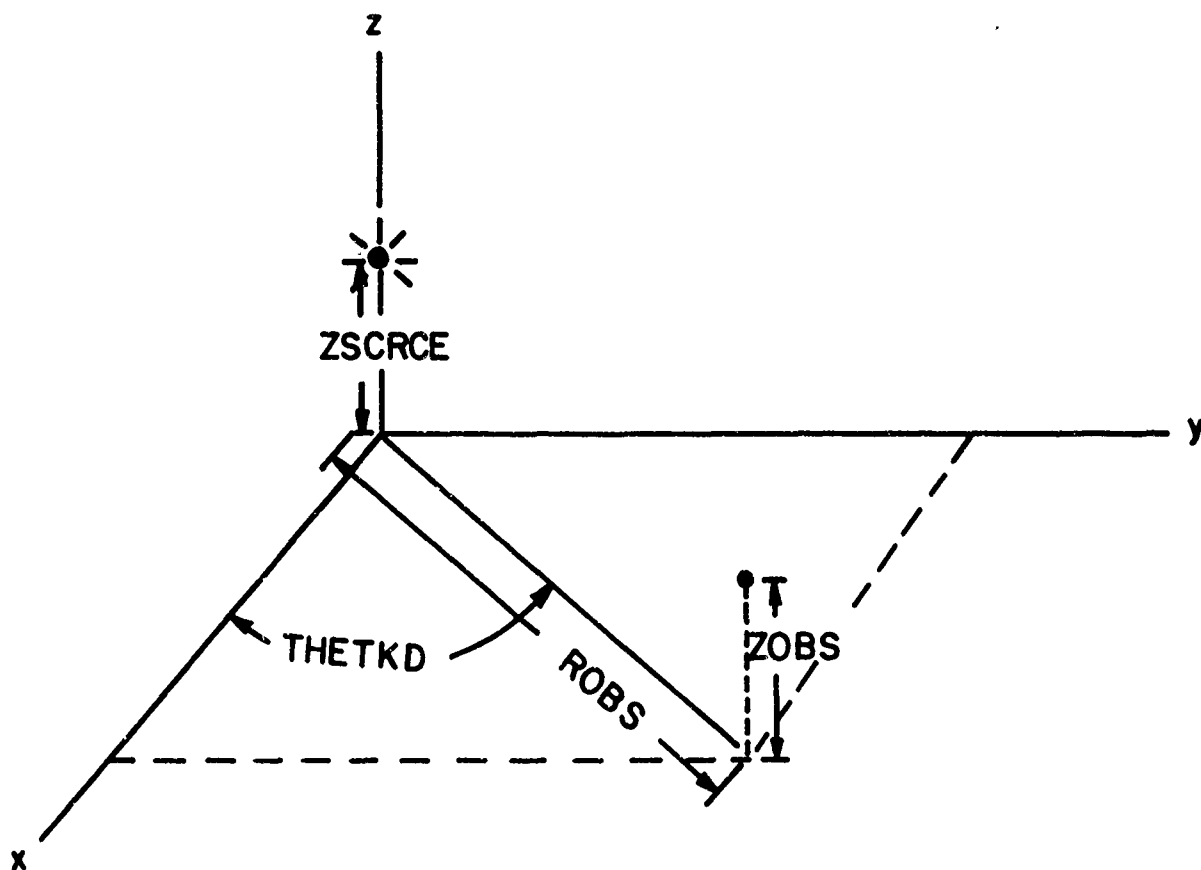


Figure 3-11. Sketch showing the geometrical meaning of some of the input data for INFRASONIC WAVEFORMS.

7) LANGLE is an integer. If it is 0 or negative, the computer is being told that set 1 (VKNTX and VKNTY) is supplied. If it is positive, set 2 (WINDY and WANGLE) is being supplied.

8) ZI(I) is the height above the ground of the top of the I-th layer of finite thickness. Its dimensions should be km. Exactlay IMAX values should be supplied.

9) HI(I) is the thickness in km of the I-th layer of finite thickness. IMAX values are supplied.

The manner in which the computer is instructed as to when set 3 rather than sets 1 or 2 is being supplied depends on the value of a control integer NSTART which has previously been input. Briefly, NSTART being 1 implies sets 1 or 2 are being input, while NSTART = 2 implies set 3 is being read. This is discussed further in Sec. 3.5.

The source model is specified by two parameters YIELD and ZSCRCE:

- 1) YIELD is the yield of the explosion in KT.
- 2) ZSCRCE is the height above the ground in km of the explosion.

The observer location is specified by parameters ROBS, ZOBS, and THETKD:

- 1) ROBS is the magnitude of the horizontal distance in km between source and observer.
- 2) THETKD is the angle in degrees, reckoned counterclockwise, which the horizontal component of the vector from source to observer makes with the x axis.
- 3) ZOBS is the height in km above the ground of the observer.

The meaning of these parameters is further illustrated by Fig. 3-11.

3.4 INPUT PARAMETERS CONTROLLING THE METHOD OF COMPUTATION AND OUTPUT

A major portion of the computation is concerned with the determination of the dispersion curves (phase velocity versus angular frequency) of the guided modes. Just which modes and which segments of modes are found and used in subsequent calculations depends on the search region in the phase velocity vs. frequency plane. This rectangular search region is specified by the following parameters:

1) OM1 is the lower angular frequency limit (rad/sec) of the search region.

2) OM2 is the upper angular frequency limit (rad/sec) of the search region.

3) V1 is the lower phase velocity limit (km/sec) of the search region.

4) V2 is the lower phase velocity limit (km/sec) of the search region.

Generally, one should select V1 to be larger than the maximum wind speed in the model atmosphere. It is also advisable that one not take OM1 identically equal to 0, as this could conceivably lead to machine overflow and termination of a given run.

The details of the search for modes within the rectangular search region depend on the initial choice of the number of points at which the sign of the normal mode dispersion function is tabulated. As is explained in Sec. 2.7, the numerical computation begins with the generation of an array of points, lying on a rectangular grid, covering the search region. The total number of intersection points in this grid is determined by two integers:

1) NOMI is the number of equally spaced constant frequency lines comprising the vertical lines of the grid.

2) NVPI is the number of equally spaced constant phase velocity lines comprising the horizontal lines of the grid.

Both NOMI and NVPI should be between 2 and 100, inclusive. In our own calculations, we have generally taken both of these integers to be 30.

The modes found in the search region are numbered consecutively, starting from the lower left corner of the region. (Phase velocity increases upwards and frequency increases to the right.) A key input parameter in this respect is the maximum number of modes MAXMOD which are to be tabulated and used in the subsequence waveform synthesis. If, for example, MAXMOD is 5, the program will not tabulate or use modes 6, 7, etc. The maximum value of MAXMOD permitted by the current version of the program is 10. In our computations, we generally use 10 unless we have some reason to believe that the higher order modes will not contribute appreciably to the wave form during the time interval of interest.

For the tabulation and graphing of pressure waveforms versus time, it is necessary to specify the time interval of the computed

waveform and the time increment between successive times at which the pressure is tabulated. The parameters specifying these quantities are listed below:

- 1) TFIRST is the earliest time in seconds relative to time of detonation at which computations are performed.
- 2) TEND is the latest time in seconds relative to time of detonation at which computations are performed.
- 3) DELTT is the time increment in seconds for which successive waveform points are tabulated.

In choosing these quantities, care should be taken to insure that the number of time points $(TEND - TFIRST) / DELTT$ is less than 1000, since otherwise incorrect values could be obtained through storage spillover. For all realistic cases which we have considered, it appears sufficient to take $DELTT > 6$ sec. It is meaningless to take $DELTT$ much less than $(1/20)$ of $1/OM2$. The choice of TFIRST and TEND is generally made with the intent of including the main pulse, which travels with speeds of the order of the sound speed at the ground. The nature of the theory suggests that the computations will generally not be too reliable at times much later than this. Some trial and error may be required to determine the optimum choice of TFIRST and TEND in relation to the observer distance ROBS. The examples treated in the following chapter may be of some assistance in this respect. Also, the experimental waveforms should in principle give a clue to the proper choice.

The nature and extent of the output is specified by a number of additional input parameters. If one wants a maximum of printout of intermediate values, he specifies $NPRNT = 1$ (or any other positive integer); if he wants a minimum, he specifies $NPRNT = 0$ (or any negative integer). In the latter case, the input is listed, the tabulation of waveform pressure versus time is given, and the waveforms are plotted. In any nonroutine operation of the program, it is probably advisable to set $NPRNT = 1$. However, this does lead to a tremendous amount of printout, and, if the current run only represents a slight modification of a previous run, one may want to set $NPRNT = -1$.

The program also allows for the option to punch out intermediate results on cards. The format of the punched cards is such that they may be used in later runs of the program as input in order to save machine time. If one desires this option, he should set $NPNCH = 1$. Otherwise, he should set $NPNCH = 0$. In general, it is advisable to avoid requesting $NPNCH = 1$ unless one plans an

immediate use of the cards. Otherwise, the bookkeeping chores of keeping track of a large number of cards may get out of hand. However, the computer time saved (which may be of the order of 10 minutes of 360 time) is not negligible, and one may sometimes wish to exercise this option.

To limit the number of plots on the CALCOMP graph and the number of modal waveforms tabulated on the printout, an input parameter IOPT is used. If IOPT is 1,2,...., 10, only the contribution to the waveform from mode number IOPT is calculated, printed, and plotted. If IOPT is 11, the computer calculates, prints, and plots all modal waveforms, as well as the total waveform. If IOPT is 12, all modal waveforms are computed, but only the total waveform is tabulated and plotted. Normally, one might wish to set IOPT = 11 and obtain all possible auxiliary results. However, in routine operation, when the qualitative properties of the individual modal waveforms are a priori known, one might set IOPT = 12. In some special cases, when one is interested in only one particular mode, he might set IOPT equal to 1,2,3,...., or 10.

3.5 PREPARATION OF THE INPUT DECK

The program is written such that all input data should be supplied in the NAMELIST format, which is a standard feature of FORTRAN IV for the IBM 360 and the IBM 7094. We find that NAMELIST is particularly convenient because it enables us to supply only the data which is needed for a given calculation and because it minimizes the possibility of keypunching errors during preparation of the input deck. For a description of NAMELIST we refer the reader to any of the FORTRAN IV manuals.

The main program has ten NAMELIST statements, each defining the data which may be read in when the computation executes a READ statement with a particular NAME. The NAMES of the possible data sets are numbered NAM1, NAM2,, NAM10.

For convenience of reference, these namelist statements are reproduced below:

```

NAMELIST /NAM1/  NSTART, NPRNT, NPNCH
NAMELIST /NAM2/  LANGLE, IMAX, T, VKNTX, VKNTY, WINDY, WANGLE, ZI
NAMELIST /NAM3/  IMAX, CI, VXI, VYI, HI
NAMELIST /NAM4/  THETKD, V1, V2, OM1, OM2, NOMI, NVPI, MAXMOD
NAMELIST /NAM5/  IMAX, CI, VXI, VYI, HI, THETKD, MDFND, KST,
1                KFIN, OMMOD, VPMOD

```

```

NAMELIST /NAM6/  ZSCRCE, ZOBS
NAMELIST /NAM7/  OMMOD, VPMOD, MDFND, KST, KFIN, AMP, ALAM, FACT
NAMELIST /NAM8/  YIELD
NAMELIST /NAM9/  MDFND, KST, KFIN, OMMOD, VPMOD, AMPLTD, PHASQ
NAMELIST /NAM10/ TFIRST, TEND, DELTT, ROBS, IOPT

```

In any given run with the program, the first card in the data card pack should be a NAM1 card. This card should generally be of the form

```

&NAM1      NSTART=      , NPRNT=      , NPNCH =      &END

```

with desired values supplied for the parameters NSTART, NPRNT, NPNCH. The order of these three quantities is irrelevant. Furthermore, the name, equal sign, and value of any of them may be omitted if one desires to use the value of 0 for any of them.

The cards following the first depend on the value of NSTART. Unless one is supplying data in the form of intermediate results computed during previous runs, he would take NSTART =1. Given that NSTART=1, the data cards following should be those corresponding to NAM2, NAM4, NAM6, NAM8, and NAM10, in that order. Only nonzero values or values of quantities which will be used in the computation need be supplied. Thus, for example, if one sets LANGLE=0 (See Sec. 3.3.), then he need not list the values of WINDY or WANGLE in the NAM2 data group. Values of subscripted variables should be listed in the format (for example)

```

VKNTX = 0.,0.,2.0,2.0,5.0,7.5,5.0,VKNTY=....

```

signifying that VKNTX(1)=0., VKNTX(2)=0., VKNTX(3)=2.0, etc. Note that, even though the first two numbers are 0, they could not be omitted, since otherwise the computer would consider 2.0 to be VKNTX(1). However, a long string of identical numbers can be abbreviated by writing 6*0., for example, for a string of six zeros. It must be emphasized that elements with indices greater than the largest index for that variable used in the computation need not be supplied. In other words, just because 100 spaces of storage are allotted to VKNTX does not imply that 100 numbers need be listed in the input.

If NSTART=2, one supplies NAM3, NAM4, NAM6, NAM8, NAM10. If NSTART=4, one supplies NAM7, NAM8, NAM10. If NSTART=5, one supplies NAM9, NAM10. This procedure is described in greater detail in the first two pages of the deck listing of the main program in Appendix B. The option of taking NSTART=2, 3, 4, or 5 simply allows one to

make use of intermediate results calculated in previous runs of the program. The data in NAM3, NAM5, NAM7, or NAM9 would generally have been punched in NAMELIST format during a previous run. We did not define all of the input variables in these latter lists in the preceding two sections since, in normal operation, those variables omitted would be specified by the computer (through the punching process) rather than by the user. The variables in NAM3 are a possible exception, as NAM3, through the NSTART=2 option, simply provides the possibility of supplying the parameters defining the model atmosphere in a manner other than that implied by NSTART=1. This has been discussed in Sec. 3.3.

A list of all possible input variables and their definitions is given on pages 3, 4, 5, of the deck listing of the main program in Appendix B.

The last card read in corresponding to a given problem is always the NAM10 card. The very next card should always be a NAM1 card. If the problem previously considered is the last problem to be computed in the run, one specifies NSTART=6 in the NAM1 list and needs not specify the values of NPRNT or NPNCH. If not, he specifies NSTART=1,2,3,4, or 5 and gives his data cards in one of the sequences and formats described above. The rules for providing data for successive problems are similar to those for the first with one exception. If the user fails to provide a value for any quantity, the value assumed by the computer for that quantity will be that value currently stored in the machine - which may not necessarily be zero. Also, even though the user may wish to use all the values previously input through a (for example) NAM2 list, he must put a NAM2 card in the portion of the deck corresponding to the current problem. Such a card would be of the form

```
&NAM2      (blanks)      &END
```

It is in successive problems that the option of taking NSTART=2, 3, 4, or 5 may find its greatest utility. For example, if one wished to study the effect of yield with all other variables fixed, he could take his data for the second and successive problems in the form

```
&NAM1      NSTART=4      &END
&NAM7                                &END
&NAM8      YIELD=2000     &END
&NAM10                                &END
```

```

&NAM1  NSTART=1,  NPRNT=1,  NPNCH= -1  &END
&NAM2
IMAX = 33.
7I=1.,2.,4.,6.,8.,10.,12.,14.,16.,18.,20.,25.,30.,35.,40.,45.,55.,
65.,75.,85.,95.,105.,115.,125.,135.,145.,155.,165.,175.,185.,195.,
205.,225.,
T=292.,288.,270.,260.,249.,236.,225.,215.,205.,198.,205.,215.,227.,
237.,249.,265.,260.,240.,205.,185.,184.,200.,250.,400.,570.,730.,
925.,1080.,1180.,1255.,1320.,1390.,1460.,1600.,
LWGLE = 1.
WINDY=3.,3.,9.,9.,19.,25.,30.,27.,21.,15.,12.,14.,4.,4.,16.,34.,42.,
42.,40.,15.,10.,20.,60.,40.,20.,9*0.,
WANGLE=-45.,-45.,6*0.,22.,45.,200.,180.,180.,7*0.,180.,13*0.,
&END
&NAM4
THETKD = 35.,
V1 = 0.25,  V2 = 0.6,
OM1 = 0.005,  OM2 = 0.1,
NOMI = 30,  NVPI = 30,
MAXMOD = 8,
&END
&NAM6  ZSCRC = 3.0,  ZOB = 0.0  &END
&NAM8  YIELD = 10.E3  &END
&NAM10  ROBS = 5600.,
TFIRST = 16.E3,  TEND = 21.E3,
DELYT = 15.,
IOPT = 11,
&END

```

Figure 3-12. A listing of the input data used in an effort to match the microbarogram recorded at Berkeley, California, following a blast at Johnson Island on 30 October, 1962. The synthesized waveform is shown in Figure 3-10 and is compared with the empirical record in Figure 4-23.

The value of YIELD would vary for successive problems. It should be noted that the above would save considerable computer time when compared with the option of taking NSTART=1.

To illustrate some of the points discussed above, a listing of a sample input deck is given in Fig. 3-12. The resulting output should include that shown in Figs. 3-1 through 3-10.

3.6 FURTHER DESCRIPTION OF THE OUTPUT

In Figs. 3-1 through 3-10 we show a selected portion of the output generated by the program with the input deck listed in Fig. 3-12. This output is representative of what might be obtained during normal usage of the program.

Figure 3-1 gives the computer printout of the basic model atmosphere used in the calculation as derived from the input data. Its format should be self-explanatory.

Figure 3-3 gives the initial table or pictorial display of the normal mode dispersion function sign at points in the phase velocity versus angular frequency plane. The +'s and -'s denote the sign, while the X's imply that the upper boundary condition could not be satisfied. The rows correspond to different values of the phase velocity VPHSE. These values in km/sec are listed in the first column. The columns correspond to different angular frequencies OMEGA. The ω values corresponding to the rows (from left to right) are listed below the figure in the order in which one would read a book (left to right, then down a row. The sequence 1234 etc. of numbers directly below the figure is intended merely to facilitate counting. The number given for the phase velocity direction should be the same as the input value of THETKD.

Figure 3-4 represents an expanded version of the display given in Fig. 3-3. This is the result of the expansion process to fully resolve the modes which was described in Sec. 2.7. Since the rows and columns are now unequally spaced, the apparent graphs of the dispersion curves are not in a uniform scale.

Figure 3-5 gives a portion of the tabulation of the dispersion curves for the modes found during the search process. Note that only the segment of a mode which lies within the search region is tabulated. It also should be noted that the increments in OMEGA (in rad/sec) and VPHSE (km/sec) are not uniform. This is an attribute of the computational process which was selected in order to obtain good resolution of both nearly horizontal and nearly vertical segments of the dispersion curves.

Figs. 3-6, 3-7 and 3-8 each give the same information as in 3-5 plus tabulations of quantities which may be of interest to the user and which vary along the dispersion curves. It may be questioned whether the printout in Fig. 3-5 is necessary, but we decided on this superfluous output because of the fact that the information in Fig. 3-5 has a wider applicability than that in Figs. 3-7 and 3-8. Also, the user might wish to display the dispersion curve tabulations alone without having to explain away the presence of other data.

As in the case of the dispersion curves, the data listed in Fig. 3-6 is a function of the model atmosphere only. PHI1 and PHI2 are the ϕ_1 and ϕ_2 , respectively, introduced in Section 2.6. The values of these "potentials" are calculated at the ground and at the top of each finite layer in the model atmosphere, and these values are used in later calculations. Since the profiles of these functions might give the user some insight into the physical significance of the various modes in the computation, Fig. 3-6 shows a tabulation of profile parameters, as defined in that printout, for each point on the dispersion curves previously tabulated.

The factor AMP , which is tabulated in Figure 3-7, is defined by Eq. (2.5.4d) and depends on heights of burst and observer but is independent of yield. Also shown in this figure are the parameters FACT and $\text{ALAM} (\lambda_0)$ which are defined in Sec. 2.5.

Figure 3-8 shows the form of the tabulation of AMPLTD and PHASE , which are defined in Sec. 2.5 and depend upon the yield of the source in addition to the atmospheric model and the heights of burst and observer.

Fig. 3-9 gives a portion of the printout of the total and modal acoustic pressures calculated in consequence of the input data shown in Fig. 3-12, while Fig. 3-10 is a reduction of the corresponding CALCOMP plot. In both the tabulation and the plot, pressures are given in microbars and time in seconds after the blast. On the plot, the modes are drawn in ascending order beginning at the top, and their total is at the bottom. The common pressure scale is determined automatically such that the maximum amplitude of the total waveform will be about 2 inches on the plot. Note that these formats for the tabulation and plot are consequences of having set $\text{IOPT} = 11$ in the namelist NAM10 . A description of other possible output formats for predicted acoustic response may be found in the last paragraph of Section 3-4.

For each input case (i.e., for each NAM10 read), the code will print all input data and will generate a tabulation and plot

of a waveform; however, whether the outputs shown in Figs. 3-1 and 3-3 through 3-8 are printed will depend upon the current values of NPRNT and NSTART. If NPRNT is less than 1, none of them will be printed; otherwise, all are printed which correspond to points in the calculation past the point of entry specified by NSTART. For example, suppose that NPRNT = 1 and NSTART = 4. The first calculations made for this case are those involving the source strength (YIELD), so that the only printouts will be the input data, a tabulation of the type shown in Fig. 3-8, and a tabulation of responses (as determined by the value of IOPT).

Chapter IV

SOME NUMERICAL STUDIES

4.1 INTRODUCTION

In this chapter, we present some numerical studies which have been made during the past year using the computer program INFRA-SONIC WAVEFORMS, which we have described in the preceding two chapters. These studies were concerned with checking out the program, comparing its predictions with previous calculations by Harkrider (1964), and in exploring some general trends. These studies are relatively modest and only scratch the surface.

In these studies we refer to the individual modes using a nomenclature devised by Press and Harkrider (1962). For convenience of reference, we review this nomenclature here. In any plot of numerically obtained dispersion curves, i.e., of phase velocity versus frequency, the modal curves fall into two clearly defined groups - regardless of the value of θ_k . A sample plot is shown in Fig. 4-1. The identification GR_0, GR_1, GR_2 , etc. for the so-called "gravity modes" and S_0, S_1, S_2 , etc. for the so-called "sound modes" should be evident from the figure. In labeling these modes the first step is always to identify GR_0 and S_0 . These are two adjacent modes which are widely separated at low frequencies, GR_0 having a low frequency phase velocity of the order of the sound speed at the ground and S_0 having one which is considerably higher, of the order of the largest sound speed in the atmospheric profile. The two modes invariably become very close at a frequency of the order of a representative Brunt frequency in the lower atmosphere. However, the two modes do not cross. (The probable reason for this absence of an intersection is explained in Sec. 2.7).

Once GR_0 and S_0 are identified, the remaining modes are labeled in the order in which they appear. Thus S_1, S_2, S_3 , etc., are the modes corresponding to curves which would be encountered by one scanning upwards and to the right starting from S_0 , while GR_1, GR_2, GR_3 , etc., are the modes encountered by one scanning downwards and to the left from GR_0 .

4.2 A COMPARISON WITH HARKRIDER'S RESULTS

Since the program is capable of synthesizing waveforms when the model atmosphere is without winds, it should in principle be

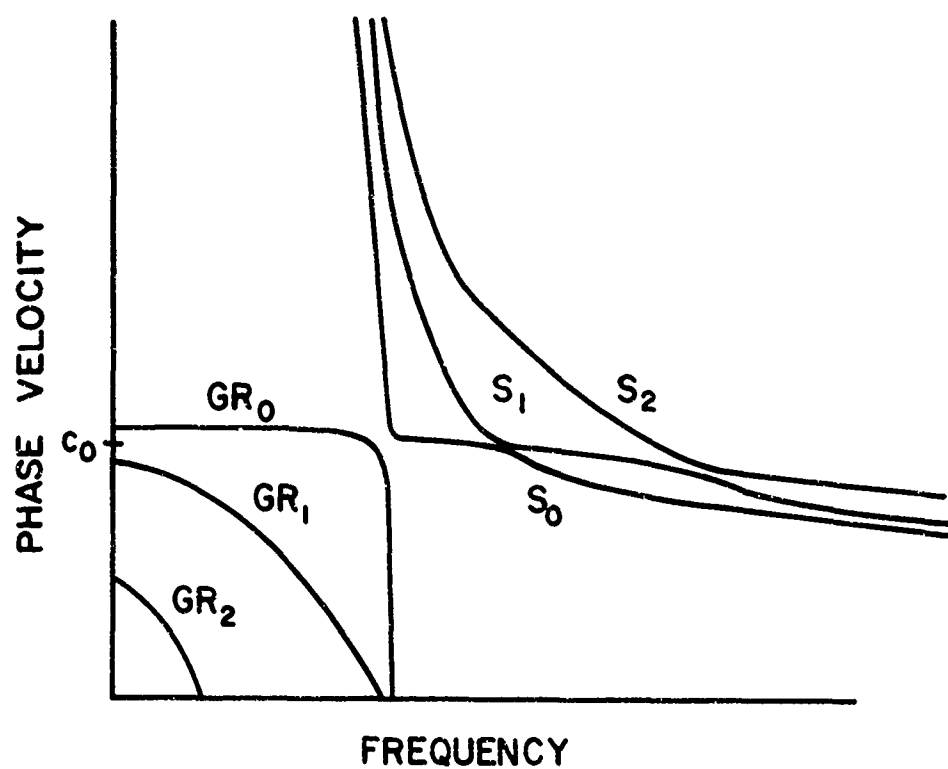


Figure 4-1. Sketch showing the labeling scheme used in this report for the acoustic-gravity modes.

capable of reproducing computations carried out by Harkrider (1964). Furthermore, any comparison of our calculations with Harkrider's should serve as a means of discovering any major coding errors in the program. We were therefore considerably disappointed when we first made such a comparison and discovered substantial discrepancies. Two fruitless months were spent in checking and rechecking the program and the theory before we finally discovered that the discrepancy was due primarily to differences in formulation. Such differences in the formulation evolve around how one incorporates a model of a nuclear explosion into the theory.

To explain this difference, we discuss below some of the differences between the mathematical expressions used by Harkrider and those presented in Chapter II. In order to avoid a lengthy review of the Harkrider theory, we use his nomenclature below. For brevity, we do not define all the symbols used, as those not defined here are defined in Harkrider's paper.

The Harkrider theory gives the pressure waveform due to any given individual mode as being of the form

$$p = \{B\} e^{-\lambda_s a_s} \rho_s^*(D) \{I_1 + I_2\} \quad (4.2.1)$$

where

$$I_1 = 2 \int_0^{\sigma_1} \{L\} \{A_A\} \{M\} \{E\} \cos[\omega(t - \tau_A)] d\omega \quad (4.2.2a)$$

$$I_2 = 2 \int_{\sigma_1}^{\infty} \{L\} \{A_A\} \{M\} \cos\{\omega[t - (\tau_A + \tau_X)]\} d\omega \quad (4.2.2b)$$

$$\{B\} = (2/\pi)^{1/2} (a_e \sin \theta)^{-1/2} a_s p_{as} \quad (4.2.2c)$$

$$\{L\} = [p_s(D)/p_o]_{H_j} \quad (4.2.2d)$$

$$\{M\} = (b_s^2 + \omega^2)^{-1} k_j^{1/2} \quad (4.2.2e)$$

$$\{E\} = \exp\{(a_s/\alpha_s)(\sigma_1^2 - \omega^2)^{1/2}\} \quad (4.2.2f)$$

The quantity σ_1 is $(\gamma/2)(g/\alpha_s)$ at the burst altitude. Note that the subscript s refers to the source and that α is the sound speed at the source. The quantity a_e is the radius of the earth, while the quantity a_s is a scaling length which increases with yield Y as $Y^{1/3}$; λ_s is σ_1/α_s ; D is source altitude.

The formula corresponding to Eq. (4.2.1) according to the formulation presented in Chapter II is

$$p = \{B\}\{1/\rho_s^\circ(D)\}\{\hat{I}\}p_s^\circ/p_o^\circ \quad (4.2.3)$$

where

$$\hat{I} = 2 \int \{K\}\{AMP\}\{M\}\cos[\omega(t - \tau_A + t_{as})]d\omega \quad (4.2.4a)$$

$$\{K\} = -(\alpha_s^2/\alpha_o)[\rho^\circ(z)\rho_s^\circ(D)]^{1/2} \quad (4.2.4b)$$

The quantity $\{AMP\}$ is as defined in Eq. (2.5.4d).

With some minor discrepancies, it would appear from a comparison of the two derivations that

$$\{K\}\{AMP\} \approx \{L\}\{A_A\} \quad (4.2.5)$$

What discrepancies do appear would be due to the fact that we use an energy source model rather than a mass source model. To check whether or not this is the case and as a check on the program, we compared our $\{AMP\}$ with Harkrider's A_A when source and observer are on the ground (i.e., $z = 0$ and $D = 0$). In this case we should have

$$A_A = -\rho_o^\circ \alpha_o \{AMP\}$$

In Fig. 4-2, we show a plot of $\{AMP\}$ in km^{-1} vs. period in minutes for this case for the U.S. Standard atmosphere with no winds. This should be compared with Fig. 7 in Harkrider's 1964 paper. Although the units are not the same, the general shapes of the curves are remarkably similar. To check on the quantitative agreement, we took $\alpha_o = 1/3 \text{ km/sec}$ and $\rho_o = 12.6 \times 10^{-4} \text{ gm/cm}^3$. The maximum value of $-\rho_o^\circ \alpha_o \{AMP\}$ for the GR_0 mode is then found from Fig. 4-2 to be $.0126 \times 10^{-3} \text{ (gm/cm}^3\text{)/sec}$. The corresponding number in Harkrider's graph (as best we can read it) is .013. Since Harkrider does not specify the units on this graph, we checked with him concerning this and found that 1 unit on the

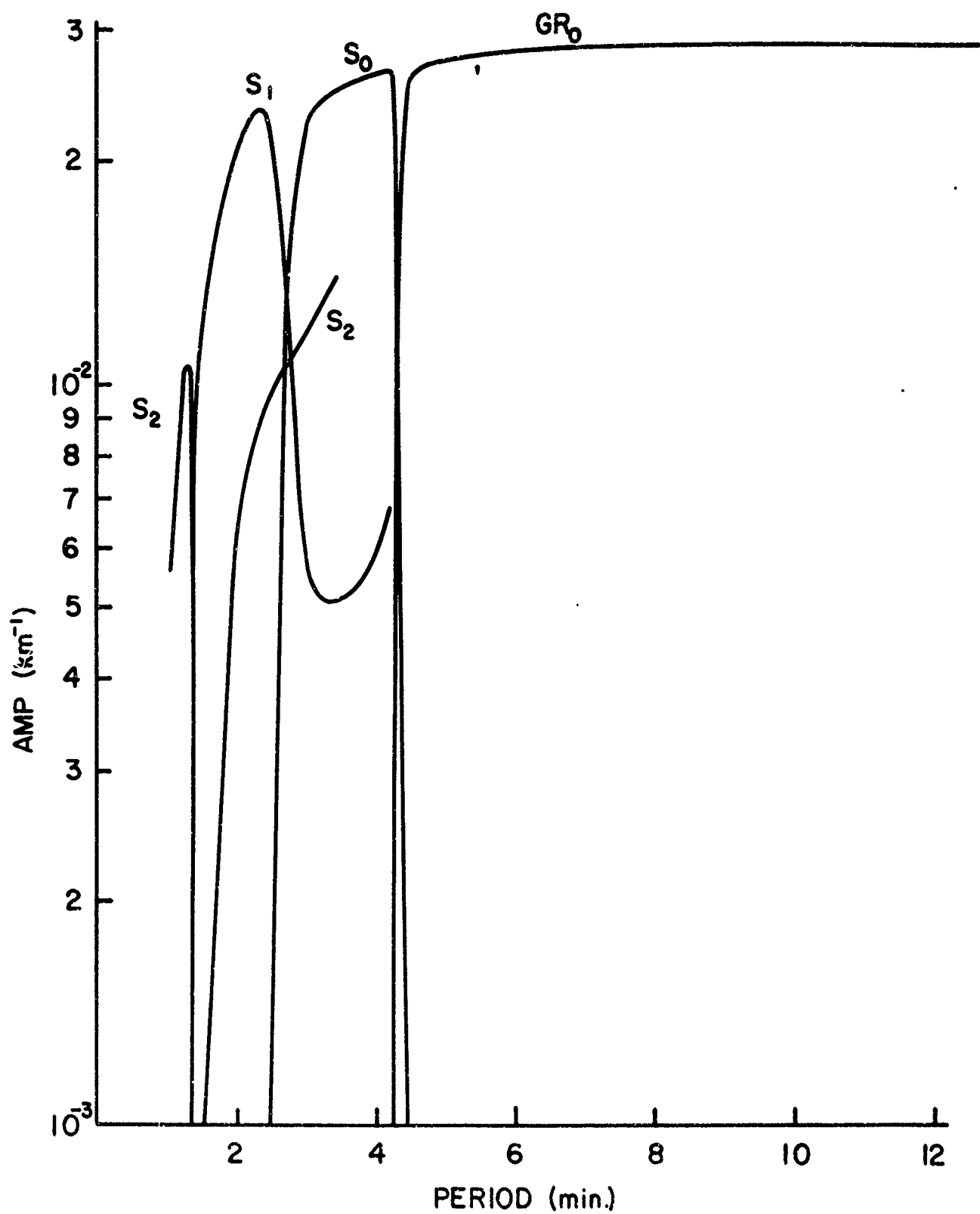


Figure 4-2. Plot of AMP vs. period for modes GR₀, S₀, S₁, and S₂. AMP is defined in Eq. (2.5.4d).

graph corresponds to 10^{-3} (gm/cm³)/sec. Thus the agreement would appear to be substantial.

The analysis above still leaves several additional formal differences which may or may not be of some importance, especially for megaton class explosions. We enumerate these explicitly below:

- 1) The factor $e^{-\lambda_s a_s}$ in Eq. (4.2.1) does not appear in Eq. (4.2.3).
- 2) The factor $\{E\}$ in Eq. (4.2.2a) for $\omega < \sigma_1$ does not appear in our Eq. (4.2.4a).
- 3) The quantity τ_χ in Eq. (4.2.4b) for $\omega > \sigma_1$ does not appear in our Eq. (4.2.4a).
- 4) The τ_A in Eqs. (4.2.2a) and (4.2.2b) is replaced by $\tau_A - \tau_{as}$ in Eq. (4.2.4a).

Each of these differences may be traced to the methods in which the source model was incorporated in the theory. In Harkrider's theory, he matched his formal solution to the cube root scaled waveform (extrapolated from 1 KT) which would be received at a distance a_s directly below the source, the distance a_s varying with cube root scaling. In the theory in Chapter II, the source was taken as a point source with a time dependence chosen such that the calculation would agree with low yield explosion data were the atmosphere homogeneous. It is difficult to say with certainty just which formulation is the more nearly correct. However, one consequence of Harkrider's method is an effective attenuation of high frequencies as yield is increased - much more so than is indicated by the available data. (This would not have been the case were the reference point at the same altitude as the source.)

We consider the fourth distinction to be of no consequence as it only changes the time origin without altering the shape of the waveform. The other three should be relatively minor for low yields but may lead to large discrepancies for megaton class explosions.

To check the assumption that the first three distinctions listed above are responsible for any major numerical discrepancies between the results computed using INFRASONIC WAVEFORMS and those given by Harkrider, we attempted to reproduce the theoretical barograms in Harkrider's Fig. 13 for the direct wave as observed at 8000 km from a 4 MT explosion at a burst height of 2.13 km. We did this first using our program with no alterations and then

modifying (temporarily) the program to include the factors 1-3 discussed above. The results are shown in Figs. 4-3 through 4-8.

Each figure shows the graphs for a given mode (or the total response) as determined by three different methods. The top graph in each figure was calculated by the unaltered Pierce-Posey code. The second graph was calculated by the Pierce-Posey code with the factors $\exp[-\lambda a]$ and E and the phase shift τ_y included as noted above. The bottom curve is the corresponding graph from Harkrider's Fig. 13.

If we compare the unaltered modes with Harkrider's we note that the mode shapes are quite similar. However, three significant differences do exist:

- 1) Our GR_0 dies off more slowly than does Harkrider's.
- 2) Our acoustic modes are much stronger relative to GR_0 than are Harkrider's.
- 3) All of our acoustic modes arrive approximately 3 minutes later relative to GR_0 than do Harkrider's.

In our altered calculations, GR_0 dies out more rapidly, the acoustic modes are weaker relative to GR_0 than before, and the acoustic modes arrive slightly earlier relative to GR_0 . This clearly indicates that the factors and phase shift considered are the major sources of differences between Harkrider's synthesized waveform and ours. Our altered total response is almost identical with his up to about 26900 sec., where our S_3 begins to dominate the sum. Since S_3 is not included in Harkrider's sum, agreement could not be expected in this region.

The fact that the altered waveforms for each mode have amplitudes of about 2/3 those reported in Harkrider's figure may be attributed in part to the absence of the factor p^o/p^o in Harkrider's original formulation. However, we understand that this has been corrected in the version of his program currently in operation. This would lower Harkrider's amplitudes by a factor of .76 and would bring the two sets of computations to a fair agreement. We are not sure of the cause of the remaining discrepancy but think it might be due to either our use of an energy source rather than a mass source or to a different choice for ambient density at the ground. The similarity in shape of the two waveforms suggests that the major cause of the discrepancy has been amply accounted for.

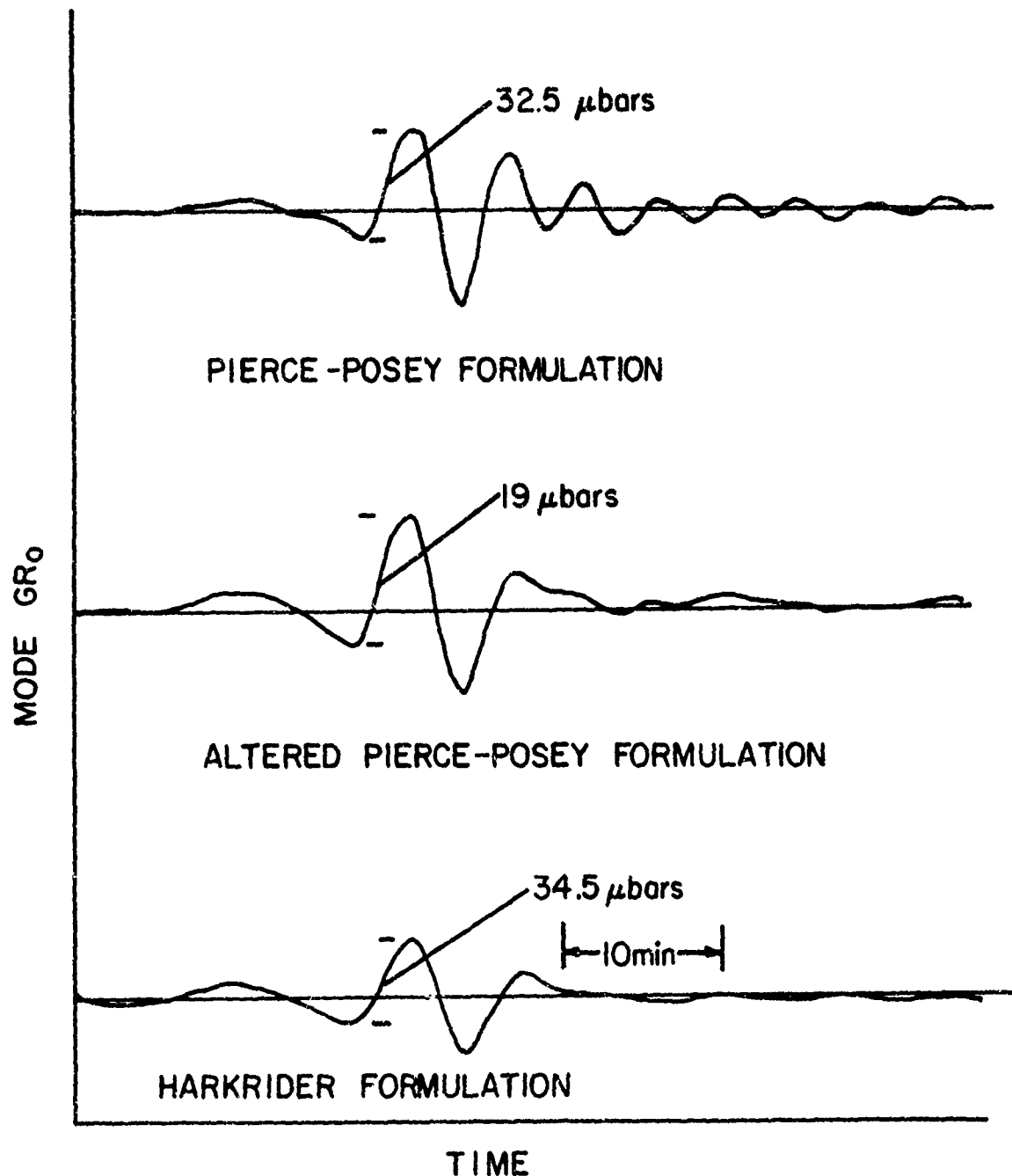


Figure 4-3. Comparison of mode GR_0 as computed by Pierce and Posey and by Harkrider. Here and in Figs. 4-4 through 4-8, no two curves are necessarily on the same pressure scale, but all use a common time scale. The value of a representative trough-to-peak pressure variation is given for each curve.

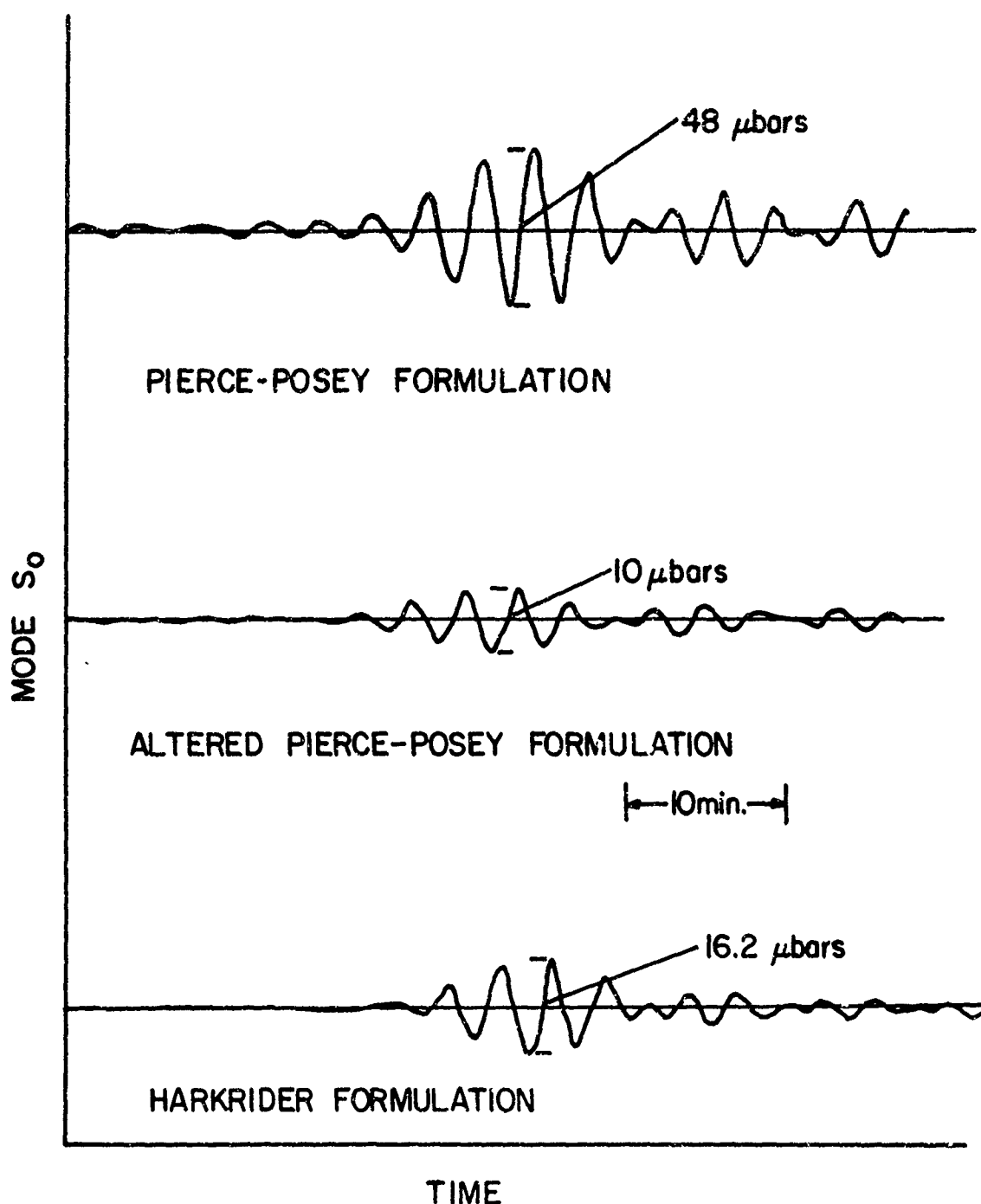


Figure 4-4. Comparison of mode S_0 as computed by Pierce and Posey and by Harkrider.

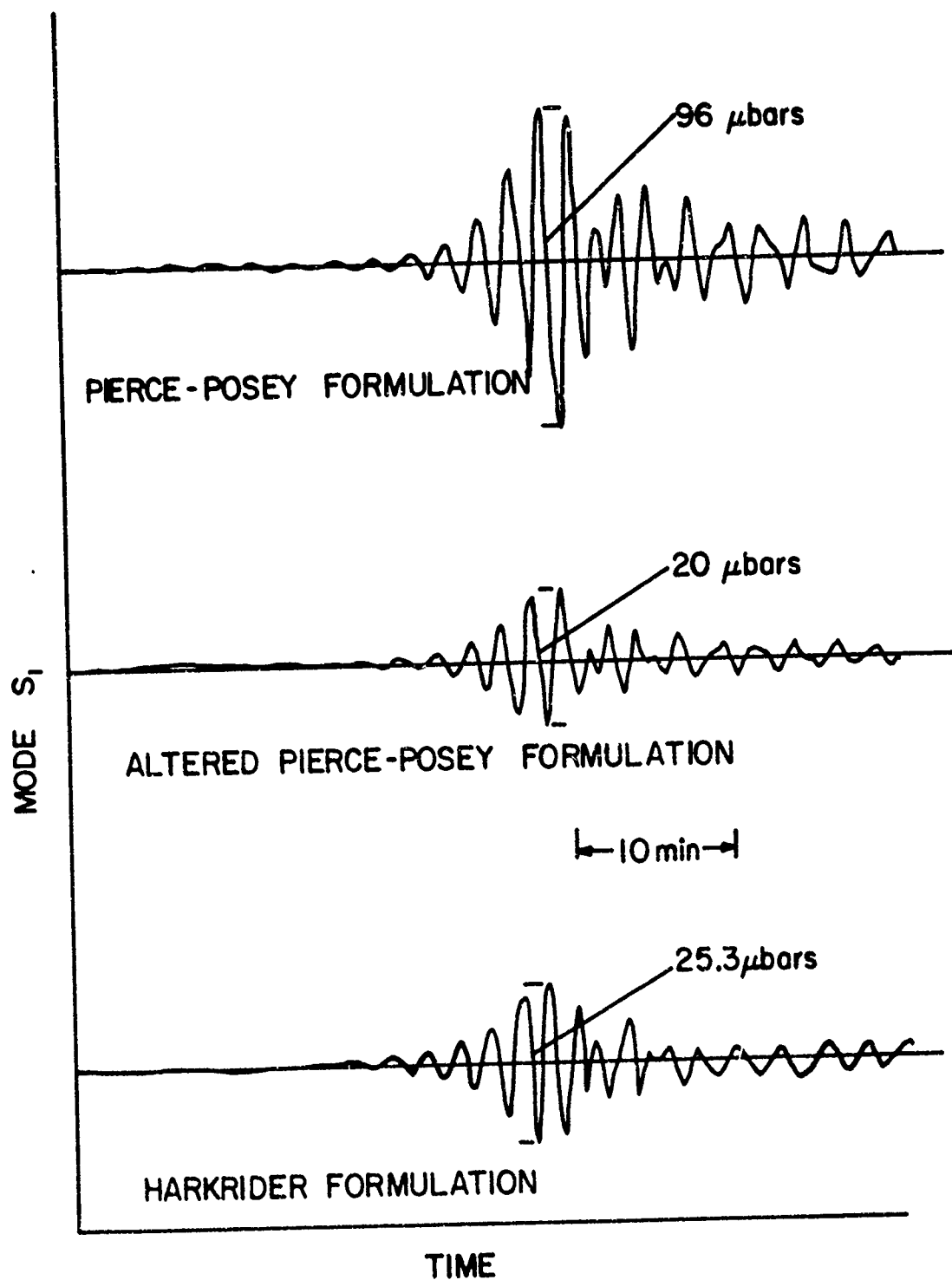


Figure 4-5. Comparison of mode S_1 as computed by Pierce and Posey and by Harkrider.

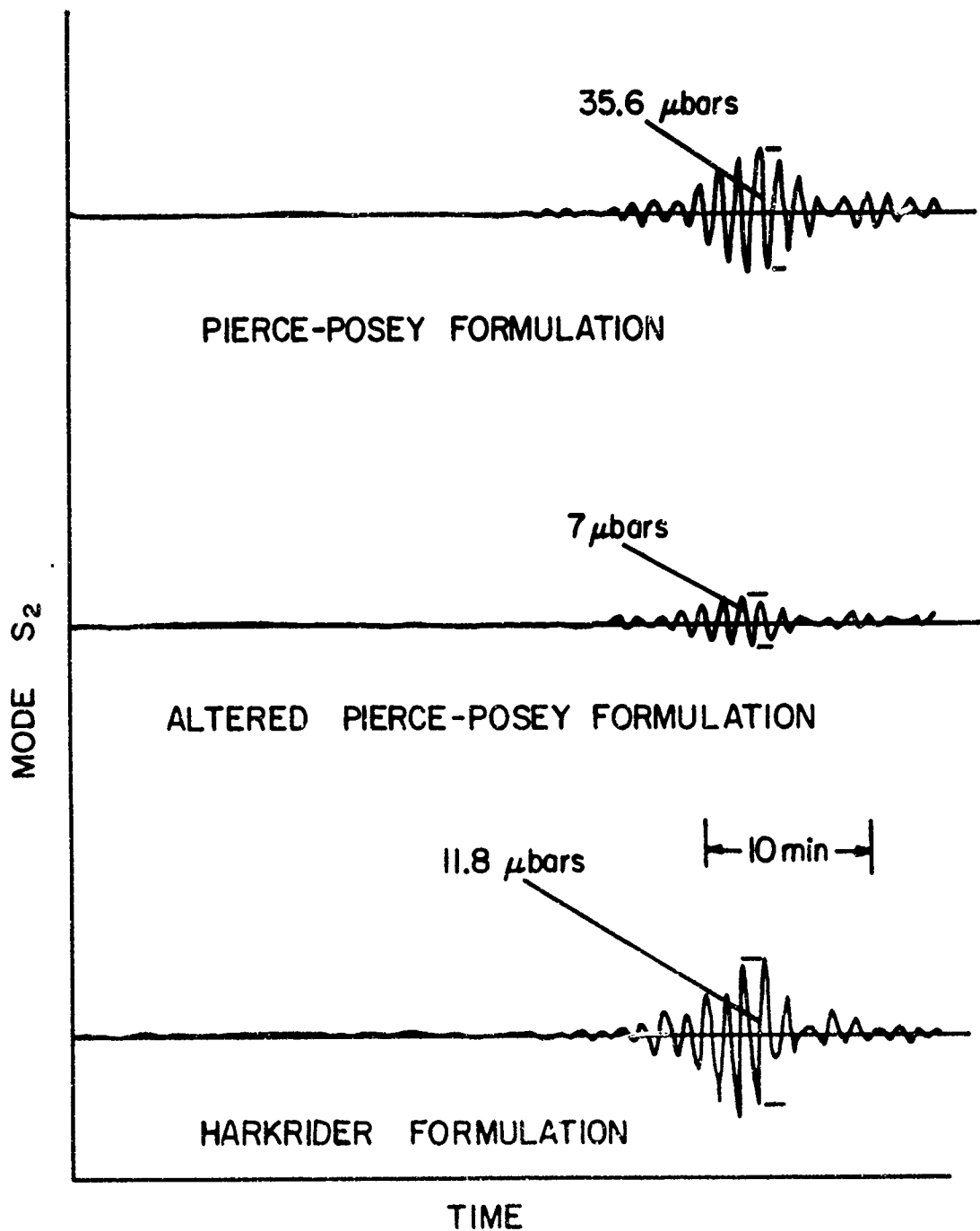


Figure 4-6. Comparison of mode S_2 as computed by Pierce and Posey and by Harkrider.

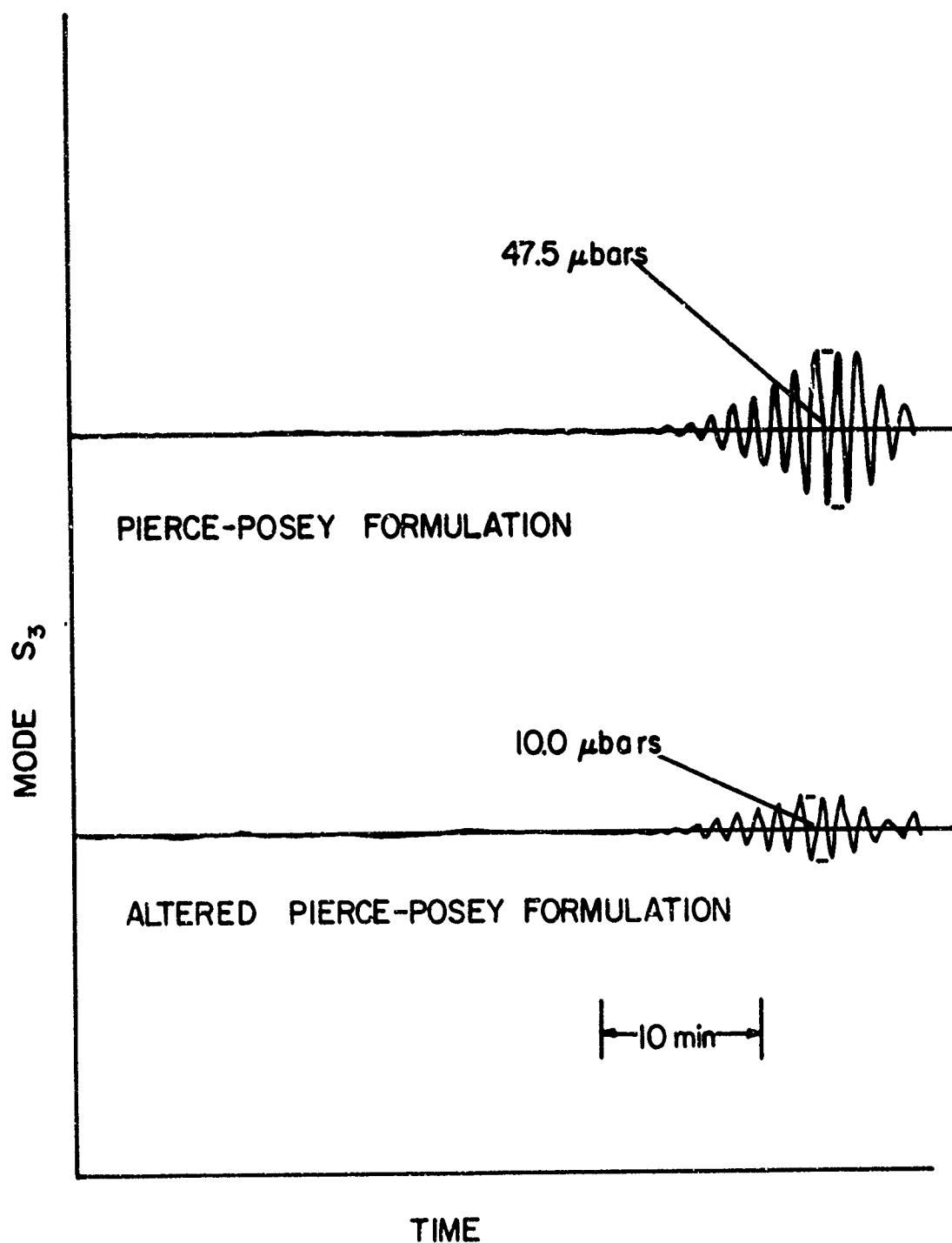


Figure 4-7. Mode S_3 as computed by Pierce and Posey.

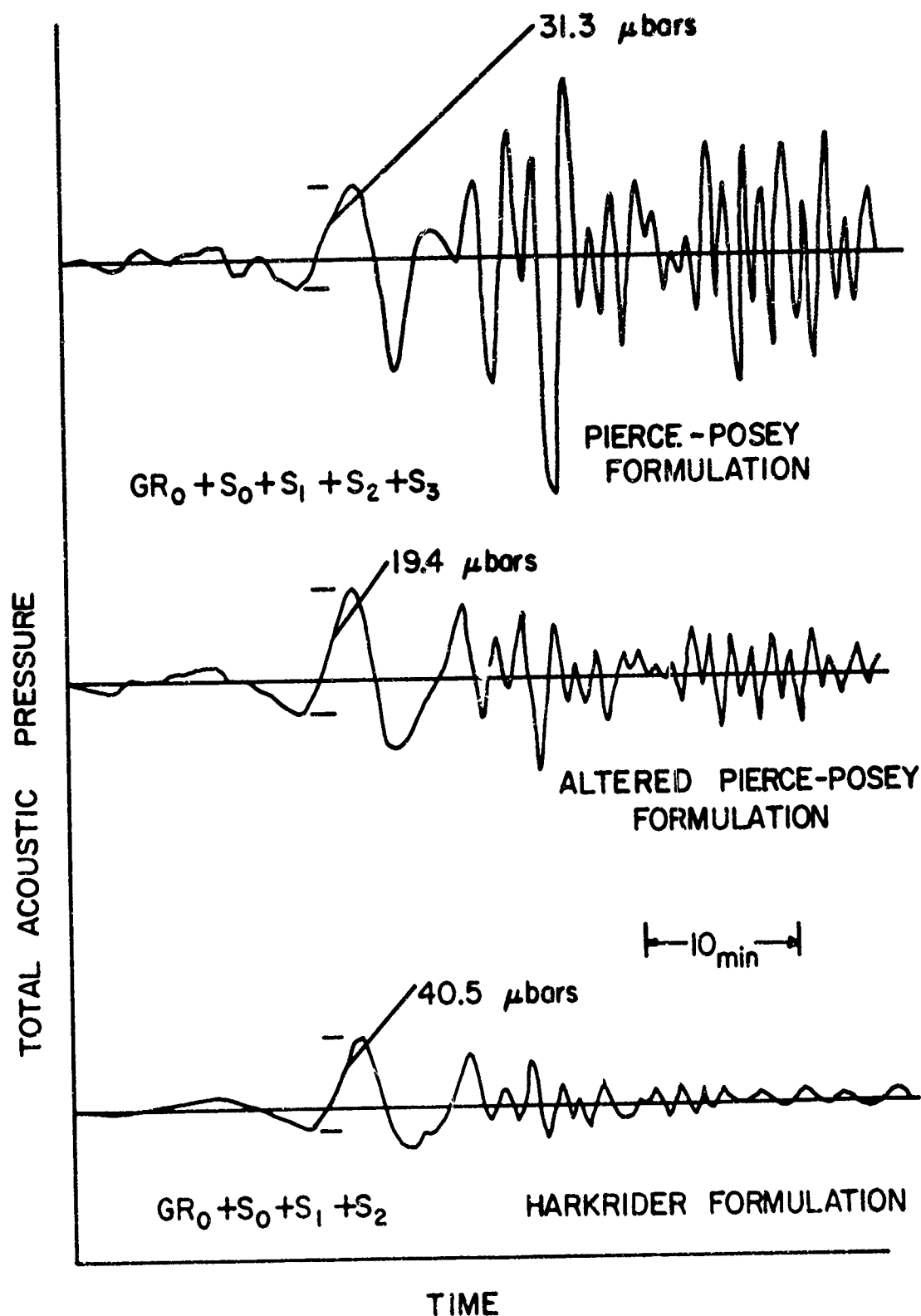


Figure 4-8. Total acoustic pressure as computed by Pierce and Posey's INFRASONIC WAVEFORMS, by the same code modified as indicated in Sec. 4.2, and by Harkrider's code.

4.3 GENERAL TRENDS

The computer code INFRASONIC WAVEFORMS has been utilized to study the effects of the various parameters of the source (yield, height of burst) and of the atmospheric model (temperature and wind profiles, upper boundary condition) upon theoretical microbarograms. For the sake of simplicity and economy, models of only four to seven finite layers plus the upper half-space were used in this initial study. The temperature profile shown in Fig. 4-9 with no winds was chosen as a standard for the purposes of comparison. Notice that this profile exhibits certain features of the ARDC standard atmosphere: there are two sound channels, one centered at 25 km and one at 85 km, with the model's minimum temperature in the upper channel. Most runs were made using a yield of 10 MT, height of burst of 3 km and a range of 2000 km. The synthesized microbarogram for the standard conditions is given in Fig. 4-10 together with graphs of the modes summed to arrive at the total response.

The Upper Boundary Condition

As the altitude increases, the composition and density of the atmosphere changes considerably. As the composition changes, the application of the perfect gas law becomes less appropriate, and as the medium becomes increasingly rarified, the equations of hydrodynamics lose their applicability. However, under the assumption that practically all of the energy of a given disturbance is below 100 km, it would seem that the details of the atmospheric structure above the ionosphere should have little effect upon the waveform observed on the ground. This hypothesis was confirmed by a series of runs in which the atmosphere below 110 km was held constant, while the temperature profile above this height was varied. While dominant frequencies and amplitudes were, in general, unaffected, the details of the individual modal waveforms did vary, and two definite trends were observed.

If only the temperature in the upper half-space (T_{∞}) is varied, one sees that the two extremes, T small ("almost" a free boundary) and T large ("almost" a rigid boundary) produce microbarograms which look very similar (Fig. 4-11). But, if one compares the tables of the normal mode dispersion function signs for the two cases (Fig. 4-12), it is clear that in the process of going from one extreme to the other, the dispersion curves have shifted, since the sign of the normal mode dispersion function at any given point in the frequency-phase velocity plane has been reversed. Examination of intermediate cases reveals that the shift has been upward for increasing T_{∞} ; i.e., the dispersion curve normally associated with the GR_0 mode in the free case moves upward and assumes the shape and position of the curve normally associated

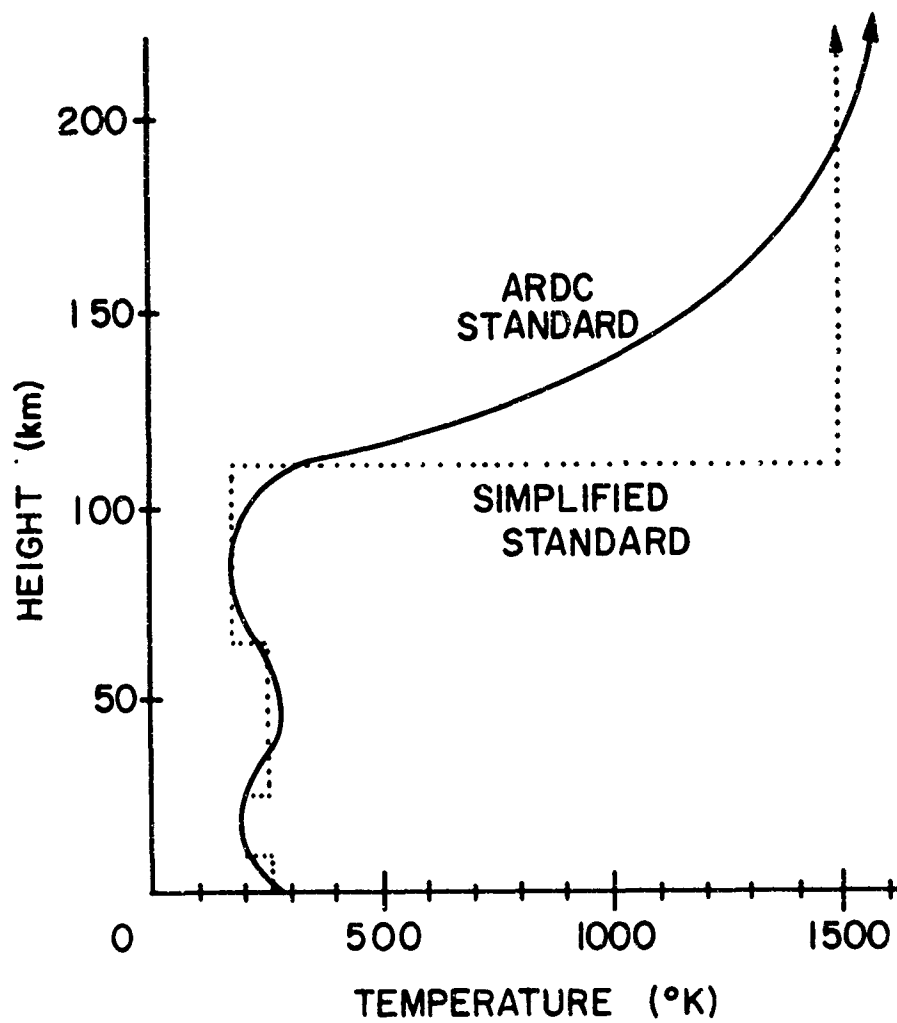


Figure 4-9. Temperature profiles of the ARDC standard atmosphere and of the standard simplified atmosphere used in Section 4.3.

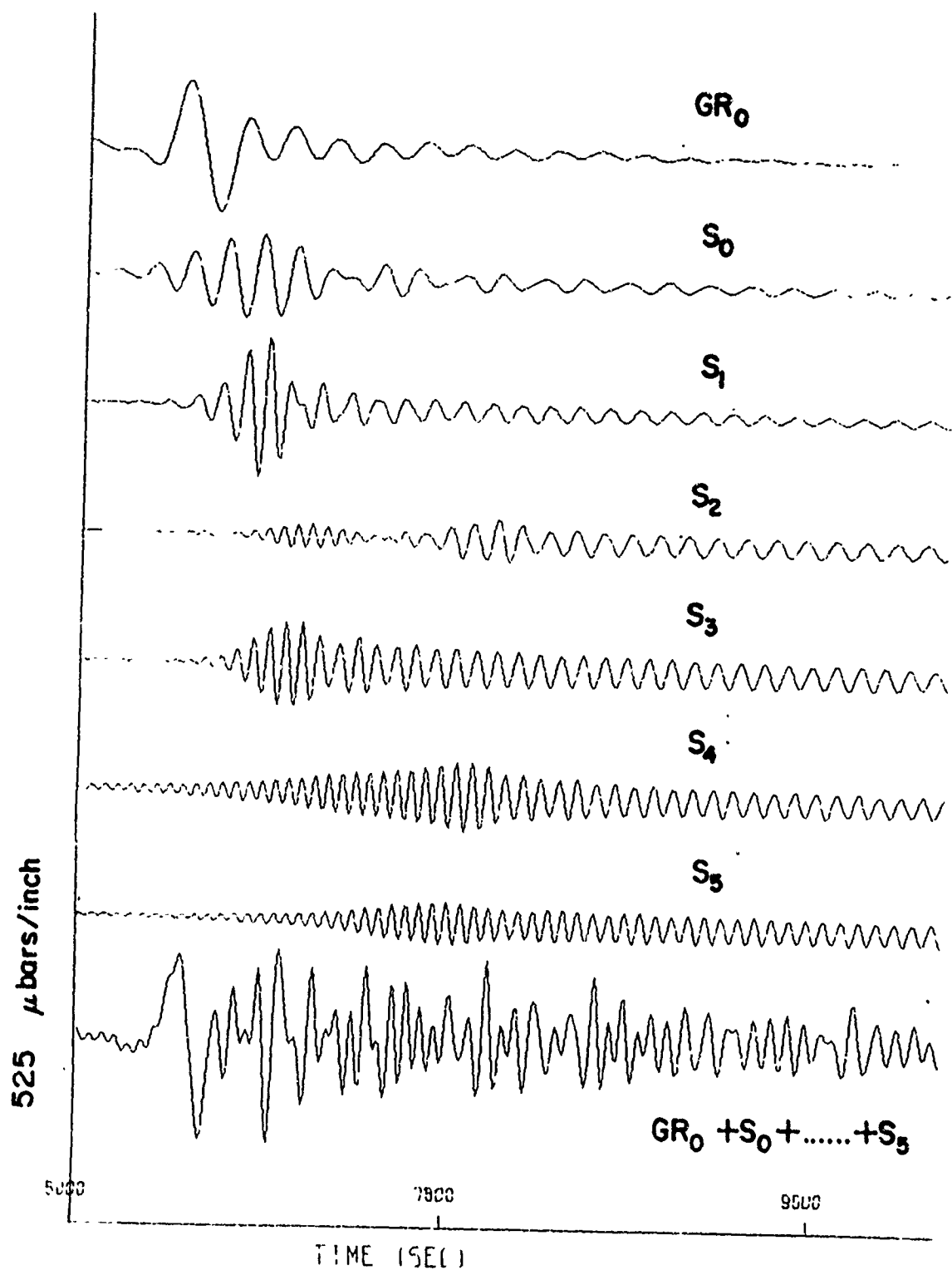


Figure 4-10. Synthesized microbarogram and modal contributions for an observer on the ground 2000 km from a 10 MT explosion 3 km above the ground in the simplified model atmosphere of Fig. 4-9.

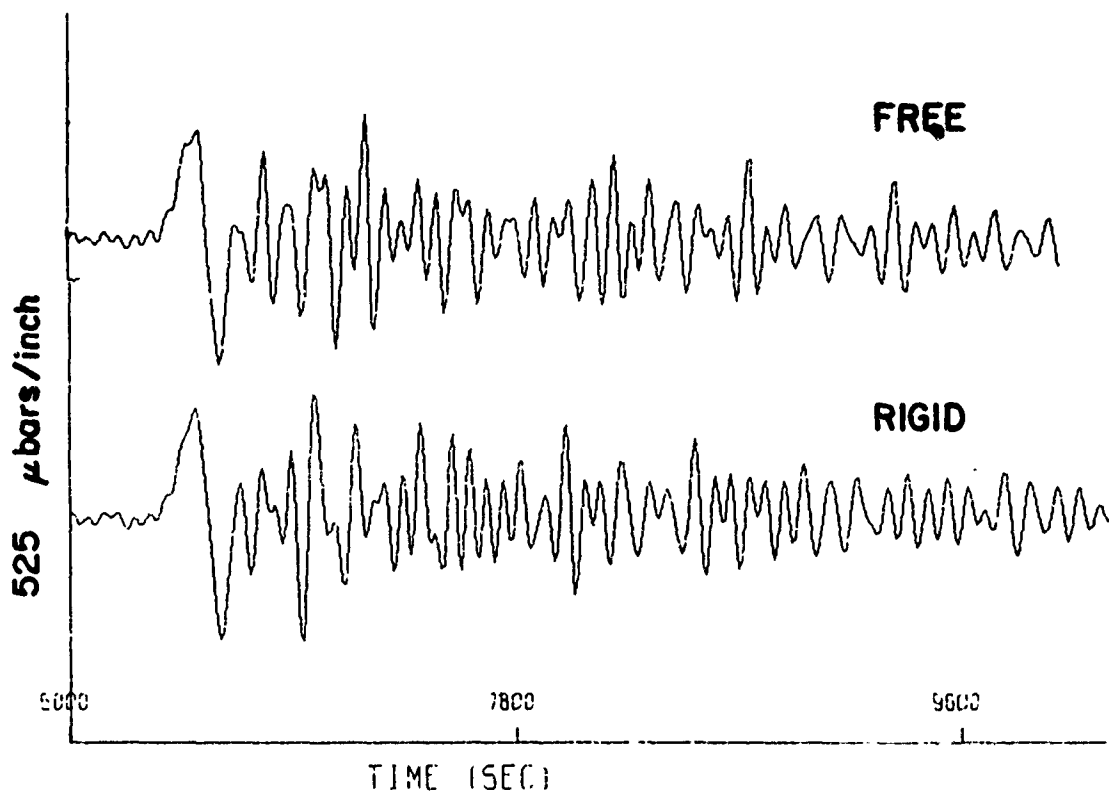


Figure 4-11. Synthesized microbarograms corresponding to free and rigid upper boundary conditions.

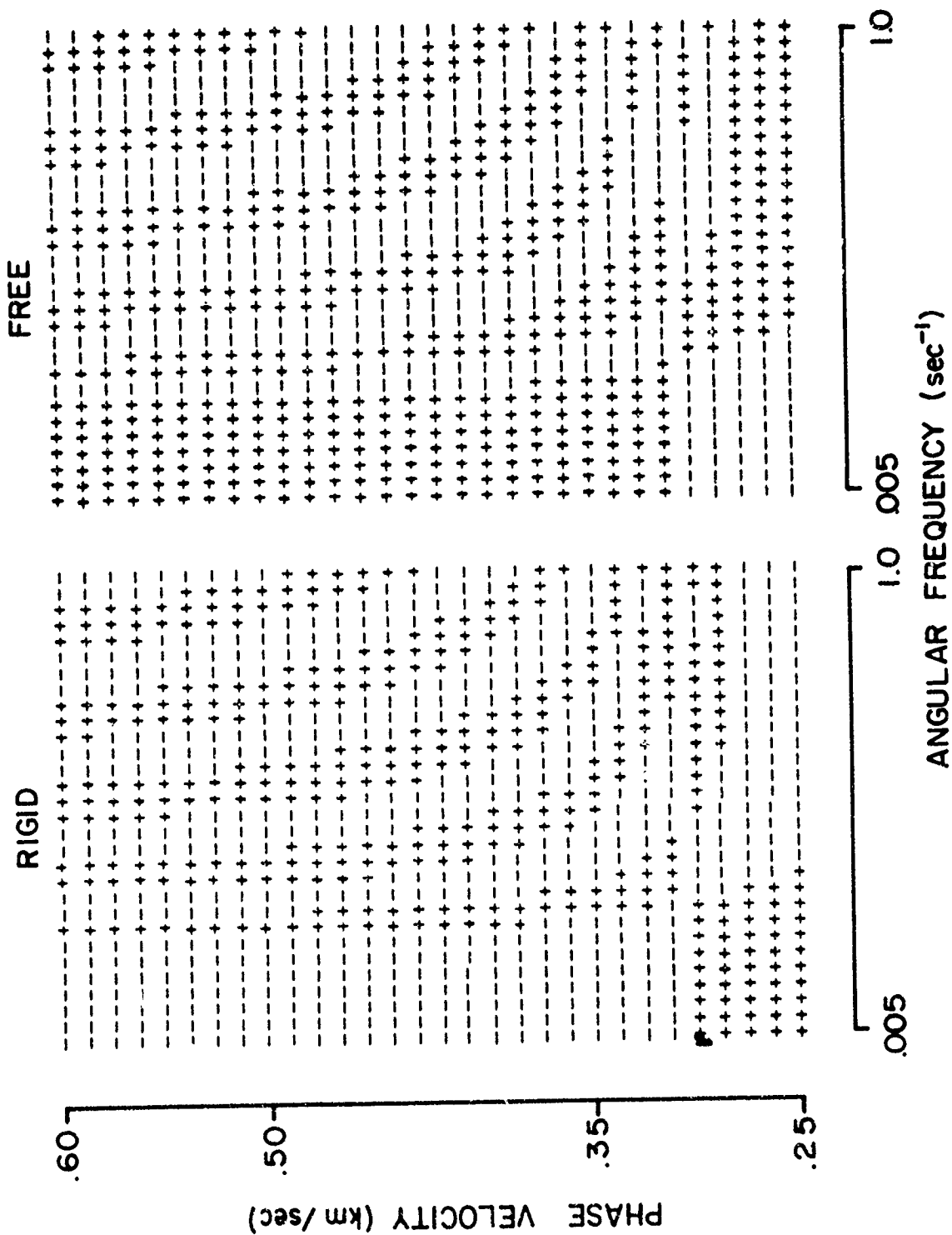


Figure 4-12. Tables of normal mode dispersion function signs for the cases of free and rigid upper boundary conditions.

with the S_0 mode. For an intermediate case, for which the modes are in the midst of their transitions, the modal disturbances bear little resemblance to those for the extremes, yet their total (Fig. 4-13) does resemble that for the extremes, at least for the first half hour. The suggestion made here is that because the dispersion curves are strongly dependent upon T_∞ , but the waveform is not, the modes which arise are primarily mathematical conveniences with limited physical significance. This contradicts the viewpoint prevalent in much of the current literature.

Secondly, as soon as the temperature in any layer is of the order of three times any relative maximum for the atmosphere below that layer, a rigid boundary condition at the bottom of that layer is approximated. For example, the two waveforms presented in Fig. 4-14 show almost negligible difference, although they were produced by two different models, one with $T_\infty = 800^\circ \text{K}$ beginning at 130 km and the other with temperatures of 800°K from 130 to 150 km, 1000°K from 150 to 200 km, and 1500°K above 200 km. It is clear that the additional layers in the second model had little effect upon the predicted microbarogram.

The Temperature Profile

The computer code being used in this study synthesizes microbarograms by summing the theoretical contributions from guided modes. In general, there are three types of ducting mechanisms which might produce guided modes: (a) Lamb mode ducting, (b) sound channel ducting, and (c) discontinuity ducting. (See Figs. 4-15 through 4-17.) A Lamb mode exists in an isothermal atmosphere due simply to the presence of the ground. Its energy density decays exponentially with altitude. A sound channel exists at any altitude where the sound speed profile has a relative minimum. The third phenomenon which might contribute to ducting is a tendency in some circumstances for wave energy to be concentrated near discontinuities or in the region of large gradients of the sound speed.

Since we are generally concerned with the waveform observed at the ground due to a source near the ground, one would guess that the most important influence would be from Lamb mode ducting, with the effect of a sound channel being to strengthen or weaken the Lamb mode, depending upon its altitude and strength. The only large sound speed gradients in the atmosphere are at great heights. Thus, our earlier consideration of the effect of the upper boundary condition tells us that this mechanism could not significantly contribute to ducting, except that a free or rigid boundary prohibits radiation of energy away from the earth and produces microbarograms (Fig. 4-11) which decay more slowly than those for intermediate cases (Fig. 4-13).

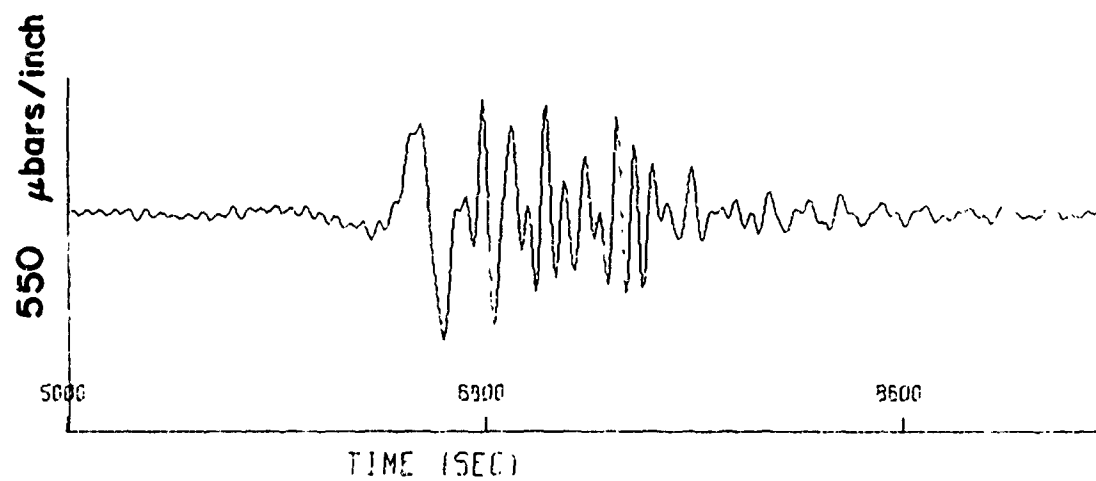


Figure 4-13. Pressure waveform for a case intermediate to the free and rigid upper boundary conditions. Here, $T_{\infty} = 300^{\circ} \text{ K}$.

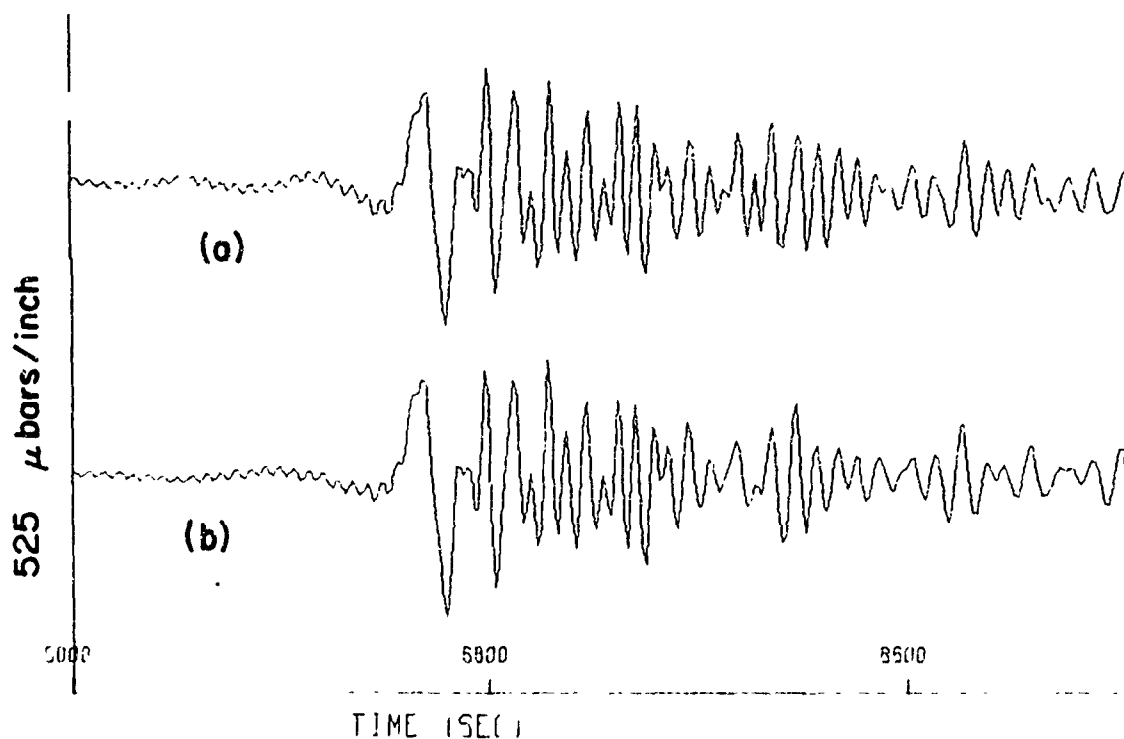


Figure 4-14. Synthesized microbarograms for (a) an atmosphere with $T_{\infty} = 800^{\circ} \text{ K}$ beginning at 130 km and (b) an atmosphere with temperatures of 800° K from 130 to 150 km, 1000° K from 150 to 200 km and 1500° K above 200 km.

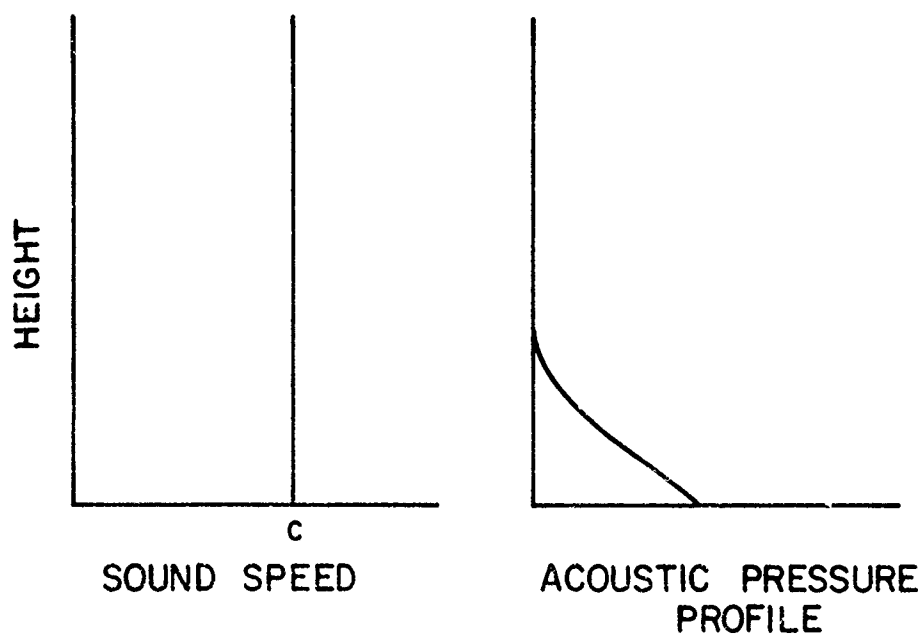


Figure 4-15. Sketch illustrating the mechanism of Lamb mode ducting. In an isothermal atmosphere, the Lamb mode has its maximum pressure at the ground and decays exponentially with height.

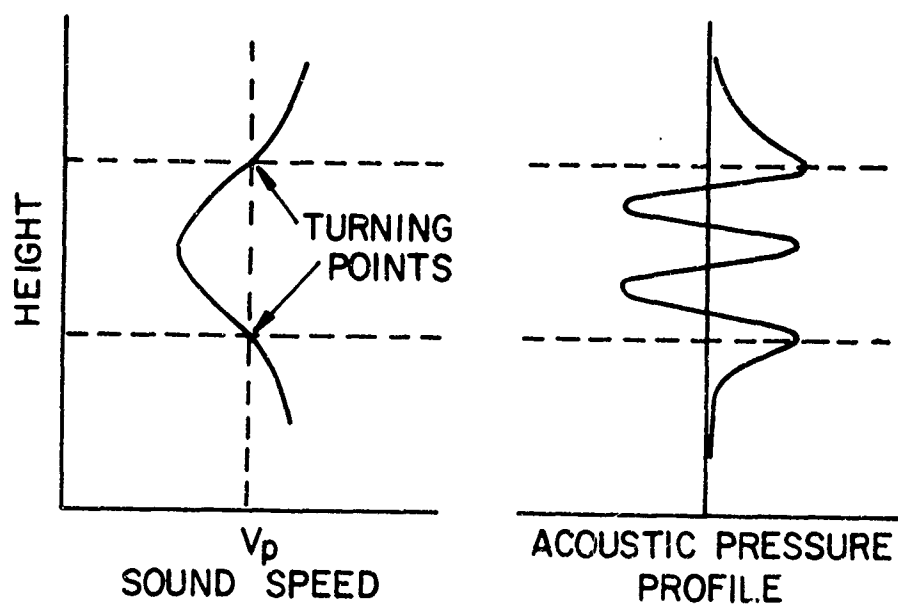


Figure 4-16. Sketch illustrating the mechanism of sound channel ducting. The energy of the disturbance is concentrated in the region of a relative sound speed minimum.

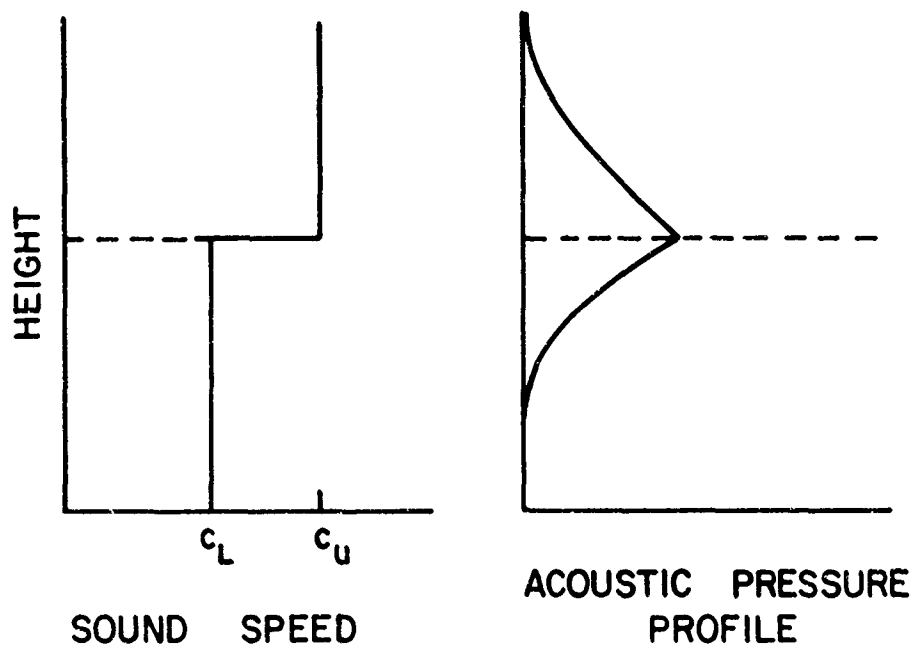


Figure 4-17. Sketch illustrating the phenomenon of discontinuity ducting. The pressure has its maximum value at the discontinuity in sound speed and decays exponentially with distance from it.

The hypothesis that Lamb mode ducting is the principal mechanism of propagation for the acoustic-gravity wave under consideration is discussed in more detail in Chapter VI. Here the effects of the two sound channels in the standard atmosphere are investigated. Since we assume that the atmosphere has constant composition and obeys the perfect gas law, the sound speed is proportional to the square root of the absolute temperature. The sound speed profile of our standard model is shown in Fig. 4-18, along with the sound channel variations studied. Variation 1 eliminates the lower channel, 2 increases the sound speed in the upper channel so that the minimum is in the lower channel, and 3 eliminates the upper channel.

Examination of the microbarograms (Figs. 4-19, 20, 21) corresponding to the three variations reveals a strong dependence upon both sound channels, since all three waveforms are different and none resembles the standard. By looking at the GR_0 , S_0 and S_1 modes, we see that their shapes and relative sizes seem to depend most strongly on the location of the minimum sound speed in the model. That is, the modes for variation 1 most resemble those of the standard, while the modal patterns of variations 2 and 3 resemble each other. Also, the entire GR_0 mode is almost the same for all cases, indicating that it is probably governed most strongly by Lamb ducting. Mode S_0 shows its largest contribution to the ground level microbarogram in the two cases, variations 2 and 3, when the model has its minimum sound speed in the lower channel. This might mean that S_0 tends to concentrate its energy near the minimum sound speed, although to put much emphasis on this possibility would be somewhat inconsistent with our earlier conclusion that the modes are of limited physical significance.

The Wind Profile

In studying the effects of winds, we find it convenient to define an equivalent sound speed, $c_e = c + \mathbf{v} \cdot \mathbf{i}$, where c is the sound speed, \mathbf{v} is the wind velocity vector, and \mathbf{i} is a horizontal unit vector in the direction of propagation. Two windy models which were used to produce theoretical microbarograms (Fig. 4-22) both have c profiles the same as the standard atmosphere, but one, variation 4, has a c_e profile equal to the c profile of variation 1, and the other, variation 5, has a c_e profile equal to the c profile of variation 2. Notice that, even though winds are actually treated in a much more sophisticated manner than simply using c_e in the place of c (see Chapter II), the results imply that the sophistication has only slight effect on the predicted waveforms. The microbarograms for the windy atmospheres, variations 4 and 5, very strongly resemble the records for the windless models having the same c_e profiles (variations 1 and 2, respectively). Thus, as long as the wind speed in every layer is

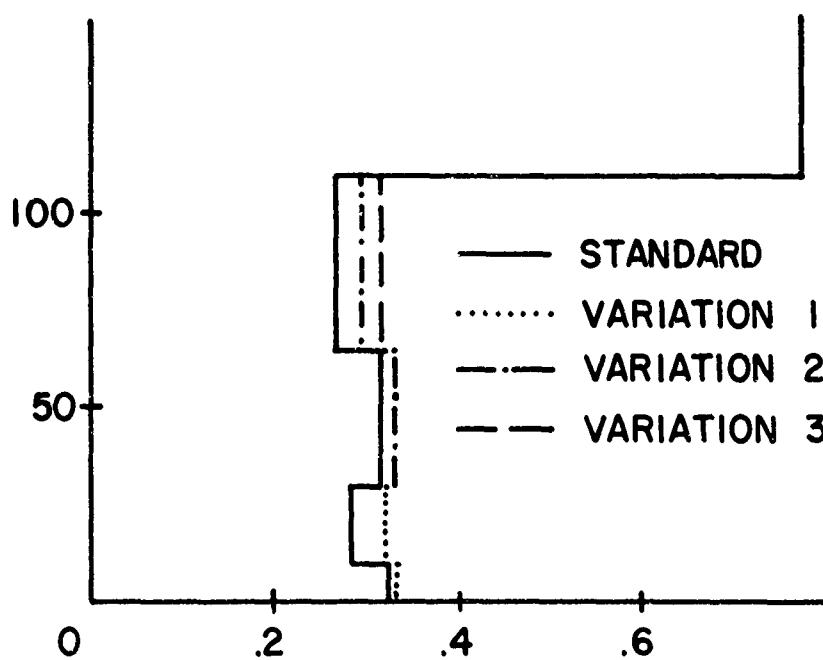


Figure 4-18. Sound speed profile for the standard temperature profile shown in Figure 4-9 and three variations studied.

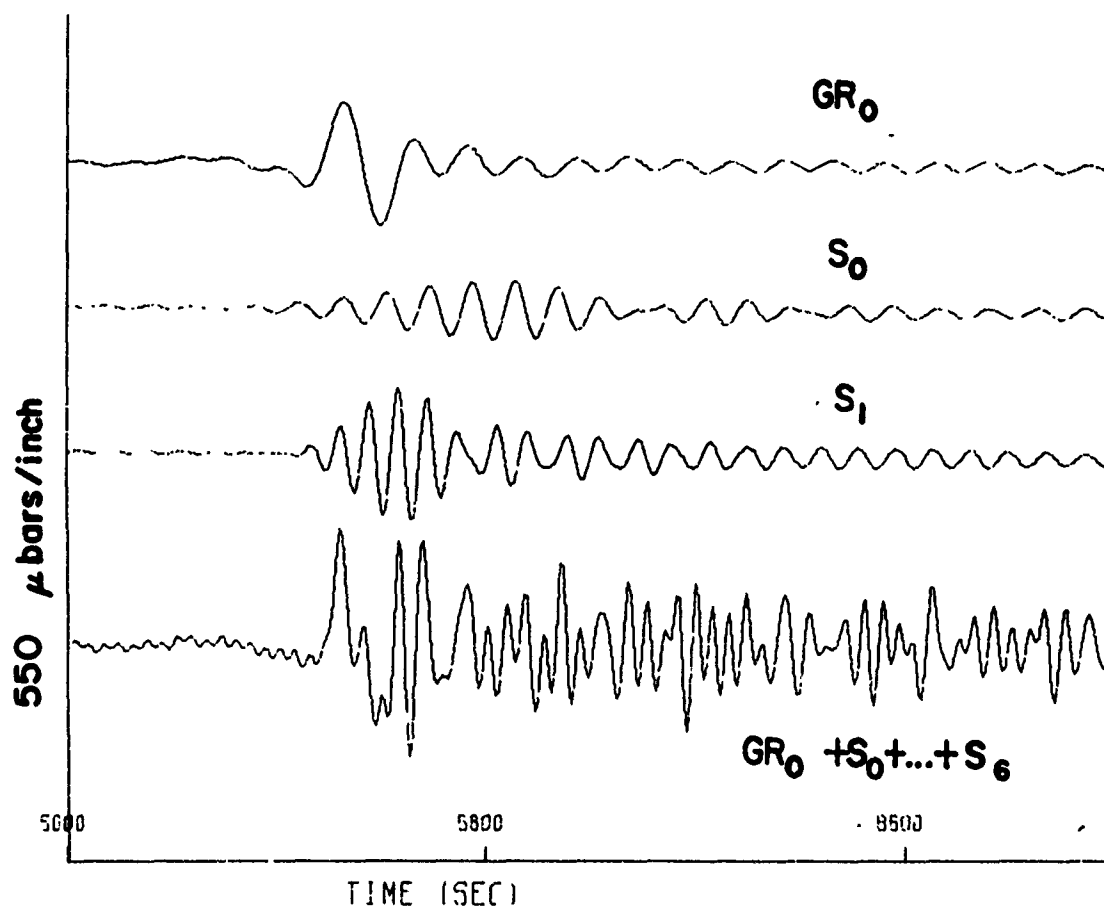


Figure 4-19. Microbarogram and three of the modes calculated using sound speed variation 1 (See Fig. 4-18).

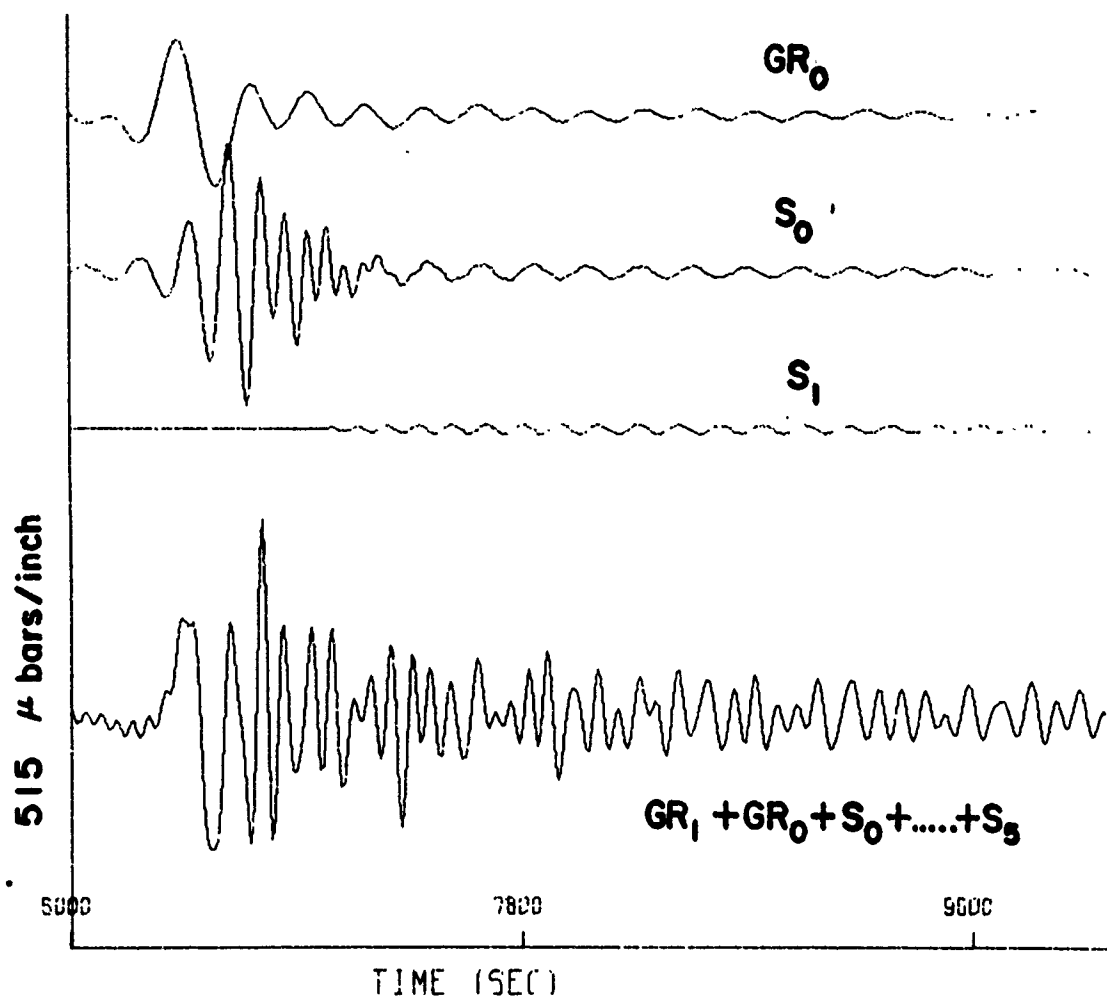


Figure 4-20. Microbarogram and three modes calculated using sound speed variation 2 (See Fig. 4-18).

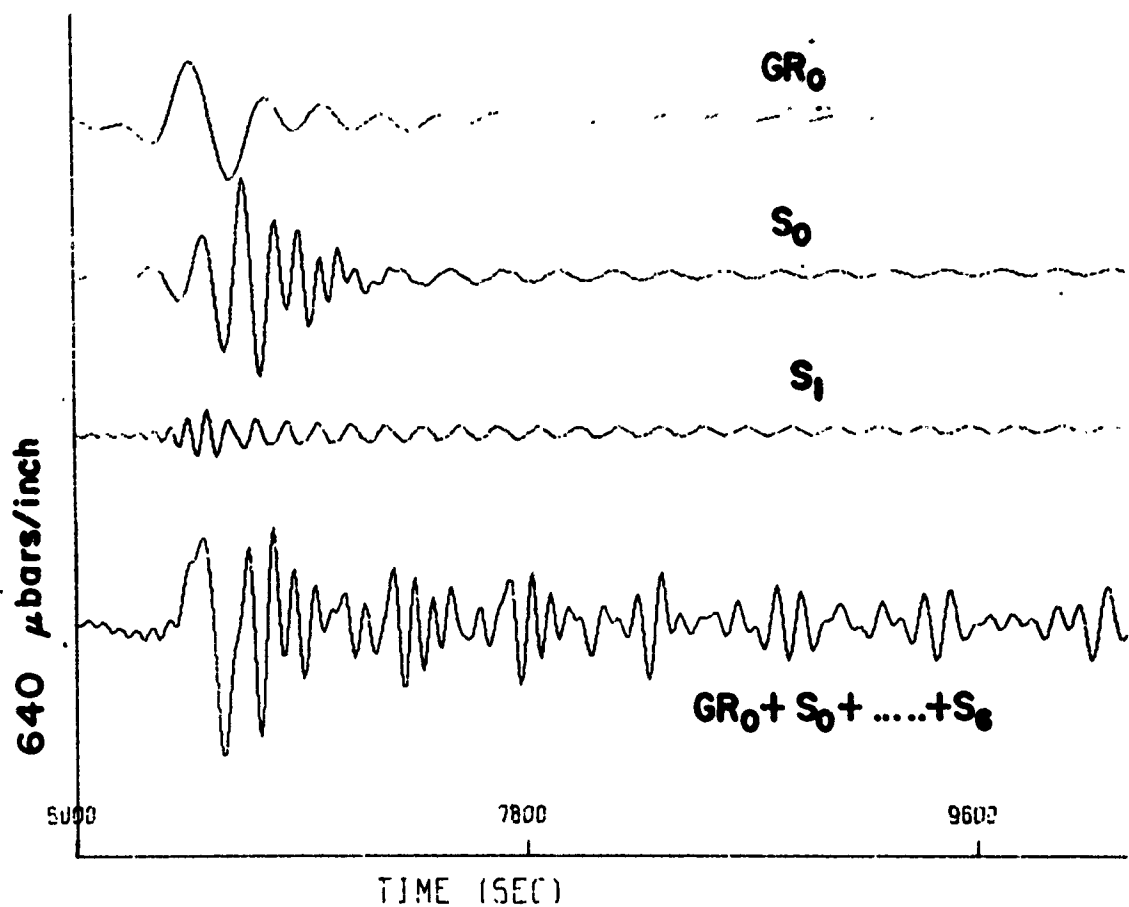


Figure 4-21. Microbarogram and three modes calculated using sound speed variation 3 (See Fig. 4-18).

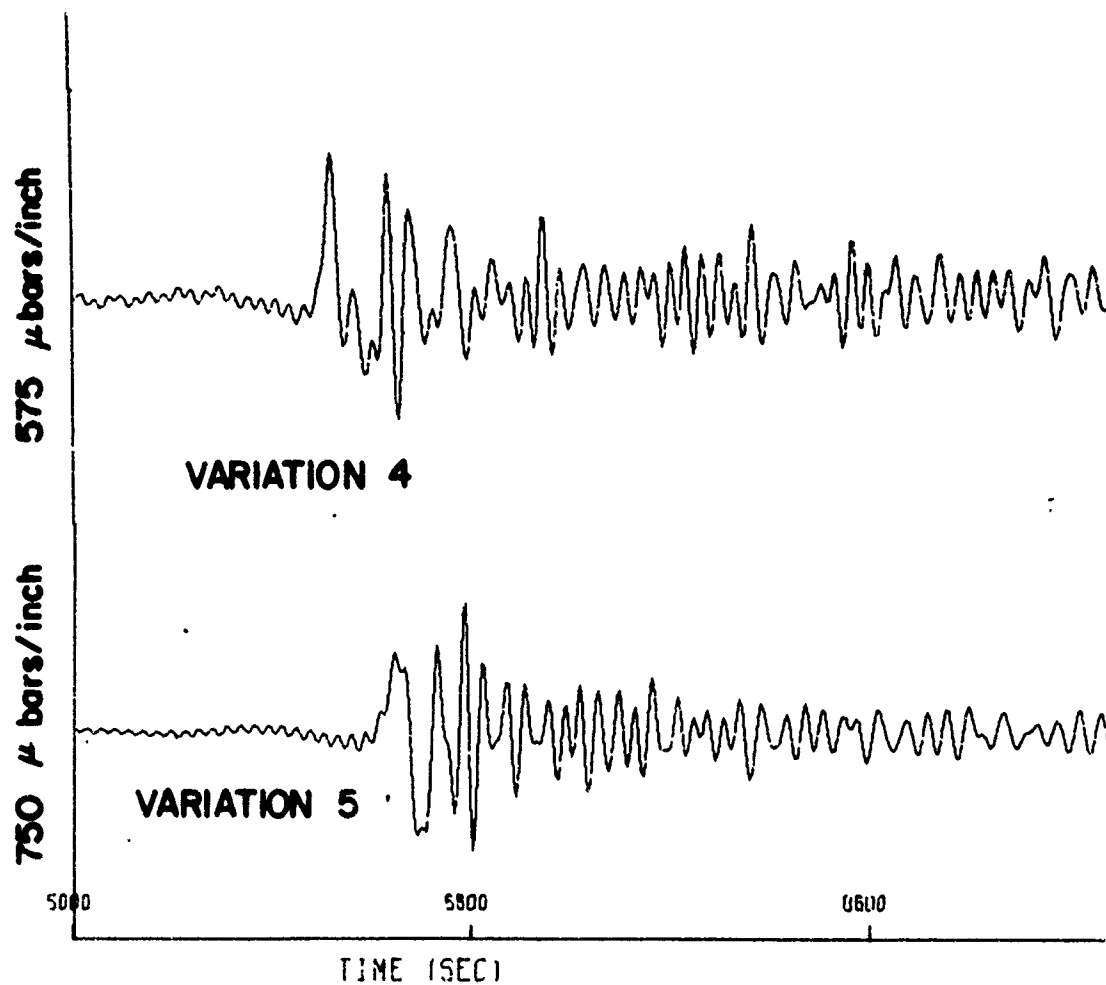


Figure 4-22. Microbarograms synthesized using the windy atmospheric models referred to as variations 4 and 5 in the text.

much less than the speed of sound for that layer, the major effect of the wind is simply to change the effective sound speed profile. In retrospect, it would appear that one need not incorporate winds into the computer code; instead he may use the device described above.

Source Parameters

Test runs in which the source parameters were varied indicated (1) that, for bursts well below the sound speed minimum in the lower channel, the height of burst has relatively little effect on the shape of the generated wave and only slight effect on its amplitude and (2) that the yield of the explosion has little influence upon either the wave's shape or the ratio $P' = (\text{wave amplitude}/\text{yield})$. In one example studied, P' fell by 4% as the yield went from 18 MT to 30 MT. Iliff (private communication, 1970) has studied both of these types of variations in considerably more detail using INFRASONIC WAVEFORMS and finds that the height of burst effect is very significant at altitudes above 10 km. In particular, the wave amplitude on the ground tends to increase with height of burst up to an altitude of the order of 40 km for megaton class explosions and then decreases with increasing altitude. Also, the ratio P' shows the smallest variation with yield for the earliest portion of the waveform; the variation may be considerable for the later arrivals.

4.4 A COMPARISON WITH EMPIRICAL DATA

On 30 October, 1962, the United States exploded a thermonuclear bomb of the megaton range near Johnson Island. The collection of observed microbarograms published by Donn and Shaw [1967] contain several records made following this blast, one of which, the Berkeley record, appears exceptionally free of noise and appears representative of what might be expected for a waveform under ideal circumstances. (This judgment is not solely that of the authors, since this waveform was chosen by others for the cover of the program of the Symposium on Acoustic Gravity Waves, Boulder, Colorado, July, 1968.) Thus, it was felt that this was the waveform which we might have the best chance of matching with a theoretical synthesis.

In preparing the input for the program INFRASONIC WAVEFORMS, the most important decision is the choice of a model atmosphere. Unfortunately, there are three categories of atmospheres which strongly affect the waveform received: the atmosphere near the source determines the relative excitation of the modes, the atmosphere along the path of the disturbance determines how the wave propagates, and the atmosphere above the observer determines

the ground strengths of the modes. Moreover, none of these atmospheres is constant over time or has winds and temperatures which are functions of altitude alone. Thus, in light of the fact that our model can neither be representative of the entire range of atmospheric profiles above the path nor display the inconsistency and horizontal inhomogeneity of any real atmosphere, exact agreement between theory and experiment would not be expected. Nevertheless, general agreement might be hoped for.

An atmospheric model was constructed to represent the average conditions between Johnson Island and Berkeley for the month of October. The temperature profile (Fig. 3-1) was taken from Valley's Handbook of Geophysics and Space Environments, Figures 2.2, 2.4, and 2.5, and the wind profile (Fig. 3-1) was taken from Valley's Figure 4.11 and Table 4.21 and from the 1965 COSPAR International Reference Atmosphere, p. 46.

Since the actual yield and height of burst for the source was not known, they were set arbitrarily at 10 MT and 3 km, respectively. A range of 5600 km and direction of propagation of 35° north of east were used. For a copy of the complete input data, see Fig. 3-12.

The synthesized waveform agrees surprisingly well with the observation (Fig. 4-23), both having the same time of arrival, a 5.5 minute period for the first major cycle, and the same dominant periods and relative amplitudes for about 35 minutes. Since Donn and Shaw did not give the amplitude of their record, an amplitude comparison cannot be made here. On the first major cycle of the synthesis, there is a variation of about 300 μ bars from peak to peak.

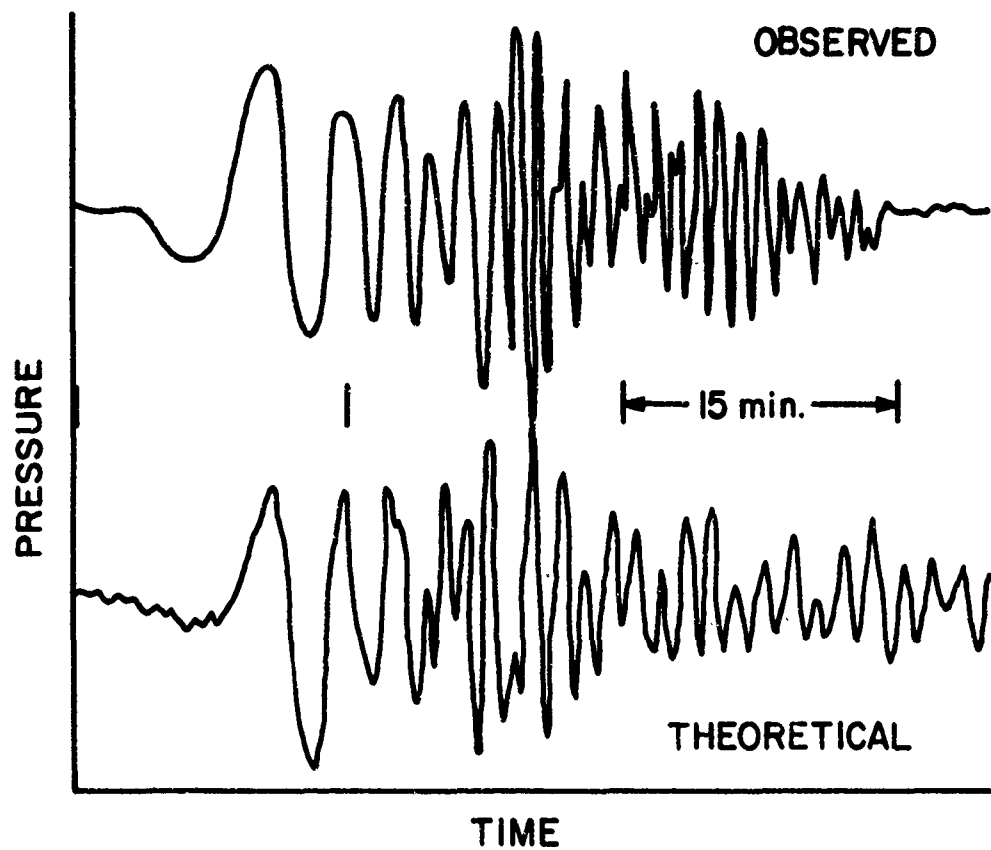


Figure 4-23. Comparison of theoretical and observed microbarograms for Berkeley, California, following a nuclear blast near Johnson Island, 30 October, 1962. A listing of the complete input data for the synthesis is given in Fig. 3-12.

Chapter V

AN APPROXIMATE METHOD BASED ON CAGNIARD'S INTEGRAL TRANSFORM TECHNIQUE

5.1 CAGNIARD'S METHOD

Cagniard's method is a technique utilizing mathematical properties of functions of a complex variable which allows one, under certain circumstances, to invert Fourier transforms. The technique dates back to Lamb's classic paper (1904) on the propagation of elastic transients on the surface of an elastic halfspace, but its significance was not realized until the 1930's when Cagniard, Pekeris, and Smirnov and Sobolov independently discovered that the technique may be applied to a much more general type of problem and developed the mathematical techniques in a more suitable form. The resulting method is generally called Cagniard's method, probably because of the fact that Cagniard's book (1939, 1962) was the first treatise on the subject to become known by the general seismological community.

Cagniard's method is generally acknowledged to be extremely complicated. This is due partly to the amount of algebra involved in using the method, to the fact that it does involve some intricate mathematical ideas, but primarily (in the authors' opinion) due to the rigorous style with emphasis on generality in Cagniard's book. The method was very little used until the mid 1950's when C.H. Dix attempted to give a simpler explanation of the method and demonstrated the fact that it leads to feasible quantitative predictions. Since the late 1950's a large number of papers have appeared on the subject with a wide scope of applications besides seismology.

In general terms, one may consider Cagniard's method to be concerned with the evaluation of integrals of the form

$$\psi(\vec{x}, t) = \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} e^{i\vec{k} \cdot \vec{x}} \int_{-\infty+i\epsilon}^{\infty+i\epsilon} e^{-i\omega t} F(\vec{k}, \omega) d\omega d^n k \quad (5.1.1)$$

where the number of dimensions of \vec{k} may, for all practical purposes, be restricted to 1 or 2. For certain restricted types of kernel functions $F(\vec{k}, \omega)$, Cagniard's method provides a sequence of mathematical manipulations which allows one to exactly transform the above to an expression of the form

$$\psi(\vec{x}, t) = \int_{-\infty}^{\infty} f(\tau) I(t - \tau, \vec{x}) d\tau \quad (5.1.2)$$

where $I(t - \tau, \vec{x})$ is a relatively simple (compared to (5.1.1)) expression to evaluate. In some cases it may be single closed expression, or it may involve one or two integrations with finite limits. This is a substantial achievement as integrals such as (5.1.1) generally defy direct numerical integration because of the infinite limits and the fact that the integrands are highly oscillatory.

Integrals of the form of Eq. (5.1.1) arise often in studies of wave propagation in stratified media -- particularly for waves generated by sources which are point and line sources. Thus one may wonder as to just what types of stratification does the method apply. As best we can tell, from an examination of cases for which the method has been applied previously, the principal restriction on $F(\vec{k}, \omega)$ is that it must be expressible as a sum of one or more terms of the form

$$F(\vec{k}, \omega) \approx \hat{f}(\omega) e^{i\omega T} D(\vec{k}, \omega) \quad (5.1.3)$$

where $\hat{f}(\omega)$ is a function of ω , and where T and D are functions of \vec{k} and ω which may each be considered (subject to some mathematical fine points) as a function of k/ω . The identification described above can be made, in particular, for a point source in a layered stratified medium, where each discrete layer is such that, were it extended to infinite thickness, propagation of any plane wave pulse in the layer would be nondispersive. This type of identification would seem evident from various mathematical formulas given in Brekhovskikh's treatise (1960) on waves in stratified media.

5.2 THE APPROXIMATION OF NEGLECT OF VERTICAL ACCELERATION

It is apparent that Cagniard's method cannot be applied to the propagation of acoustic-gravity waves per se since these waves are inherently dispersive. The counterpart of a homogeneous medium for such waves is an isothermal atmosphere and it was demonstrated by Hines (1960) that plane waves in such a medium are dispersed. Thus, Cagniard's method would appear inapplicable to an integral such as that appearing in Eq. (2.3.1).

However, it appears that there is one rather simple approximation under which Eq. (2.3.1) may be put into a form which is amenable to Cagniard's method. This is where one neglects the vertical acceleration term ρD_w in Eq. (2.1.4a). Whether or not neglecting this term is justified is somewhat debatable. However, its neglect leads to such considerable simplification that one feels compelled to explore its consequences.

We were led to the observation described above by a paper written by Row in 1966. Row sought to obtain the transient wave generated by a point source in an unbounded isothermal atmosphere with the neglect of the effects of the ground. The theory developed led to a single integral over angular frequency where the integrand was essentially the Green's function determined by Pierce (1963) and by Dikiĭ (1962) for a harmonic point source. In order to evaluate the integral, Row used the artifice of formally equating $\omega_B = (\gamma - 1)^{1/2} g/c$ and $\omega_A = \gamma g/(2c)$, which amounts to taking $\gamma = 2$. On examination of Row's result, we found that it could also be approximately interpreted as arising from the neglect of vertical acceleration. It was natural then to ask if this idea could not also be used in other situations where the atmosphere was not isothermal. Pursuing this point led to the discovery that Cagniard's method applied when the vertical acceleration is neglected.

While the approximation may seem somewhat drastic, there are several factors involved which suggest that the physical significance of the results may not be entirely negated and that its inherent inadequacies may be offset by the fact that it leads to a theory which does not necessitate using some of the approximations peculiar to the multi-mode theory described in Chapter II (such as neglect of branch line integrals, neglect of leaky modes, and the truncation of integrals over ω).

In the first instance, the approximation of neglecting vertical acceleration would seem to be most appropriate at lower frequencies. Since the first major cycle in empirical waveforms normally has a period in the range of 5 minutes, it would seem that some low-frequency approximation might be applicable in the calculation of the earliest portion of the wavetrain.

While the approximation does lead (as is demonstrated in subsequent sections) to an instantaneous propagation in the vertical direction (which is clearly wrong), we might consider this shortcoming to be not too serious since we are concerned with propagation to large horizontal distances. Furthermore, what calculations we have performed for the theory outlined in Chapter II suggest that the vertical acceleration near the ground at large distances are very small compared to the longitudinal accelerations for the earliest part of the wave. One clear cut advantage of the method is that it leads to a calculable solution which is clearly causal -- which is not true for the theory embodied in the computer program INFRASONIC WAVEFORMS. This would also suggest that we might do better for the earliest part of the waveform with the Cagniard's method theory. Of course, the final test of this would be in the comparison of results with experiment.

5.3 FORMAL DESCRIPTION OF CAGNIARD'S METHOD FOR ACOUSTIC-GRAVITY WAVES

We consider the same problem as posed in Sec. 2.1. The only distinction is that we replace Eq. (2.1.4a) by the two equations

$$\rho_0 [D_t \vec{u}_H + (\vec{u} \cdot \nabla) \vec{v}] = -\nabla_H p \quad (5.3.1a)$$

$$dp/dz = -g\rho \quad (5.3.1b)$$

corresponding to the approximation discussed in the preceeding section. Then, the solution of (5.3.1), (2.1.4b), and (2.1.4c) is of the form, for acoustic pressure p ,

$$p = \int_{-\infty}^{\infty} f_E(\tau) G(t - \tau, x, y, z, z_0) d\tau \quad (5.3.2)$$

where the Green's function G represents the response to a point impulsive source ($x_0 = 0, y_0 = 0$).

The Fourier integral expression for G is essentially the same as Eq. (2.3.1), one distinction being that $f_E(\omega)$ is replaced by $1/2\pi$.

Thus we have

$$G(t, x, y, z, z_0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{i\vec{k} \cdot \vec{x}} \left\{ \int_{-\infty-i\epsilon}^{\infty+i\epsilon} \hat{G}(\omega, \vec{k}, z, z_0) e^{-i\omega t} d\omega \right\} dk_x dk_y \quad (5.3.3)$$

where

$$\hat{G} = \left[\frac{\rho_0(z)}{\rho_0(z_0)} \right]^{\frac{1}{2}} \frac{1}{\pi[\omega - \vec{k} \cdot \vec{v}(z_0)]} \left\{ \frac{\Psi(z, z_0)}{Z_\ell(0) Y_u(0)} \right\} \quad (5.3.4)$$

with

$$\Psi(z, z_0) = [Z_u(z_0) - gY_u(z_0)] Z_\ell(z) \quad z_0 > z \quad (5.3.5a)$$

$$= [Z_\ell(z_0) - gY_\ell(z_0)] Z_u(z) \quad z_0 < z \quad (5.3.5b)$$

$$Y = \phi_1/c \quad (5.3.6a)$$

$$Z = g\phi_1/c - c\phi_2 \quad (5.3.6b)$$

Subscripts ℓ and u have been omitted from the last two equations for brevity. The above are essentially the same as Eqs. (2.3.12), (2.3.13), and (2.6.1). The only formal appearance of the effect of the neglect of

vertical acceleration is in the ordinary differential equations (residual equations) satisfied by ϕ_1 and ϕ_2 . These are

$$\frac{d}{dz} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} \quad (5.3.7)$$

where

$$A_{11} = gk^2/\Omega^2 - \gamma g/2c^2 \quad (5.3.8a)$$

$$A_{12} = 1 - c^2 k^2/\Omega^2 \quad (5.3.8b)$$

$$A_{21} = g^2 k^2/\Omega^2 c^2 \quad (5.3.8c)$$

$$A_{22} = -A_{11}$$

Note that these are the same as Eqs. (2.6.3) except that A_{21} does not have the term $-\Omega^2/c^2$ present in Eq. (2.6.3c). The quantities (ϕ_{1l}, ϕ_{2l}) and (ϕ_{1u}, ϕ_{2u}) are particular solutions of the above residual equations -- only that the first satisfy the upper boundary condition ($\phi_{1l} = 0$ at $z = 0$) while the second set satisfies the upper boundary (ϕ_1 and ϕ_2 analytic and bounded for $\omega_I > \epsilon$, k real, and all $z > 0$).

The analysis preceding Eqs. (2.4.2) and (2.4.3) shows that we may select $G(\omega, \vec{k})$ to be such that

$$\hat{G}(\omega, \vec{k})^* = \hat{G}(-\omega^*, -\vec{k}^*) \quad (5.3.9a)$$

$$\hat{G}(\omega, \vec{k}) = -\hat{G}(-\omega, -\vec{k}) \quad (5.3.9b)$$

A third symmetry property follows from the fact that the new set of coefficients depend on ω and k only through the combination k^2/Ω^2 , or alternately, only through the combination k/ω . If we examine the consequences of this we find that we may take

$$\hat{G} = \frac{1}{[\omega - \vec{k} \cdot \vec{v}(z_0)]} D(\omega, \vec{k}) \quad (5.3.10)$$

where

$$D(\omega^*, \vec{k}^*) = D(\omega, \vec{k})^* \quad (5.3.11a)$$

$$D(-\omega, -\vec{k}) = D(\omega, \vec{k}) \quad (5.3.11b)$$

$$D(\alpha\omega, \alpha\vec{k}) = D(\omega, \vec{k}) \quad (5.3.11c)$$

for any real α .

Other relevant properties of D are that it is finite or else zero as $\omega \rightarrow \infty$ for all real \vec{k} . None of its branch lines in the ω plane (when \vec{k} is fixed and real) extend to infinity. One may show that the only branch points are those associated with the upper half-space and are located for real k at $\omega = \omega_1$ and $\omega = \omega_2$ where

$$\omega_{1,2} = \vec{k} \cdot \vec{v} \mp 2c_\infty [(\gamma-1)^{1/2}/\gamma] |\vec{k}| \quad (5.3.12)$$

Thus, there are only two branch points -- both on the real axis. The branch line is taken as extending directly along the real axis between the two points.

The poles of D in the ω plane for real \vec{k} may be denoted by $\omega_n(\vec{k})$. Near any such pole,

$$D \approx \frac{D_n(\vec{k})}{\omega - \omega_n(\vec{k})} \quad (5.3.13)$$

where $D_n(\vec{k})$ is the residue. The quantity $\omega_n(\vec{k})$ should be of the form

$$\omega_n(\vec{k}) = |\vec{k}| v_n(\theta_k) \quad (5.3.14)$$

where θ_k is the direction of k . This follows from Eq. (5.3.11c). Also, Eq. (5.3.11b) would imply that

$$v_n(\theta_k + \pi) = -v_n(\theta_k) \quad (5.3.15)$$

The Eq. (5.3.11a) would imply that v_n is entirely real. Finally, we can show that D_n is real, and that it is of the form

$$D_n = |\vec{k}| A_n(\theta_k) \quad (5.3.16)$$

where

$$A_n(\theta_k) = A_n(\theta_k)^* \quad (5.3.17)$$

$$A_n(\theta_k + \pi) = -A_n(\theta_k) \quad (5.3.18)$$

If D_a is D on the real axis just above the branch line and D_b is D on the real axis just below the branch line, we may show (k real) that

$$D_a(\omega, \vec{k}) = D_b^*(\omega, \vec{k})$$

If we use polar coordinates for \vec{k} , then

$$D_a(-\omega, k, \theta_k) = D_a(\omega, k, \theta_k + \pi)$$

follows from Eq. (5.3.11b)

With these preliminaries, we may now describe Cagniard's method (or at least the authors' version of the method) as it applies to the problem. For $t < 0$, G vanishes. For $t > 0$ we deform the integration contour to enclose the entire lower halfspace in the clockwise sense. This contour is then shrunk to enclose all poles and the branch line (Fig. 5-1) and the residue theorem is utilized to pick up the contribution from the poles. This gives us

$$G = \left[\frac{\rho_o(z)}{\rho_o(z_o)} \right]^{1/2} \frac{1}{\pi} [I_{BL} + \sum_n I_n] \quad (5.3.19)$$

where the branch line contribution is given by

$$I_{BL} = \int_0^\infty k dk \int_0^{2\pi} d\theta_k e^{i\vec{k} \cdot \vec{x}} \int_{\omega_1}^{\omega_2} e^{-i\omega t} \left\{ \frac{i(D_a - D_a^*)}{\omega - \vec{k} \cdot \vec{v}(z_o)} \right\} d\omega \quad (5.3.20)$$

and a particular pole contribution is given by

$$I_n = 2\pi \int_0^\infty k dk \int_0^{2\pi} d\theta_k e^{i\vec{k} \cdot \vec{x}} \left\{ \frac{k A_n e^{i k v_n t}}{k v_n - \vec{v}(z_o) \cdot \vec{k}} \right\} \quad (5.3.21)$$

In the above we neglect the pole associated with the zero of $\omega - \vec{k} \cdot \vec{v}(z_o)$. The integral along the branch line is interpreted as a principal value.

As for the branch line contribution, we let $\omega = vk$ and change the ω variable of integration to one over v . Then we perform the k integration first. Doing this gives

$$I_{BL} = \int_0^{2\pi} d\theta_k \int_{v_1}^{v_2} dv \left\{ \frac{i(D_a - D_a^*)}{v - \vec{e}_k \cdot \vec{v}(z_o)} \right\} \int_0^\infty e^{ik[R \cos(\theta - \theta_k) - vt]} k dk \quad (5.3.22)$$

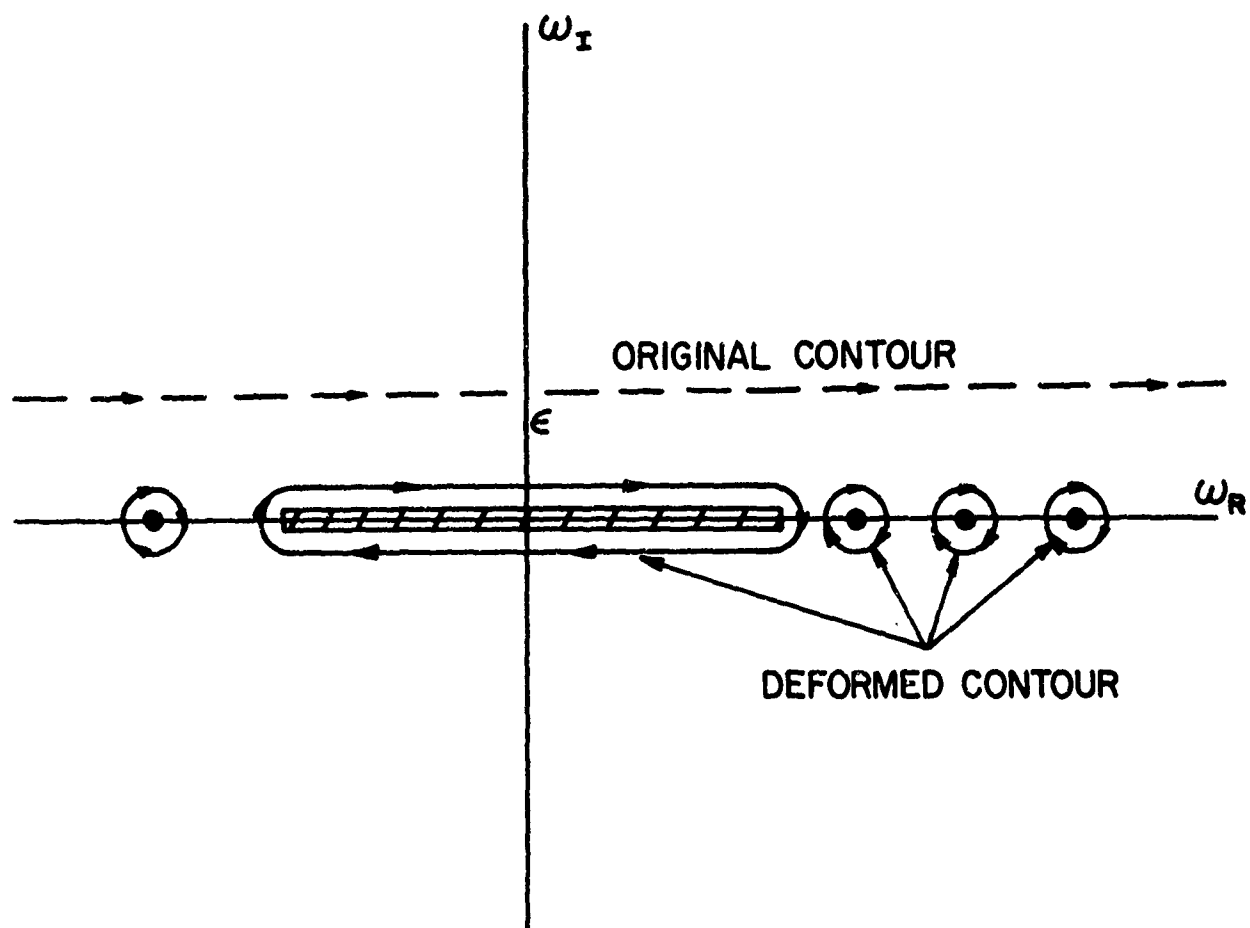


Figure 5-1. Sketch showing contour deformation in the complex w plane for positive times.

where

$$v_{1,2} = \vec{e}_k \cdot \vec{v}_\infty \mp 2c_\infty [(\gamma-1)^{1/2}/\gamma] \quad (5.3.23a)$$

$$\vec{e}_k = (\cos \theta_k) \vec{e}_x + (\sin \theta_k) \vec{e}_y \quad (5.3.23b)$$

It should be noted that v_1 and v_2 are functions of θ_k . Also, D_a is a function of v and θ_k .

The pole contributions may similarly be expressed as

$$I_n = 2\pi \int_0^{2\pi} d\theta_k \frac{A_n}{[v_n - \vec{v}(z_0) \cdot \vec{e}_k]} \int_0^\infty e^{ik[R \cos(\theta - \theta_k) - v_n t]} k dk \quad (5.3.24)$$

Here R is the net horizontal distance from the source.

Next, we may show, using various properties described above, that the contribution to the integrand in either (5.3.22) or (5.3.24) from $\theta_k + \pi$ is just the complex conjugate of that from θ_k . Thus we set

$$I_{BL} = 2 \operatorname{Re} \int_{-\pi/2}^{\pi/2} d\theta_k \int_{v_1}^{v_2} dv \left\{ \frac{i(D_a - D_a^*)}{v - \vec{e}_k \cdot \vec{v}(z_0)} \right\} \int_0^\infty e^{ik[R \cos(\theta - \theta_k) - vt]} k dk \quad (5.3.25)$$

$$I_n = 4\pi \operatorname{Re} \int_{-\pi/2}^{\pi/2} d\theta_k \frac{A_n}{[v_n - \vec{v}(z_0) \cdot \vec{e}_k]} \int_0^\infty e^{ik[R \cos(\theta - \theta_k) - v_n t]} k dk$$

At this point we introduce some minor approximations which would not be approximations at all were there no winds. We formally replace

$$e^{-ikvt} \rightarrow \cos(kvt)$$

in Eq. (5.3.25). The justification for this is that $(D_a - D_a^*)/v$ is even in the absence of winds. Also, for the pole contribution, the terms I_n can be paired (I_n, I_{-n}) in the absence of winds where $v_{-n} = v_n$. The residues A_n would have the property that $A_n = A_{-n}$ and thus we might interpret the quantity A_n/v_n as being even in n , and consequently we might let

$$\sum_n (A_n/v_n) e^{-ikv_n t} \rightarrow \sum_{n \geq 0} 2(A_n/v_n) \cos kv_n t$$

These ideas lead to the expressions

$$I_{BL} = 2 \operatorname{Re} \int_{\theta-\pi/2}^{\theta+\pi/2} d\theta_k \int_{v_1}^{v_2} dv \frac{i(D_a - D_a^*)}{v - \vec{e}_k \cdot \vec{v}(z_0)} \int_0^\infty e^{ikR \cos(\theta-\theta_k)} \cos(kvt) k dk$$

$$I_n + I_{-n} = 8\pi \operatorname{Re} \int_{\theta-\pi/2}^{\theta+\pi/2} d\theta_k \frac{A_n}{[v_n - \vec{v}(z_0) \cdot \vec{e}_k]} \int_0^\infty e^{ikR \cos(\theta-\theta_k)} \cos(kv_n t) k dk$$

Another approximation we introduce in the same spirit is to set $\theta_k = \theta$ in the argument of v_1 , v_2 , D_a , \vec{e}_k , v_n , and A_n . This is justified in the absence of winds and would seem to be appropriate with winds included since the integrand contribution is heaviest near $\theta_k = \theta$. With this approximation we have

$$I_{BL} = \int_{v_1}^{v_2} dv \left[\frac{i(D_a - D_a^*)}{v - \vec{e}_k \cdot \vec{v}(z_0)} \right]_{\theta_k = \theta} M(R, vt)$$

$$I_n + I_{-n} = 4\pi \left[\frac{A_n}{[v_n - \vec{v}(z_0) \cdot \vec{e}_k]} \right]_{\theta_k = \theta} M(R, v_n t)$$

where

$$\begin{aligned} M(R, vt) &= 2 \operatorname{Re} \int_0^\pi \int_0^\infty e^{ikR \sin \theta} \cos(kvt) k dk d\theta \\ &= v^{-1} (d/dt) \left\{ 2 \operatorname{Re} \int_0^\pi \int_0^\infty e^{ikR \sin \theta} \sin(kvt) dk d\theta \right\} \quad (5.3.26) \end{aligned}$$

The indicated integral can be shown to be

$$2 \operatorname{Re} \int_0^\pi \int_0^\infty e^{ikR \sin \theta} \sin(kvt) dk d\theta = \frac{2\pi U(vt - R)}{[(vt)^2 - R^2]^{\frac{1}{2}}} \quad (5.3.27)$$

where U is the Heaviside step function.

Finally, we combine the results above and obtain

$$P = P_{BL} + \sum_{n \neq 0} P_n \quad (5.3.28)$$

where

$$p_{BL} = -2 \left[\frac{\rho_o(z)}{\rho_o(z_o)} \right]^{\frac{1}{2}} \int_{v_1}^{v_2} \frac{1}{|v|} \left\{ \frac{1(D_a - D_a^*)}{v - \vec{e}_k \cdot \vec{v}(z_o)} \right\} \theta \left\{ \int_{-\infty}^{t-R/|v|} \frac{f'_E(\tau) d\tau}{[v^2(t-\tau)^2 - R^2]^{\frac{1}{2}}} \right\} dv \quad (5.3.29a)$$

$$p_n = -8\pi \left[\frac{\rho_o(z)}{\rho_o(z_o)} \right]^{\frac{1}{2}} \frac{1}{v_n} \left\{ \frac{A_n}{v_n - \vec{v}(z_o) \cdot \vec{e}_k} \right\} \theta \int_{-\infty}^{t-R/v_n} \frac{f'_E(\tau) d\tau}{[v_n^2(t-\tau)^2 - R^2]^{\frac{1}{2}}} \quad (5.3.29b)$$

In the above expression, v_n is considered as being positive and the sum over n is over only those "modes" having positive phase velocities.

The physical interpretation of the above solution is that the total waveform is the sum of a "lateral wave" (the branch line integral) plus a sum of guided mode waveforms. Each guided mode is nondispersive and has a speed v_n . The shapes of various guided mode waveforms are similar.

The relative simplicity of the results must be emphasized. The τ integration is over finite limits and should be easily performed on a digital computer. The only lengthy problem would be that of finding the v_n and A_n for the guided modes. However, this could be done with only a slight modification to the existing program INFRASONIC WAVEFORMS. The lateral wave might be more difficult to evaluate (since it involves two integrations) but we would expect its contribution to be small for most cases of interest. We should also point out that there is no apparent restriction on the atmospheric profiles for which the above theory might be applied.

5.4 THE ISOTHERMAL ATMOSPHERE AS AN EXAMPLE OF THIS METHOD

The only example which we have explored in any depth using the method of the previous section is that where the ambient atmosphere is isothermal. In this event the function D appearing in Eq. (5.3.10) is given by

$$D = -i \left\{ M e^{\frac{i\mu}{c} |z-z_o|} + N e^{\frac{i\mu}{c} |z+z_o|} \right\} \quad (5.4.1)$$

where

$$M = \frac{g}{2\mu} \left\{ \frac{w_A}{c} \pm i\mu \right\} \quad (5.4.2a)$$

$$N = \frac{g}{2\mu} \frac{[i\mu - c^{-1}(\omega_A^2 - \omega_B^2)^{\frac{1}{2}}]}{[-i\mu - c^{-1}(\omega_A^2 - \omega_B^2)^{\frac{1}{2}}]} [i\mu + \omega_A/c] \quad (5.4.2b)$$

$$\mu = \left[\frac{\omega_B^2 k^2}{\omega^2} - \frac{\omega_A^2}{c^2} \right]^{\frac{1}{2}} \quad (5.4.2c)$$

The plus sign in (5.4.2a) corresponds to $z_0 > z$, while the minus sign corresponds to $z > z_0$. The quantity μ has a branch line between $-(\omega_B/\omega_A)c|k|$ and $(\omega_B/\omega_A)c|k|$. Its phase is between 0 and π in the upper half of the ω plane.

The only poles are at $\omega = \pm c|k|$ and lie on the real axis to the left and right of the branch line. The residue at the positive pole is $|k|A_1$ where

$$A_1 = g(1 - \gamma/2)e^{-(1 - \gamma/2)(g/c^2)|z + z_0|} \quad (5.4.3)$$

Thus, from Eqs. (5.3.28) and (5.3.29), we have

$$p = p_{BL} + p_1 \quad (5.4.4)$$

where

$$p_{BL} = -4 \left[\frac{\rho_0(z)}{\rho_0(z_0)} \right]^{\frac{1}{2}} \int_0^{(\omega_B/\omega_A)c} Q \left\{ \int_{-\infty}^{t-R/v} \frac{f'_E(\tau) d\tau}{[v^2(t - \tau)^2 - R^2]^{\frac{1}{2}}} \right\} dv \quad (5.4.5a)$$

$$p_1 = -8\pi \left[\frac{\rho_0(z)}{\rho_0(z_0)} \right]^{\frac{1}{2}} (g/c) (1 - \gamma/2)e^{-(1 - \gamma/2)(g/c^2)|z + z_0|} \\ \times \int_{-\infty}^{t-R/c} \frac{f'_E(\tau) d\tau}{[c^2(t - \tau)^2 - R^2]^{\frac{1}{2}}} \quad (5.4.5b)$$

Here

$$Q = (2/v^2) \text{Re} \left\{ \bar{M} e^{-iK|z - z_0|} + \bar{N} e^{-iK|z + z_0|} \right\} \quad (5.4.6)$$

where

$$\bar{M} = -\frac{g}{2K} \{ (\gamma/2)(g/c^2) \mp iK \} \quad (5.4.7a)$$

$$\bar{N} = -(g/2K) \left\{ \frac{-iK - (1 - \gamma/2)g/c^2}{iK - (1 - \gamma/2)g/c^2} \right\} [-iK + (\gamma/2)g/c^2] \quad (5.4.7b)$$

$$K = g[(\gamma - 1)/v^2 - (\gamma/2)/c^2]^{1/2}/c \quad (5.4.7c)$$

In the event the presence of the ground is neglected, there is no guided wave p_1 and the term with \bar{N} does not appear in (5.4.6).

We have carried out a modest amount of calculations using the above formulas, taking $f_E(t)$ to be a delta function. The quantity p_1 then, for $t > R/c$, has a t and R dependence given by

$$p_1 \approx \frac{ct}{[c^2t^2 - R^2]^{3/2}} \quad (5.4.8)$$

and thus falls off as $1/t^2$ at large t . The direct wave is oscillatory in general. The nature of the oscillation can be described if we let $v = R/t$ and examine the factor $\exp[-iK|z - z_0|]$. Thus

$$p_{BL} \approx \{\text{amplitude}\} \cos \{g|z - z_0|[(\gamma - 1)t^2/R^2 - (\gamma/2)c^{-2}]^{1/2} + \text{phase factor}\} \quad (5.4.9)$$

The angular frequency as a function of time is then

$$\omega \approx \frac{g|z - z_0|(\gamma - 1)t/R^2}{c[(\gamma - 1)t^2/R^2 - (\gamma/2)c^{-2}]^{1/2}} \quad (5.4.10)$$

which is large at early times and which asymptotically (large t and fixed R) approaches

$$\omega \approx \frac{g}{c} \frac{|z - z_0|}{R} (\gamma - 1)^{1/2} \quad (5.4.11)$$

which is essentially the same as Row's ω . The difference is that the R above is horizontal distance rather than total slant path distance. As long as $|z - z_0|/R \ll 1$, our result agrees with Row's.

Chapter VI

THE SINGLE MODE THEORY

6.1 INTRODUCTION

The calculations presented in Chapter IV and by previous authors (Scorer, Pekeris, Harkrider, etc.) suggest that the earliest portion of the waveform (say, the first three cycles) received at large distances may be considered as associated with a single composite mode. This point of view has, in particular, been espoused with considerable eloquence by Garrett and by Bretherton in some very recent papers on the subject. We find this point of view to be appealing by virtue of the fact that it may lead to a satisfactory method for taking into account some effects which are neglected in the formulation of the multi-mode theory presented in Chapter II. Such effects would include far field nonlinear effects, departures of the atmosphere from perfect stratification, attenuation by viscosity and thermal conduction, and large scale irregularities in the earth's terrain.

Another virtue of a single-mode theory would be its inherent simplicity. The computational procedure represented by INFRA-SONIC WAVEFORMS, regardless of how good one regards the theory on which it is based, is sufficiently complicated that its consequences can only be explored by numerical experiment. The large number of possible parameters which must be specified in order to construct a single waveform make it very difficult to draw any succinct simple cause and effect relationships between any one of these parameters (for example, yield) and particular features of the waveform. This would probably be a minor handicap from a practical standpoint, given the existence of the computer program, if we possessed a reasonable knowledge of the atmosphere's state at the time the explosion took place. In practice, however, this is not the case, as the atmosphere is always imperfectly known at any given time. The usual experimental situation is where a number of waveforms are recorded at various points and where a limited knowledge of the explosion and of the atmosphere is possessed. The typical analysis problem would be to use this data and whatever else is known to determine a refined description of the atmosphere and/or the explosion. Borrowing a term from exploration geophysics, this might be designated the inverse problem of infra-sonic wave propagation. In principle, given an adequate theory and a numerical procedure for synthesizing waveforms, we can

solve this inverse problem (or at least find a possible range of solutions and a most probable solution) by trial and error in repetitive calculations with systematic variation of input parameters. Obviously, this could be a very expensive and time-consuming process. Thus, a strong case can be made for an attempt to find a simple model where the number of input parameters is greatly reduced.

Insofar as the theory embodied in INFRASONIC WAVEFORMS is concerned, the possibility of using it to solve the inverse problem is severely limited by the fact that it is restricted to perfectly stratified atmospheres. The data showing amplitude variations with observer location exhibited by Wexler and Hass following the largest Soviet explosion strongly suggest that departures from stratification are of considerable significance. On the other hand, the present theory is already so complicated that it appears prohibitively difficult to extend it to include departures from stratification. A possibility would be a tradeoff - altering the theory to take the non-stratification into account at the expense of the accuracy which might be expected were the atmosphere perfectly stratified. In this respect, the single mode theory might represent a very convenient compromise.

6.2 LAMB'S MODE

In 1910, Horace Lamb demonstrated that a single guided mode exists for the isothermal atmosphere with no winds. In retrospect, the existence of this mode is very curious as the normal criterion for ducting in conventional (gravity neglected) acoustics would seemingly preclude its existence.

The formulas for Lamb's mode are trivially extended to include constant horizontal wind. For convenience of reference, we summarize the result here. The acoustic pressure p , density ρ , horizontal fluid velocity deviation u , and vertical fluid velocity w are given by

$$p = e^{-gz/c^2} F(\vec{x}_H, t) \quad (6.2.1a)$$

$$\rho = c^{-2} e^{-gz/c^2} F(\vec{x}_H, t) \quad (6.2.1b)$$

$$\vec{u} = e^{-gz/c^2} \vec{U}(\vec{x}_H, t) / \rho_0(z) \quad (6.2.1c)$$

$$w = 0 \quad (6.2.1d)$$

where F and \vec{U} satisfy

$$[\partial/\partial t + \vec{v} \cdot \nabla_H] \vec{U} = -\nabla_H F \quad (6.2.2a)$$

$$[\partial/\partial t + \vec{v} \cdot \nabla_H] F + c^2 \nabla_H \cdot \vec{U} = 0 \quad (6.2.2b)$$

or

$$[\partial/\partial t + \vec{v} \cdot \nabla_H]^2 F - c^2 \nabla_H^2 F = 0 \quad (6.2.3)$$

which is the two dimensional wave equation for nondispersive propagation. In the above, x_H is horizontal displacement and ∇_H is the horizontal component of the gradient. Note that c and \vec{v} (horizontal wind) are considered constant in the above.

The plane wave solution of (6.2.3) is

$$F = F_0 e^{-i[\omega t - \vec{k} \cdot \vec{x}]}$$

where ω and \vec{k} satisfy the dispersion relation

$$(\omega - \vec{k} \cdot \vec{v})^2 = c^2 k^2$$

Since this gives \vec{k}/ω as being independent of frequency, the propagation is nondispersive.

The relative simplicity of Eqs. (6.2.2) and (6.2.3) must be emphasized. Although the disturbance is in a three dimensional space, these equations only involve two spatial coordinates. Furthermore, the coefficients in these equations are constant - a substantial simplification for propagation in an inhomogeneous medium.

It would appear that, if a single-mode theory of infrasonic propagation were to be developed, the mode selected should be that which, for more realistic atmospheres, is the counterpart of Lamb's mode for an isothermal atmosphere. This follows since the principal disturbance contributing to the waveform observed at ground level is one which moves very nearly with the ground speed, which is only slightly dispersive, and which has very little vertical movement (as contrasted with horizontal movement) associated with it. Garrett and Bretherton have succeeded in finding this mode for a stratified atmosphere which is nearly isothermal and which has nearly constant winds. We give a modified derivation (with slightly different results) below:

The residual equations, (2.3.8), for disturbances of given \vec{k} and ω may be rewritten in the form

$$\frac{d}{dz} (Z\phi) = \phi S_{12} Y \quad (6.2.4a)$$

$$\frac{d}{dz} (Y\phi^{-1}) = \phi^{-1} S_{21} Z \quad (6.2.4b)$$

where ϕ may be taken as

$$\phi = p_0^{-1/\gamma(z)} \rho_0^{1/2}(z) \quad (6.2.5)$$

Since $Y = 0$ at the ground altitude z_g , we may place these in the form of coupled integral equations as

$$Z = \phi^{-1} \left[F + \int_{z_g}^z \phi S_{12} Y dz \right] \quad (6.2.6a)$$

$$Y = \phi \int_{z_g}^z \phi^{-1} S_{21} Z dz \quad (6.2.6b)$$

where F is independent of z . To the above we add, as a restriction on ω and k , the guided condition that $Y\phi^{-1} \rightarrow 0$ as $z \rightarrow \infty$, i.e.

$$\int_{z_g}^{\infty} \phi^{-1} S_{11} Z dz = 0 \quad (6.2.7)$$

By successive iteration starting with $Y = 0$ in Eq. (6.2.6a) we find that these have the formal solution

$$Z = \phi^{-1} [1 + L_{12} L_{21} + L_{12} L_{21} L_{12} L_{21} + L_{12} L_{21} L_{12} L_{21} L_{12} L_{21} + \dots] F \quad (6.2.8a)$$

$$Y = \phi [L_{21} + L_{21} L_{12} L_{21} + L_{21} L_{12} L_{21} L_{12} L_{21} + \dots] F \quad (6.2.8b)$$

$$[L_{21} + L_{21} L_{12} L_{21} + L_{21} L_{12} L_{21} L_{12} L_{21} + \dots]_{z=\infty} = 0 \quad (6.2.8c)$$

where L_{12} and L_{21} are operators, defined such that for any function

$Q(z)$ appearing to their right

$$L_{12}Q = \int_{z_g}^z \phi^2 S_{12}Q \, dz \quad (6.2.9a)$$

$$L_{21}Q = \int_{z_g}^z \phi^{-2} S_{21}Q \, dz \quad (6.2.9b)$$

The subscript ($z=\infty$) in Eq. (6.2.8c) implies that the upper limit of the last integration is ∞ .

The lowest order approximation to the dispersion relation would be $[L_{12}]_{\infty} = 0$ or

$$\int_{z_g}^{\infty} \phi^{-2} [(k^2/\Omega^2) - c^{-2}] \, dz = 0 \quad (6.2.10)$$

To further approximate this, we expand

$$\frac{k^2}{\Omega^2} = \frac{k^2}{(\omega - \vec{k} \cdot \vec{v})^2} \approx \frac{k^2}{\Omega_L^2} [1 + 2\vec{k} \cdot (\vec{v} - \vec{v}_L)/\Omega_L] \quad (6.2.11)$$

where

$$\Omega_L = \omega - \vec{k} \cdot \vec{v}_L$$

and where \vec{v}_L is any representative wind speed. We consider \vec{v}_L independent of z . Since we have some latitude in the definition of \vec{v}_L , we define \vec{v}_L such that

$$\int_{z_g}^{\infty} \phi^{-2} (\vec{v} - \vec{v}_L) \, dz = 0$$

or

$$\vec{v}_L = \frac{\int \phi^{-2} \vec{v} dz}{\int \phi^{-2} dz} \quad (6.2.12)$$

Then, substituting (6.2.11) into (6.2.10), we find

$$k^2/\Omega_L^2 = 1/c_L^2 \quad (6.2.13)$$

where

$$\frac{1}{c_L^2} = \frac{\int \phi^{-2} c^{-2} dz}{\int \phi^{-2} dz} \quad (6.2.14)$$

In what follows we refer to \vec{v}_L as the average wind velocity and to c_L as the average sound speed for the Lamb mode.

One should note that eq. (6.2.13) is exactly the same dispersion relation as was obtained for Lamb's mode in an isothermal atmosphere with constant winds.

From Eqs. (6.2.8a,b), keeping just the first order terms in $c^2 - c_L^2$ and $\vec{v} - \vec{v}_L$ and using (6.2.13), we find

$$Z = \phi^{-1} [1 + (\Omega_L^2 - \omega_{BL}^2) (A + \vec{B} \cdot \vec{k} / \Omega_L)] F \quad (6.2.15a)$$

$$Y = \phi [C + \vec{D} \cdot \vec{k} / \Omega_L] F \quad (6.2.15b)$$

where A , \vec{B} , C , \vec{D} are functions of z , given by

$$A = \int_{z_g}^z \phi^2 C dz \quad (6.2.16a)$$

$$\vec{B} = \int_{z_g}^z \phi^2 \vec{D} dz \quad (6.2.16b)$$

$$C = \int_{z_g}^z \phi^{-2} (c_L^{-2} - c^{-2}) dz \quad (6.2.16c)$$

$$\vec{D} = (2/c_L^2) \int_{z_g}^z \phi^{-2} (\vec{v} - \vec{v}_L) dz \quad (6.2.16d)$$

We have also defined the average Brunt's frequency ω_{BL} for the Lamb mode by the relation

$$\omega_{BL}^2 = (\gamma-1)g^2/c_L^2 \quad (6.2.17)$$

If we examine the next highest order correction (which is second order in $c^2 - c_L^2$ and $\vec{v} - \vec{v}_L$) to the dispersion relation (6.2.8c), we find, after some algebra and the use of the definitions of v_L and c_L , that

$$\begin{aligned} & [k^2/\Omega_L^2 - c_L^{-2}] \int_{z_g}^{\infty} \phi^{-2} dz + (3c_L^{-2}\Omega_L^{-2}) \int_{z_g}^{\infty} \phi^{-2} [\vec{k} \cdot (\vec{v} - \vec{v}_L)]^2 dz \\ & - (\Omega_L^2 - \omega_{BL}^2) \int_{z_g}^{\infty} \phi^2 [C + \vec{D} \cdot \vec{k}/\Omega_L]^2 dz = 0 \end{aligned} \quad (6.2.18)$$

This is essentially the same as the dispersion relation derived by Garrett. It should be noted that the presence of the last term makes the mode dispersive. The integral should be convergent since $C(\infty)$ and $\vec{D}(\infty)$ are both zero.

To the same order of approximation, we may write the dispersion relation for a wave traveling with wave normal in the direction of \vec{k} in the form

$$\omega = k(c_L + v_{Lk} + a_{kk}) - k(k^2 - k_{BL}^2)h_{kk} \quad (6.2.19)$$

where

$$v_{Lk} = \vec{v}_L \cdot \vec{e}_k \quad (6.2.20a)$$

$$a_{kk} = [3/(2c_L)] \frac{\int [\vec{v} - \vec{v}_L] \cdot \vec{e}_k]^2 \phi^{-2} dz}{\int \phi^{-2} dz} \quad (6.2.20b)$$

$$k_{BL}^2 = \omega_{BL}^2 / c_L^2 \quad (6.2.20c)$$

$$h_{kk} = (1/2)c_L \frac{\int \phi^2 [C + \vec{D} \cdot \vec{e}_k / c_L]^2 dz}{\int \phi^{-2} dz} \quad (6.2.20d)$$

Here \vec{e}_k is the unit vector in the direction of \vec{k} . One should note that a_{kk} and h_{kk} may each be considered as the Cartesian components of a tensor.

If we had a hypothetical pulse propagating in the \vec{e}_k direction, such that all frequency components can be considered as being plane (or line) waves (in the horizontal plane) with the same wave number direction \vec{e}_k , this pulse could be represented as a Fourier integral in the form

$$\psi(t,s) = 2 \operatorname{Re} \int_0^\infty \hat{\psi}(k) e^{-i(\omega t - ks)} dk \quad (6.2.21)$$

where $s = \vec{e}_k \cdot \vec{x}$ is distance in the direction of \vec{k} and ω is considered as a function of k . Then, if $\omega(k)$ is given by (6.2.19), it must follow that the wave variable ψ satisfies the equation

$$\partial\psi/\partial t + [c_L + v_{Lk} + a_{kk}] \partial\psi/\partial s + h_{kk} (\partial^2/\partial s^2 + k_{BL}^2) (\partial\psi/\partial s) = 0 \quad (6.2.22)$$

which may be recognized as an equation which in many other contexts is generally called the linearized Korteweg-de Vries equation.

6.3 FAR FIELD NONLINEAR EFFECTS

In this section we generalize the linearized Korteweg-de Vries equation governing pulse propagation in the Lamb mode to include accumulative nonlinear effects. We assume at the outset that such effects are weak and that their primary effect is to distort the waveform. In this respect, we consider that the only appreciable nonlinear effect is represented by the fact that c_L and v_L should

be the height averaged sound speed and wind velocity, given the fact that the ambient medium is altered by the presence of the wave disturbance. Thus we replace

$$c_L \rightarrow c_L^{NL} \quad (6.3.1a)$$

$$v_{Lk} \rightarrow v_{Lk}^{NL} \quad (6.3.1b)$$

in the dominant terms (zeroth order in $\vec{v} - \vec{v}_L$ and $c^2 - c_L^2$) in (6.2.22).

To determine c_L^{NL} we set

$$c^2 \rightarrow \frac{\gamma(p_0 + p)}{(\rho_0 + \rho)} \quad (6.3.2)$$

in Eq. (6.2.14) such that, to the first order,

$$(c_L^{NL})^{-2} = \frac{1}{c_L^2} + \frac{\int \phi^{-2} \left\{ \frac{1}{c^2} \left[\frac{p}{p_0} - \frac{\rho}{\rho_0} \right] \right\} dz}{\int \phi^{-2} dz}$$

The fact that ϕ also depends on c is not important as it leads to nonlinear terms of first order in $c^2 - c_L^2$ which are considered small. To obtain the lowest order nonlinear correction, we may approximate

$$\frac{1}{c^2} \left[\frac{p}{p_0} - \frac{\rho}{\rho_0} \right] = \frac{1}{c^2} \frac{(\gamma - 1)}{\gamma} \frac{p}{p_0} \approx \frac{1}{c^2} \frac{(\gamma - 1)}{\gamma} F \cdot \frac{1}{\rho_0} \frac{1}{\gamma - 1}$$

using Eqs. (2.3.4a), (6.2.5), (6.2.15a) and various relations appropriate to the case when the atmosphere is isothermal. Using some additional approximations, we find

$$c_L^{NL} = c_L \{ 1 + [(\gamma - 1)/(2\gamma)] v_p(z_g)/p_0(z_g) \} \quad (6.3.3)$$

where

$$v = \frac{\int_{z_g}^{\infty} [p_o(z)/p_o(z_g)]^{(3/\gamma - 2)} dz}{\int_{z_g}^{\infty} [p_o(z)/p_o(z_g)]^{(2/\gamma - 1)} dz} \quad (6.3.4)$$

In a similar manner, we compute

$$v_{Lk}^{NL} = \frac{\int_{z_g}^{\infty} (\vec{v} + \vec{u}) \cdot \vec{e}_k \phi^{-2} dz}{\int_{z_g}^{\infty} \phi^{-2} dz} \approx v_{Lk} + \frac{\int_{z_g}^{\infty} [p/c\rho_o] \phi^{-2} dz}{\int_{z_g}^{\infty} \phi^{-2} dz}$$

$$\approx v_{Lk} + c_L \gamma^{-1} v[p(z_g)/p_o(z_g)] \quad (6.3.5)$$

where v is the same as in Eq. (6.3.4).

With Eqs. (6.3.3) and (6.3.5), the modified pulse propagation equation (6.2.22) becomes

$$\partial\psi/\partial t + \{c_L + v_{Lk} + a_{kk} + c_L[(\gamma + 1)/2\gamma]v[p(z_g)/p_o(z_g)]\}\partial\psi/\partial s$$

$$+ h_{kk}(\partial^2/\partial s^2 + k_{BL}^2)(\partial\psi/\partial s) = 0 \quad (6.3.6)$$

This equation will be nonlinear, since $p(z_g)$ is a function of ψ , regardless of what we choose ψ to represent. [For example, we could take ψ to be $p(z_g)$.] The above equation is generally referred to as the Korteweg-de Vries equation.

6.4 DISSIPATION EFFECTS

We next consider the modification to Eq. (6.3.6), i.e., the Korteweg-de Vries equation, due to the dissipation caused by viscosity and thermal conduction in the atmosphere. Specifically, we derive an extra term which represents the correction due to the effects of these phenomena. In carrying through the deri-

vation, we neglect nonlinear effects - with the assumption that terms which are both nonlinear and which involve viscosity and thermal conduction are of negligible influence on the waveform.

There are essentially two broad types of dissipation which may be considered - bulk dissipation and wall dissipation. The former is the dissipation which occurs when any wave propagates in an unbounded medium, while wall dissipation is that which occurs due to the presence of the ground. The former takes place primarily at high altitudes because of the decrease of ambient density with height, while the latter takes place close to the ground in a thin boundary layer. A priori, we assume that bulk dissipation is the more important and we accordingly neglect wall dissipation. We have not, however, investigated this quantitatively as yet, and we plan to do so in later studies. In what follows we proceed on the assumption of negligible wall dissipation.

The general procedure we adopt is to first write out the equations of hydrodynamics with viscosity and thermal conduction included and then derive the linearized first order equations for acoustic perturbations to an ambient state. This ambient state is taken to be height stratified and time independent as described in Sec. 2-1. The source term is neglected at the outset, since we are here concerned with propagation at distances somewhat removed from the source location.

The modified equations then become

$$\rho_0 [D_t \vec{u} + \vec{u} \cdot \nabla \vec{v}] = -\nabla p - g \rho \vec{e}_z + (\partial \sigma'_{ij} / \partial x_i) \vec{e}_j \quad (6.4.1a)$$

$$D_t \rho + \nabla \cdot (\rho_0 \vec{u}) = 0 \quad (6.4.1b)$$

$$(D_t p + \vec{u} \cdot \nabla p_0) - c^2 (D_t \rho - \vec{u} \cdot \nabla \rho_0) = D_E \quad (6.4.1c)$$

where

$$\sigma'_{ij} = \eta [\partial u_i / \partial x_j + \partial u_j / \partial x_i - (2/3) \delta_{ij} \nabla \cdot \vec{u}] + \zeta \delta_{ij} \nabla \cdot \vec{u} \quad (6.4.2a)$$

$$D_E = 2\eta(\gamma - 1) (\partial \vec{v} / \partial z) \cdot [\partial \vec{u} / \partial z + \nabla u_z] + (\gamma c_v)^{-1} \nabla \cdot \{ \kappa \nabla [(\gamma p - c^2 \rho) / \rho_0] \} \quad (6.4.2b)$$

Here η is the dynamic viscosity, ζ is the bulk viscosity, c_v is the specific heat per unit mass at constant volume and κ is the thermal conductivity. The above result neglects fluctuations in

c_v , η , ζ , and κ due to the presence of the disturbance. The remaining symbols have the same meaning as used in Chapter 2.

If we next consider a planar disturbance of fixed angular frequency ω and fixed horizontal wave number k , such that

$$p = \hat{p}(z)e^{-i\omega t}e^{i\vec{k}\cdot\vec{x}} \quad (6.4.3)$$

with analogous relations for density ρ , vertical fluid velocity \hat{w} , etc., and impose the condition $\hat{w}(z) = 0$ at $z = z_g$, we find, after a lengthy analysis, that

$$-\gamma(\hat{w}/\hat{u})p_0^{-1/\gamma} = \int_{z_g}^z p_0^{(1/\gamma - 1)} \{-1[1 - c^2k^2/\Omega^2]\hat{p} - \hat{Q}\} dz \quad (6.4.4)$$

where

$$\begin{aligned} \hat{Q} = & (c^2/\Omega^2)\{(\partial/\partial z)[\eta(ik^2\hat{w} + \partial[\vec{k}\cdot\vec{u}]/\partial z)] \\ & - [(4/3)\eta + \zeta]k^2\vec{k}\cdot\vec{u} + i\zeta k^2\partial w/\partial z\} \\ & + (1/\Omega)\{2\eta(\gamma - 1)(\partial\vec{v}/\partial z) \cdot [\partial\vec{u}/\partial z + i\vec{k}\hat{w}] \\ & + (\gamma c_v)^{-1}(\partial/\partial z)[\kappa(\partial/\partial z)\{\gamma\hat{p} - c^2\hat{\rho}\}/\rho_0]\} \\ & - (\gamma c_v)^{-1}k^2\kappa(\gamma\hat{p} - c^2\hat{\rho})/\rho_0 \end{aligned} \quad (6.4.5)$$

A priori, we expect \hat{Q} to be small. Thus it would seem appropriate to neglect all quantities in this expression which are known to be small for the unattenuated Lamb mode. In particular, we neglect all terms involving \hat{w} . Also, since we expect

$$\left| \frac{\partial\vec{v}}{\partial z} \right| \ll kc$$

we neglect all terms involving $\partial\vec{v}/\partial z$. In addition, it would appear to be sufficient to take the plane wave relations

$$\hat{\rho} = \hat{p}/c^2 \quad ; \quad \vec{k}\cdot\vec{u} = \Omega\hat{p}/\rho_0$$

and thus to express \hat{Q} entirely in terms of \hat{p} . Thus we obtain

$$\begin{aligned} \hat{Q} = & (c^2/\Omega^2)\{(\partial/\partial z)\eta[\partial(\Omega\hat{p}/\rho_0)/\partial z] - [(4/3)\eta + \zeta]k^2\Omega\hat{p}/\rho_0\} \\ & + [(\gamma - 1)/(\Omega c_v)]\{\partial/\partial z[\kappa\partial/\partial z(\hat{p}/\rho_0)] - k^2\kappa\hat{p}/\rho_0\} \end{aligned}$$

Next, since η and κ are relatively slowly varying with height, and Ω is also slowly varying, and since, in the Lamb mode,

$$\frac{\partial}{\partial z}(\hat{p}/\rho_0) \approx [(\gamma - 1)g/c^2]\hat{p}/\rho_0$$

we can further approximate the above by

$$\begin{aligned} \hat{Q} = & \{[c^2/\Omega]\{[(\gamma - 1)^2 g^2/c^4]\eta - [(4/3)\eta + \zeta]k^2\} \\ & + [(\gamma - 1)/(\Omega c_v)]\{[(\gamma - 1)^2 g^2/c^4] - k^2\}\kappa\}\hat{p}/\rho_0 \end{aligned} \quad (6.4.6)$$

If we desire a dispersion relation for the Lamb mode which includes dissipation, we may obtain one by simply taking the guided condition $\hat{p}_0 \rightarrow 0$ as $z \rightarrow \infty$. From Eq. (6.4.4) we would have

$$\int_0^\infty p_0^{(1/\gamma)-1} \{-1[1 - c^2 k^2/\Omega^2]\hat{p} - \hat{Q}\} dz = 0 \quad (6.4.7)$$

Then, to obtain a lowest order dispersion relation, we simply set $\Omega = \Omega_L$ and take

$$\hat{p}(z) = (D)(1/c^2)e^{-\int_0^z (g/c^2) dz} \quad (6.4.8)$$

as is appropriate to the Lamb mode in lowest order. Here D is any constant. In this manner, we obtain

$$1 - c_L^2 k^2/\Omega_L^2 = -12\mu_d(k^2 - k_d^2)/\Omega_L \quad (6.4.9)$$

where

$$2\mu_d = \frac{\int \{c^2[4/3 \eta + \zeta] + (\gamma - 1)\kappa/c_v\}\phi^{-2}\rho_0^{-1} dz}{\int \phi^{-2} dz} \quad (6.4.10a)$$

$$k_d^2 = \frac{\int (\gamma - 1)^2 (g^2/c^2)[\eta + (\gamma - 1)\kappa/c_v]\phi^{-2}\rho_0^{-1} dz}{\int \{c^2[(4/3)\eta + \zeta] + (\gamma - 1)\kappa/c_v\}\phi^{-2}\rho_0^{-1} dz} \quad (6.4.10b)$$

and where

$$\phi^{-2} = (1/c^2) e^{-\frac{2}{\gamma} \int_0^z (g/c^2) dz} \quad (6.4.11)$$

is the same as used previously.

Then to first order in μ_d we find

$$\Omega_L = c_L k - i\mu_d(k^2 - k_d^2) \quad (6.4.12)$$

which corresponds to the wave equation

$$\frac{\partial \psi}{\partial t} + (c_L + v_{Lk}) \frac{\partial \psi}{\partial s} - \mu_d \left(\frac{\partial^2}{\partial s^2} + k_d^2 \right) \psi = 0 \quad (6.4.13)$$

This should be compared with Eq. (6.3.6). It should be noted that the term

$$-i\mu_d \left(\frac{\partial^2}{\partial s^2} + k_d^2 \right) \psi$$

represents the presence of dissipation. Thus we have a correction term to add to that equation. The general relation would be

$$\begin{aligned} \frac{\partial \psi}{\partial t} + \{ (c_L + v_{Lk} + a_{kk} + c_L[(\gamma + 1)/2\gamma] v_p(0)/p_0(0)) \} \frac{\partial \psi}{\partial s} \\ + h_{kk} (\partial^2 / \partial s^2 + k_{BL}^2) (\partial \psi / \partial s) - \mu_d (\partial^2 / \partial s^2 + k_d^2) \psi = 0 \end{aligned} \quad (6.4.14)$$

In analogy with the usual nomenclature, we might term this the Korteweg-de Vries-Burgers' equation for propagation in the Lamb mode.

The presence of the term k_d^2 is an interesting byproduct of the height stratification of the Lamb mode. Formally, it represents a negative damping and arises from the fact that there is a continuous transfer of energy from high altitudes to low altitudes (or conversely, depending on the wave's phase) due to the fact that the amplitudes of u and p/ρ_0 increase with altitude. In order for the wave to maintain the stratification associated with the Lamb mode, one must assume that this energy is continuously being extracted from the ambient medium. This k_d^2 term is important only for very low frequency propagation and would seem to imply that the mode is weakly unstable at sufficiently low frequencies.

We cannot ascertain whether this is a real instability or merely a fiction of our mathematical technique. However, in any event, the growth of the instability, if it did exist, would be of such a slow rate that it probably would not be possible to detect it in practice.

6.5 HORIZONTAL RAY PATHS

The discussion up to now has assumed the ambient atmosphere to be independent of horizontal coordinates x and y . If this is not so, we expect that much of the preceding can be salvaged if the variation with these coordinates is sufficiently slow. The propagation at long distances would still locally appear as propagation of planar waves (almost constant direction for the horizontal wave numbers k). Thus we might assume the energy (or whatever we might associate with the wave) propagates along horizontal ray paths.

Let us consider a particular characteristic feature of the waveform which is received at some time $\tau(\vec{x})$ at locations having a position \vec{x} on the ground. (The vector \vec{x} has only x and y components.) A line of constant $\tau(\vec{x})$ may be termed a wavefront. In the absence of dispersion, dissipation, and nonlinear effects (all of which we assume to be small) this wavefront moves out from the source with a speed c_L (the height-averaged speed) when viewed by someone moving with the local height-averaged wind velocity \vec{v}_L . Thus if someone moved with speed

$$\frac{d\vec{x}}{dt} = c_L \vec{e}_k + \vec{v}_L \quad (6.5.1)$$

he would always be on a wavefront (assuming he was initially on a wavefront). Here \vec{e}_k is the unit outward pointing normal to the wavefront

$$\vec{e}_k = \nabla\tau / |\nabla\tau| \quad (6.5.2)$$

Since, for small dt , one must have

$$\tau(\vec{x}) + dt = \tau(\vec{x} + [d\vec{x}/dt]dt)$$

from the identification of $d\vec{x}/dt$ as wavefront velocity, it follows that

$$\nabla\tau \cdot d\vec{x}/dt = 1$$

or

$$\nabla\tau \cdot \{c_L \nabla\tau / |\nabla\tau| + \vec{v}_L\} = 1$$

This gives us the following partial differential equation for τ
(the eikonal equation)

$$(\nabla\tau)^2 = \frac{(1 - \nabla\tau \cdot \vec{v}_L)^2}{c_L^2} \quad (6.5.3)$$

or, if we abbreviate

$$\vec{k} = \nabla\tau \quad (6.5.4)$$

we have

$$k^2 = (1 - \vec{k} \cdot \vec{v}_L)^2 / c_L^2 \quad (6.5.5).$$

Since \vec{k} has the units of inverse velocity, we refer to it as the wave slowness vector.

We next consider just how this parameter \vec{k} would vary with time when viewed by someone moving with the speed $c_L \vec{e}_k + \vec{v}_L$. We note that

$$\begin{aligned} \frac{d\vec{k}}{dt} &= \left(\frac{d\vec{x}}{dt} \cdot \nabla \right) \vec{k} = \{ (c_L \vec{e}_k + \vec{v}_L) \cdot \nabla \} \nabla\tau \\ &= \left\{ \frac{c_L^2 \nabla\tau}{1 - \nabla\tau \cdot \vec{v}_L} \right\} \cdot \nabla(\nabla\tau) + \vec{v}_L \cdot \nabla(\nabla\tau) \end{aligned}$$

where we have used Eqs. (6.5.2) and (6.5.3). Let us note that

$$\begin{aligned} [\nabla\tau \cdot \nabla] \nabla\tau &= \frac{\partial\tau}{\partial x_\alpha} \frac{\partial^2\tau}{\partial x_\alpha \partial x_\beta} \vec{e}_\beta = \frac{1}{2} \frac{\partial}{\partial x_\beta} [\nabla\tau]^2 \vec{e}_\beta \\ &= \frac{1}{2} \nabla [(\nabla\tau)^2] = \frac{1}{2} \nabla \left\{ \frac{(1 - \nabla\tau \cdot \vec{v}_L)^2}{c_L^2} \right\} \end{aligned}$$

$$= - \frac{(1 - \nabla \tau \cdot \vec{v}_L)^2}{c_L^3} \nabla c_L - \frac{(1 - \nabla \tau \cdot \vec{v}_L)}{c_L^2} \nabla [\nabla \tau \cdot \vec{v}_L]$$

Thus,

$$\begin{aligned} \frac{d\vec{k}}{dt} &= - \frac{(1 - \vec{k} \cdot \vec{v}_L)}{c_L} \nabla c_L - \nabla (\vec{k} \cdot \vec{v}_L) + \vec{v}_L \cdot \nabla \vec{k} \\ &= - \frac{(1 - \vec{k} \cdot \vec{v}_L)}{c_L} \nabla c_L - k_\alpha \nabla v_{L\alpha} + v_{L\alpha} [\partial k_\alpha / \partial x_\beta - \partial k_\beta / \partial x_\alpha] \vec{e}_\beta \end{aligned}$$

The last term vanishes since $\nabla \times \vec{k} = 0$. Using an identity from vector analysis, we then obtain

$$\frac{d\vec{k}}{dt} = - \frac{(1 - \vec{k} \cdot \vec{v}_L)}{c_L} \nabla c_L - (\vec{k} \cdot \nabla) \vec{v}_L - \vec{k} \times (\nabla \times \vec{v}_L) \quad (6.5.6)$$

The above relation plus the Eq. (6.5.1), which we rewrite as

$$\frac{d\vec{x}}{dt} = \frac{c_L^2 \vec{k}}{1 - \vec{k} \cdot \vec{v}_L} + \vec{v}_L \quad (6.5.7)$$

gives us two coupled vector equations (or four coupled scalar equations) which, given \vec{v}_L and c_L as functions of \vec{x} , and, given \vec{k} and \vec{x} at some time t_0 , enable us to determine a ray trajectory $\vec{x}(t)$, $\vec{k}(t)$, as a function of time t . The curve $\vec{x}(t)$ represents what we might term a horizontal ray path in the x, y plane. There are, in actuality, a family of such paths. We distinguish various members of the family by a parameter θ (whose precise definition is deferred to later) and accordingly write $\vec{k}(t, \theta)$ and $\vec{x}(t, \theta)$.

The basic assumption we make here is that propagation along a horizontal ray path is such that the dispersion, nonlinear distortion, and dissipation of the pulse is governed by only the state of the atmosphere immediately above the path. Thus we set the acoustic variables as being of the form

$$p = P(s, \theta, z) \psi(s, t, \theta) \quad (6.5.8a)$$

$$\vec{u} = \vec{U}(s, \theta, z) \psi(s, t, \theta) \quad (6.5.8b)$$

$$\rho = Q(s, \theta, z) \psi(s, t, \theta) \quad (6.5.8c)$$

where s is a parameter characterizing distance along the path (although not precisely equal to distance) and where ψ satisfies the partial differential equation (6.4.14) (i.e., the Korteweg-de Vries-Burgers equation) with the coefficients being considered as functions of s and θ .

A definition of s may be obtained from the fact that an increment ds represents a distance in the direction of \vec{e}_k . Thus, if one follows a horizontal ray path with the speed given by (6.5.7) one should have s changing at the rate

$$\frac{ds}{dt} = \vec{e}_k \cdot \frac{d\vec{x}}{dt} = c_L + v_{Lk}$$

Thus, if $d\ell$ is the increment of distance along the path, we have

$$\frac{ds}{d\ell} = \frac{c_L + v_{Lk}}{|\vec{c}_L \vec{e}_k + \vec{v}_L|} = \frac{c_L + v_{Lk}}{[c_L^2 + 2c_L v_{Lk} + v_L^2]^{1/2}} \quad (6.5.9)$$

It would appear that, in the usual case where $v_L^2 \ll c_L^2$, it would be adequate to take $ds/d\ell = 1$.

The remaining question we need consider is how the amplitude quantities P , \bar{U} , and Q vary with height z and with the parameter s . It would appear that the former variation should be that appropriate in the lowest order for the Lamb mode, at the appropriate point on the ground. Thus we might take

$$P = p_0^{1/\gamma} A(s, \theta) \quad (6.5.10a)$$

$$\bar{U} = [p_0^{1/\gamma} / \rho_0] \{ \vec{k}(s, \theta) / (1 - \vec{v}_L \cdot \vec{k}) \} A(s, \theta) \quad (6.5.10b)$$

$$Q = [p_0^{1/\gamma} / c_L^2] A(s, \theta) \quad (6.5.10c)$$

where the ambient pressure and density are considered as functions of z , s , and θ . The quantity $\vec{k}(s, \theta)$ is the wave slowness vector computed for the point in question from the ray tracing equations. One should note that we have assumed the ratios of \bar{U} , P , and Q to be always appropriate for a planar wave propagating in the Lamb mode. This would seem to be adequate at moderate distances from the source.

The s variation of the remaining factor $A(s, \theta)$, is determined from the geometrical acoustics law recently espoused by Bretherton and Garrett that a wave propagating in slowly varying inhomogeneous

moving media should propagate such as to conserve wave action, in the absence of nonlinear effects and dissipation. By wave action, one means simply the wave energy divided by the frequency of the wave, as would be seen by someone moving with the fluid. If dispersion is small this law simply means that

$$\nabla_H \cdot \left\{ \frac{(\vec{v}_g + \vec{v}_L) \langle E \rangle}{1 - \vec{k} \cdot \vec{v}_L} \right\} = 0 \quad (6.5.11)$$

where \vec{v}_g is the group velocity and E is the energy density (per unit area of earth surface) in the wave as would be computed for a homogeneous medium by someone moving with the local wind speed. The brackets imply a time average. Neglecting dispersion, we have

$$\vec{v}_g = [c_L^2 / (1 - \vec{k} \cdot \vec{v}_L)] \vec{k} \quad (6.5.12)$$

$$E = \int_{z_g}^{\infty} \{ (1/2) \rho_0 \vec{u}^2 + (1/2) p^2 / [\rho_0 c^2] \} dz \quad (6.5.13)$$

with the neglect of the very small kinetic energy of vertical motion associated with the Lamb mode and with the neglect of the correspondingly small change in gravitational potential energy. Here z is the height of the earth's surface. Since, in the absence of nonlinear terms and dissipation, ψ is just a constant times $\cos [\omega t - ks + \chi]$ where χ is a constant phase for constant frequency waves and since the remaining factors are independent of time, it would appear that $\langle E \rangle$ may be taken proportional to

$$\begin{aligned} \langle E \rangle &\approx A^2 \int_{z_g}^{\infty} [p_0^{2/\gamma} / (\rho_0 c_L^2)] dz \\ &\approx A^2 c_L^{-2} \int_{z_g}^{\infty} [p_0^{2/\gamma} / \rho_0] dz \end{aligned} \quad (6.5.14)$$

Thus we have

$$\nabla_{\parallel} \cdot \left\{ \frac{A^2 (\vec{v}_g + \vec{v}_L) c_L^{-2}}{1 - \vec{k} \cdot \vec{v}_L} \int_{z_g}^{\infty} [p_o^{2/\gamma} / \rho_o] dz \right\} = 0$$

If we now integrate this relation over a narrow segment of a ray tube bounded by two adjacent rays and apply Gauss' theorem, we obtain (after some algebra)

$$\frac{d}{ds} \left\{ \left[\int_{z_g}^{\infty} [p_o^{2/\gamma} / \rho_o] dz \right] (A^2 / c_L^3) (c_L + v_{Lk})^2 J \right\} = 0 \quad (6.5.15)$$

where J is the Jacobian

$$J = \left| \frac{\partial x}{\partial s} \frac{\partial y}{\partial \theta} - \frac{\partial x}{\partial \theta} \frac{\partial y}{\partial s} \right| \quad (6.5.16)$$

in the event the earth's surface is considered flat.

We may also extend the above analysis to take into account earth curvature by simply replacing J by

$$J = r_e \sin(r/r_e) |[(\partial r / \partial s)(\partial \phi / \partial \theta) - (\partial r / \partial \theta)(\partial \phi / \partial s)]| \quad (6.5.17)$$

where r is great circle distance from the source, r_e is the radius of the earth, and ϕ is the azimuth angle location of the observer, taking the source as being on the axis.

Once J is determined as a function of s for fixed θ , we may regard Eq. (6.5.15) as an ordinary differential equation; the quantity in braces should be a constant along the ray path and, following a terminology used in sonic boom studies, may be termed the Blokhintzev invariant for the propagation. Note that z could be a function of x and y. Thus the formalism also takes into account the possibility of gradual variations in ground elevation.

6.6 EXCITATION OF LAMB'S MODE

In order to solve the various approximate equations derived in the previous section, some initial conditions are required. The various equations we have derived allow us to: (1) determine the horizontal ray paths given an initial point on the path; (2) determine the amplitude factor $A(s, \theta)$ given its value at the start

of the path; and (3) determine the waveform function $\psi(s, \theta, t)$ given its s dependence at some initial time. In the present section we seek to determine these various initial conditions for waves launched by a low altitude nuclear explosion.

Let the explosion be on the z axis such that $x = 0, y = 0$ at the source. Since all waves must originate from the source, it is clear that all horizontal ray paths should ensue from the point $(0, 0)$. The initial path direction could be any angle between 0 and 2π . We accordingly choose the angle θ to be the initial direction of \vec{k} , reckoned counterclockwise with respect to the x axis. The initial value of \vec{k} for any given path is then found from Eq. (6.5.5) to be given by

$$\vec{k}(0, \theta) = \frac{\vec{n}_o(\theta)}{c_L + \vec{v}_L \cdot \vec{n}_o(\theta)} \quad (6.6.1)$$

where $\vec{n}_o(\theta)$ is that unit vector making an angle of θ with the x axis. The appropriate values of c_L and \vec{v}_L should be those corresponding to the source location. The remaining initial condition is

$$\vec{x}(0, \theta) = 0 \quad (6.6.2)$$

for all θ . Thus, the horizontal ray paths are completely determined with the integration of Eqs. (6.5.6) and (6.5.7).

The initial conditions on $A(s, \theta)$ and $\psi(s, \theta, t)$ are obtained with reference to the intermediate field solution for a point source in a temperature- and wind-stratified atmosphere. Since the viewpoint adopted in the present chapter is that the only principal effect of the temperature variation and wind profile variations with altitude is to cause the Lamb mode to be dispersed and since the dispersion does not have appreciable effect until relatively large distances, it would appear sufficient to calculate the intermediate field on the supposition that the atmosphere is isothermal and has constant winds. In this event, the Eqs. (2.1.4) reduce to

$$\begin{aligned} D_t^2(D_t^2 + \omega_A^2)(p/\sqrt{\rho_o}) - c^2 \nabla_H^2(D_t^2 + \omega_B^2)(p/\sqrt{\rho_o}) - c^2 D_t^2(\partial^3/\partial z^2)(p/\sqrt{\rho_o}) \\ = (4\pi c^2/\sqrt{\rho_o}) D_t [D_t^2 - g(\partial/\partial z)] [f_E(t) \delta(\vec{r} - \vec{r}_o)] \end{aligned} \quad (6.6.3)$$

with the boundary condition

$$dp/dz + (g/c^2)p = 0 \quad \text{at } z = 0 \quad (6.6.4)$$

To isolate the Lamb mode, we write the overpressure as

$$p = p_0^{1/\gamma}(z)F(x,y,t) + \psi \quad (6.6.5)$$

where the first term represents the contribution from the Lamb mode and ψ represents any remaining contribution. An orthogonality condition is readily derived which guarantees that

$$\int_{z_g}^{\infty} \psi p_0^{1/\gamma}/\rho_0 dz = 0 \quad (6.6.6)$$

Thus we find the function F satisfies

$$\begin{aligned} (D_t^2 + \omega_{\perp}^2)(D_t^2 - c^2 \nabla_H^2)F = \\ \frac{\int_{z_g}^{\infty} (4\pi c^2)(p_0^{1/\gamma}/\rho_0)D_t(D_t^2 - g(\partial/\partial z))[f_E(t)\delta(\vec{r} - \vec{r}_0)] dz}{\int_{z_g}^{\infty} (p_0^{2/\gamma}/\rho_0) dz} \end{aligned}$$

or

$$(D_t^2 - c^2 \nabla_{\perp}^2)F = 4\pi c^2 Q D_t[f_E(t)\delta(x - x_0)\delta(y - y_0)] \quad (6.6.7)$$

where

$$Q = \frac{p_0^{1/\gamma}(z_0)/\rho_0(z_0)}{\int_{z_g}^{\infty} [p_0^{2/\gamma}/\rho_0] dz} \quad (6.6.8)$$

To simplify the analysis, we assume that the source is drifting horizontally with the wind speed. In a coordinate system moving with the source, the solution of (6.6.7) is readily found to be

$$F = 2Qc \int_{-\infty}^{t-R/c} \frac{f'_E(\tau) d\tau}{[c^2(t-\tau)^2 - R^2]^{1/2}} \quad (6.6.9)$$

where R is the net horizontal distance from the source. The only distinction for the fixed coordinate system is that we replace R by R^* where

$$(R^*)^2 = (\vec{x} - \vec{v}t)^2 \quad (6.6.10)$$

Since the excited pulse is of relatively short duration, we may consider the dominant contribution in the integration to come from values of τ near where $c(t-\tau) \approx R$. In this event, Eq. (6.6.9) simplifies to

$$F = (2c/R)^{1/2} Q \int_{-\infty}^{t-R/c} \frac{f'_E(\tau) d\tau}{[(t-\tau) - R/c]^{1/2}} = (2c/R)^{1/2} Q G(t - R/c) \quad (6.6.11)$$

where

$$G(t) = \int_{-\infty}^t \frac{f'_E(\tau) d\tau}{[t-\tau]^{1/2}} \quad (6.6.12)$$

is similar to the Whitham F -function utilized in the theory of sonic boom propagation.

The function $G(t)$ can be evaluated with recourse to Eq. (2.1.5b). We find that

$$G(t) = Y_{KT}^{1/2} [p_o(z_o)/p_o(0)]^{1/2} [c(0)/c(z_o)]^{1/2} L_s P_s t_s^{1/2} M(t/T_Y)$$

where

$$T_Y = [c(0)/c(z_0)][p_0(0)/p_0(z_0)]^{1/3} \frac{1}{Y^{1/3}} \frac{1}{KT} t_s$$

$$M(X) = \int_0^X (1 - \xi) e^{-\xi} [X - \xi]^{-1/2} d\xi U(X)$$

$$= \{ \sqrt{X} + (1 - 2X) e^{-X} \int_0^{\sqrt{X}} e^{y^2} dy \} U(X) \quad (6.6.13)$$

The various symbols used above have the same meaning as in Chapter 2.

If the source time duration T_Y is sufficiently short or if the winds are sufficiently weak, such that at moderate distances R , one has $R \gg |\vec{v}_L| T_Y$, then it would appear sufficient to take

$$R^* = c_L s / (c_L + v_{Lk}) \quad (6.6.14)$$

where s is the distance parameter defined by Eq. (6.5.9).

Since, with the above approximation, the only t dependence is the function M , it would appear that we can choose

$$\psi(t, s, \theta)_{t=0} = M(-s / [(c_L + v_{Lk}) T_Y]) \quad (6.6.15)$$

where the range of s is formally considered as encompassing negative as well as positive values.

The remaining quantity of interest, the amplitude factor A , is given by Eq. (6.5.15) as

$$A = \frac{B c_L^{3/2}}{J^{1/2} (c_L + v_{Lk}) I^{1/2}} \quad (6.6.16)$$

where J is the Jacobian and I is the integral

$$I = \int_{z_g}^{\infty} [p_0^{2/\gamma} / \rho_0] dz \quad (6.6.17)$$

The constant B is a quantity which we may obtain with detailed comparison with the intermediate range solution. Since, for small s, we can show that

$$J = \frac{sc_L}{c_L + v_{Lk}} \quad (6.6.18)$$

we identify

$$B = \left[\left\{ \frac{KY_{KT}^{1/2} (c_L + v_{Lk})}{I^{1/2} c_L^{3/2}} \right\} p_o^Y(z_o) / \rho_o(z_o) \right]_{x=0, y=0} \quad (6.6.19)$$

where

$$K = \{ \sqrt{2} L_s p_s (c_L t_s)^{1/2} [c(0)/c(z_o)]^{1/2} [p_o(z_o)/p_o(0)]^{1/2} \}_{x=0, y=0} \quad (6.6.20)$$

Thus, in summary, we have the acoustic pressure given by

$$p = KY_{KT}^{1/2} D(\vec{x}) D(\vec{x}_o) [\Lambda(\vec{x}) / \Lambda(\vec{x}_o)] \psi(t, s, \theta) / J^{1/2} \quad (6.6.21)$$

where

$$D(\vec{x}) = \frac{p_o / \rho_o^{1/2}}{\left\{ \int_{z_g}^{\infty} [p_o^{2\gamma} / \rho_o] dz \right\}^{1/2}} \quad (6.6.22a)$$

$$\Lambda(\vec{x}) = \rho_o^{1/2} [c_L^{3/2} / (c_L + v_{Lk})] \quad (6.6.22b)$$

The quantity K is given by Eq. (6.6.20) while the Jacobian J is given by Eq. (6.5.16) or Eq. (6.5.17). The quantity ψ satisfies the Eq. (6.4.14), with the initial condition (6.6.15).

6.7 SOLUTION OF THE LINEARIZED KORTEWEG-DE VRIES EQUATION

The determination of the waveform profile $\psi(t,s,\theta)$ is probably the chief computational obstacle to the procedure outlined in the preceding sections. Here we outline the method of solution in the neglect of nonlinear and dissipation terms. This may be an adequate approximation for all cases of interest, although one cannot say this with certainty until he has some quantitative estimates of the effects of the neglected terms.

With the neglect of dissipation and nonlinear terms the Eq. (6.4.14) becomes

$$\partial\psi/\partial t + c_e \partial\psi/\partial s + D \partial^3\psi/\partial s^3 = 0 \quad (6.7.1)$$

where

$$c_e = c_L + v_{Lk} + a_{kk} + h_{kk} k_{BL}^2 \quad (6.7.2a)$$

$$D = h_{kk} \quad (6.7.2b)$$

or, to the same approximation,

$$\partial\psi/\partial t + c_e \partial\psi/\partial s - (D/c_e^3) \partial^3\psi/\partial t^3 = 0 \quad (6.7.3)$$

To put this in a form appropriate to the case where c_e and D are slowly varying functions of s , we consider ψ to be a function of parameters \bar{t} and \bar{s} , where

$$\bar{t} = t - \int_0^s (1/c_e) ds \quad (6.7.4a)$$

$$\bar{s} = \int_0^s (D/c_e^4) ds \quad (6.7.4b)$$

such that Eq. (6.7.3) becomes

$$\frac{\partial\psi}{\partial\bar{s}} - \frac{\partial^3\psi}{\partial\bar{t}^3} = 0 \quad (6.7.5)$$

If ψ is specified when $\bar{s} = 0$ as $\psi_0(\bar{t})$, then the solution to Eq. (6.7.5) may be found, after some analysis, to be given by

$$\psi(\bar{s}, \bar{t}) = \frac{1}{\sqrt{\pi} (3\bar{s})^{1/3}} \int_{-\infty}^{\infty} \text{Ai} \left(\frac{\bar{t}_0 - \bar{t}}{(3\bar{s})^{1/3}} \right) \psi_0(\bar{t}_0) d\bar{t}_0 \quad (6.7.6)$$

where $\text{Ai}(x)$ is the Airy function defined by

$$\text{Ai}(x) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} \cos(v^3/3 + xv) dv \quad (6.7.7)$$

In terms of s and t we have

$$\psi(s, t) = \frac{1}{\sqrt{\pi} \tau_D} \int_{-\infty}^{\infty} \text{Ai} \left(\frac{t_0 + \tau_c - t}{\tau_D} \right) \psi_0(t_0) dt_0 \quad (6.7.8)$$

where τ_c and τ_D are functions of s given by

$$\tau_c = \int_0^s (1/c_e) ds \quad (6.7.9a)$$

$$\tau_D = \left\{ 3 \int_0^s (D/c_e^4) ds \right\}^{1/3} \quad (6.7.9b)$$

For the problem of interest, we find from reasoning similar to that which leads to Eq. (6.6.15), that

$$\psi_0(t_0) = M(t_0/T_Y)$$

where the function M is as given by (6.6.13). Thus

$$\begin{aligned}
\psi(s, \theta, t) &= \frac{1}{\sqrt{\pi} \tau_D} \int_{-\infty}^{\infty} \text{Ai} \left(\frac{t_0 + \tau_c - t}{\tau_D} \right) M(t_0/T_Y) dt_0 \\
&= \frac{1}{\sqrt{\pi} \tau_D} \int_0^{\infty} \text{Ai} \left(\frac{\tau_c - t}{\tau_D} + \mu \frac{T_Y}{\tau_D} \right) M(\mu) d\mu \quad (6.7.10)
\end{aligned}$$

with a change of integration variable and recognition of the fact that $M = 0$ for $\mu < 0$.

It is of some interest to examine Eq. (6.7.10) in the limit of small yields as it is not immediately apparent that the well-known law of yield-amplitude proportionality holds for the single mode model. The correct behavior in this limit is somewhat subtle since the function $M(\mu)$ is not integrable. With reference to Eq. (6.6.13), we write

$$M(\mu) = \int_0^{\mu} [f(\xi)/(\mu - \xi)^{1/2}] d\xi \quad (6.7.11)$$

where

$$f(\xi) = (1 - \xi)e^{-\xi} \quad (6.7.12)$$

Then Eq. (6.7.10) becomes

$$\begin{aligned}
\psi &= \frac{T_Y/\tau_D}{\sqrt{\pi}} \int_0^{\infty} d\mu \int_0^{\infty} d\xi \text{Ai} \left(\frac{\tau_c - t}{\tau_D} + \mu \frac{T_Y}{\tau_D} \right) f(\xi) \frac{U(\mu - \xi)}{(\mu - \xi)^{1/2}} \\
&= \frac{T_Y/\tau_D}{\sqrt{\pi}} \int_0^{\infty} f(\xi) \int_{\xi}^{\infty} \text{Ai} \left(\frac{\tau_c - t}{\tau_D} + \mu \frac{T_Y}{\tau_D} \right) \frac{1}{(\mu - \xi)^{1/2}} d\mu d\xi
\end{aligned}$$

In the integral over μ we set $\mu - \xi = (\alpha^2)(\tau_D/T_Y)$ and obtain

$$\psi = \frac{2(T_Y/\tau_D)^{1/2}}{\sqrt{\pi}} \int_0^{\infty} f(\xi) \left\{ \int_0^{\infty} A1 \left[\frac{\tau_c - t}{\tau_D} + \xi \frac{T_Y}{\tau_D} + \alpha^2 \right] d\alpha \right\} d\xi \quad (6.7.11)$$

We next expand the integrand in a power series in T_Y/τ_D . Since

$$\int_0^{\infty} f(\xi) d\xi = 0 \quad (6.7.12)$$

the first non-zero term is

$$\psi = - \frac{2(T_Y/\tau_D)^{3/2}}{\sqrt{\pi}} \left\{ \int_0^{\infty} f(\xi) \xi d\xi \right\} \left\{ \int_0^{\infty} A1' \left[\frac{\tau_c - t}{\tau_D} + \alpha^2 \right] d\alpha \right\}$$

or

$$\psi = (2/\sqrt{\pi}) (T_Y/\tau_D)^{3/2} PP([t - \tau_c]/\tau_D) \quad (6.7.13)$$

where $PP(x)$ is a function defined by

$$PP(x) = \int_0^{\infty} A1' (\alpha^2 - x) d\alpha \quad (6.7.14)$$

According to Eq. (6.6.12), the overpressure varies with yield as $Y_{KT}^{1/2}$. However, the above shows that ψ varies with yield as $T_Y^{3/2}$. But T_Y varies with yield as $Y_{KT}^{1/3}$. Thus the amplitude is directly proportional to yield and the waveform shape is independent of yield in the limit of small yields. The limit applies, strictly speaking, when $T_Y/\tau_D \ll 1$ and is accordingly more appropriate at larger distances and for propagation with strong dispersion (large D).

6.8 SUMMARY

Although the results are reasonably simple the derivation of the single mode theory given in the present chapter is somewhat intricate. It would therefore seem appropriate to pause here and summarize the various results scattered throughout the chapter from an operational point of view.

The basic assumption is that the earliest portion of the wave which arrives with transit speeds of the order of the sound speed at the ground may be interpreted as being caused by a single guided mode, which is the real atmosphere's counterpart of Lamb's guided mode for the isothermal atmosphere. The dispersion of this mode is small, but important. However, this dispersion is neglected in determining the ground projected ray paths along which the mode travels.

The determination of these ground projected paths is the same as for two dimensional acoustics in a medium having sound speed c_L (L for Lamb) and wind velocity v_L . Both c_L and v_L are in general functions of position on the earth's surface and are averages over height z of the sound speed and wind velocity profiles. The manner in which these averages should be taken turns out to be

$$c_L^2 = \frac{\int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o] dz}{\int_{z_g}^{\infty} \{p_o^{2/\gamma}/[\rho_o c^2]\} dz} \quad (6.8.1)$$

$$\vec{v}_L = \frac{\int_{z_g}^{\infty} \vec{v} [p_o^{2/\gamma}/\rho_o] dz}{\int_{z_g}^{\infty} [p_o^{2/\gamma}/\rho_o] dz} \quad (6.8.2)$$

where p_o and ρ_o are ambient pressure and density, γ is the specific heat ratio, and z_g is the ground level.

The rays all start out on a point on the ground directly below the source and are distinguished from each other by a parameter θ which ranges from 0 to 2π . A given ray may be characterized by giving the position $x_H(s, \theta)$ and wave slowness vector $k(s, \theta)$ as functions of s , where s is a function of distance along the path (which is the same as distance in the limit of no winds).

Along or above a given path the acoustic pressure p is given by

$$p = KY_{KT}^{1/2} D(\vec{x}) D(\vec{x}_0) [\Lambda(\vec{x}) / \Lambda(\vec{x}_0)] \psi(t, s, \theta) / J^{1/2} \quad (6.8.3)$$

where

$$K = \{\sqrt{2} L_s P_s (c_L t_s)^{1/2} [c(0)/c(z_0)]^{1/2} [p_0(z_0)/p_0(0)]^{1/2}\}_{\text{source}} \quad (6.8.4a)$$

$$D(\vec{x}) = \frac{p_\bullet^\gamma / \rho_o^{1/2}}{\left[\int_{z_g}^{\infty} [p_o^{2/\gamma} / \rho_o] dz \right]^{1/2}} \quad (6.8.4b)$$

$$\Lambda(\vec{x}) = \rho_o^{1/2} [c_L^{3/2} / (c_L + v_{Lk})] \quad (6.8.4c)$$

$$L_s = 1 \text{ kilometer} \quad (6.8.4d)$$

$$P_s = 1.61 \times 34.45 \times 10^3 \text{ dynes/cm}^2 \quad (6.8.4e)$$

$$t_s = 0.48 \text{ sec.} \quad (6.8.4f)$$

$$J = r_e \sin(r/r_e) [\partial r / \partial s (\partial \phi / \partial \theta) - (\partial r / \partial \theta) (\partial \phi / \partial s)] \quad (6.8.4g)$$

$$r = \text{great circle distance from source} \quad (6.8.4h)$$

$$\phi = \text{azimuth angle of observer location} \quad (6.8.4i)$$

$$v_{Lk} = \vec{v}_L \cdot \vec{k} / |\vec{k}| \quad (6.8.4j)$$

The quantity $\psi(t, s, \theta)$ satisfies a partial differential equation known as the Korteweg-de Vries-Burgers' equation

$$\partial \psi / \partial t + [c_e + \beta] \partial \psi / \partial s + D \partial^3 \psi / \partial s^3 - \mu_d [\partial^2 / \partial s^2 + k_d^2] \psi = 0 \quad (6.8.5)$$

where

$$c_e = c_L + v_{Lk} + a_{kk} + h_{kk} k_{BL}^2 \quad (6.8.6a)$$

$$\beta = c_L [(\gamma + 1) / (2\gamma)] v_p(z_g) / p_o(z_g) \quad (6.8.6b)$$

$$D = h_{kk} \quad (6.8.6c)$$

The remaining quantities a_{kk} , h_{kk} , v , μ_d , \vec{k}_d , and \vec{k}_{BL} are defined in Eqs. (6.2.20), (6.3.4) and (6.4.10).

In the derivation of Eq. (6.8.5), it was assumed that the terms with coefficients β , D , and μ_d were all relatively small. It was also assumed that all the coefficients were slowly varying. Thus to the same order of approximation, one may set $\partial/\partial s = -c_e^{-1} \partial/\partial t$ in the higher order terms and obtain the alternate form

$$\begin{aligned} \partial\psi/\partial s + c_e^{-1} \partial\psi/\partial t - (\beta/c_e^2) \partial\psi/\partial t - (D/c_e^4) \partial^3\psi/\partial t^3 \\ - (\mu_d/c_e^3) (\partial^2/\partial t^2 + c_e^2 k_d^2) \psi = 0 \end{aligned} \quad (6.8.7)$$

Either of the forms (6.8.5) or (6.8.7) may be used.

If one chooses to use Eq. (6.8.7), then $\psi(s, \theta, t)$ must be specified when $s = 0$. The choice prescribed by the analysis is

$$\psi(0, \theta, t) = M(t/T_Y) \quad (6.8.8)$$

where

$$M(x) = \left\{ \sqrt{x} + (1 - 2x) e^{-x} \int_0^{\sqrt{x}} e^{y^2} dy \right\} U(x) \quad (6.8.9a)$$

$$T_Y = \left\{ [c(z_g)/c(z_o)] [p_o(z_g)/p_o(z_o)]^{1/3} Y_{KT}^{1/3} t_s \right\}_{\text{source}} \quad (6.8.9b)$$

We succeeded in solving the initial value problem when $\beta = \mu_d = 0$ and found

$$\psi = \frac{T_Y/\tau_D}{\sqrt{\pi}} \int_0^\infty A1 \left(\frac{\tau_c - t}{\tau_D} + \mu \frac{T_Y}{\tau_D} \right) M(\mu) d\mu \quad (6.8.10)$$

where

$$\tau_c = \int_0^s (1/c_e) ds \quad (6.8.11a)$$

$$\tau_D = \left\{ 3 \int_0^s (D/c_e^4) ds \right\}^{1/3} \quad (6.8.11b)$$

Furthermore, in the limit of small yields, we find

$$\psi = (2/\sqrt{\pi}) (T_Y/\tau_D)^{3/2} PP([t - \tau_c]/\tau_D) \quad (6.8.12)$$

where

$$PP(x) = \int_0^\infty \Lambda i'(\alpha^2 - x) d\alpha \quad (6.8.13)$$

which demonstrates yield-amplitude proportionality.

In conclusion, the authors state that this approach appears extremely promising. An initial test of the theory will be to see how well its predictions agree with the computations performed using INFRASONIC WAVEFORMS for stratified atmospheres. If this works out, then we may expect to have a number of interesting areas to explore. One hope is that we may be able to explain data such as presented by Wexler and Hass in detail.

Appendix A

BIBLIOGRAPHY ON INFRASONIC WAVES

The following bibliography is a compendium of papers and books which have come to the attention of the authors as having some relation (either direct or indirect) to the long range propagation of mechanical radiation through the atmosphere. While no claims are made as to its completeness, it is the most comprehensive and up-to-date bibliography specifically restricted to this topic of which the authors are currently aware.

For ease of referral, the subject matter has been broken down into a number of categories as follows:

1. Books on acoustics, wave propagation, hydrodynamics and mathematical physics
2. Meteorology, including data on atmospheric structure
3. Theoretical papers on acoustic-gravity waves and gravity waves in the atmosphere
4. Theoretical papers on higher frequency atmospheric waves
5. Observations of infrasonic waves in the lower atmosphere
6. Observations of infrasonic waves in the ionosphere
7. Data concerning the properties of nuclear explosions
8. Related papers on the mathematical theory of wave propagation, and on mathematical techniques useful in wave propagation
9. Nonlinear effects on wave propagation, including shock waves
10. Instrumentation
11. Data analysis techniques

The classification scheme is not mutually exclusive, although we have generally classified each reference under only one heading.

This bibliography is an updated and expanded version of one given previously in 1967 by Pierce and Moo. Since we hope at some later date to issue, in turn, a revised version of the present bibliography, we request that readers notify the authors of any neglected papers, errors, etc.

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A.3 THEORETICAL PAPERS ON ACOUSTIC-GRAVITY WAVES AND GRAVITY WAVES IN THE ATMOSPHERE

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Appendix B

Deck Listing

of

INFRASONIC WAVEFORMS


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CENAM6 YIELD= SEND MAIN 71
CENAM10 TFIRST= , TEND= , DELTT= , ROBS= , IOPT= SEND MAIN 72
C MAIN 73
C *****NSTART=4**** MAIN 74
CENAM7 OMNOD= , , .... VPMOD= , .... MODFO= , ETC. MAIN 75
CENAM8 YIELD= SEND MAIN 76
CENAM10 TFIRST= , TEND= , DELTT= , ROBS= , IOPT= SEND MAIN 77
C MAIN 78
C *****NSTART=5**** MAIN 79
CENAM9 MODFND= , KST= , .... KFIN= , .... OMNOD= , , ETC. MAIN 80
CENAM10 TFIRST= , TEND= , DELTT= , ROBS= , IOPT= SEND MAIN 81
C MAIN 82
C *****NSTART=6**** MAIN 83
C (NO ADDITIONAL DATA IS NEEDED. COMPUTATION TERMINATES.) MAIN 84
C MAIN 85
C FOR A COMPLETE LIST OF VARIABLES THAT ARE INCLUDED IN A GIVEN MAIN 86
C NAMELIST GROUP, SEE NAMELIST STATEMENTS IN PROGRAM. NOTE THAT MAIN 87
C DATA INPUT BY READ(5,NAM1), READ(5,NAM2), ETC., NEED NOT INCLUDE MAIN 88
C VALUES OF ALL VARIABLES IN THE CORRESPONDING NAMELIST GROUP. ONE MAIN 89
C NEED ONLY INPUT THOSE VALUES NEEDED FOR THE CALCULATION AND WHICH MAIN 90
C ARE NOT ALREADY IN STORAGE. MAIN 91
C MAIN 92
C DATA ASSOCIATED WITH NAM3, NAM5, NAM7, AND NAM9 SHOULD IN GENERAL MAIN 93
C NOT BE SUPPLIED ARBITRARILY, BUT MAY BE OBTAINED FROM PREVIOUS MAIN 94
C RUNS OF THE PROGRAM. IF NSTART=1, NPNCH=1, DATA CARDS FOR NAM3, MAIN 95
C NAM5, NAM7, AND NAM9 ARE AUTOMATICALLY PUNCHED. IF NSTART=2, MAIN 96
C NPNCH=1, DATA CARDS FOR NAM5, NAM7, AND NAM9 ARE PUNCHED. IF MAIN 97
C NSTART=3, NPNCH=1, DATA CARDS FOR NAM7 AND NAM9 ARE PUNCHED. IF MAIN 98
C NSTART=4, NPNCH=1, DATA CARDS FOR NAM9 ARE PUNCHED. MAIN 99
C THE NEXT BATCH OF DATA AFTER NAM10 SHOULD BE NAM1. THE LAST DATA MAIN 100
C CARD SHOULD BE NAM1 WITH NSTART=6. MAIN 101
C MAIN 102
C -----EXTERNAL SUBROUTINES REQUIRED----- MAIN 103
C MAIN 104
C SUBROUTINE TYPE CALLED BY MAIN 105
C AAAA SUB ELINT,MMM,MAMPDE,NMODF MAIN 106
C AKI SUB THPT MAIN 107
C ALLMOD SUB MAIN 108
C AMBNT SUB MAMPDE MAIN 109
C ATMOS SUB MAIN 110
C AXIS1 SUB THPT (M.I.T. CALCOMP ROUTINE) MAIN 111
C BRRB SUB ELINT MAIN 112
C CAI FUNC BRRB,MMM MAIN 113
C DODY1 SUB THPT (M.I.T. CALCOMP ROUTINE) MAIN 114
C ELINT SUB TOTINT MAIN 115
C ENDPLT SUB THPT (M.I.T. CALCOMP ROUTINE) MAIN 116
C FNMOD1 FUNC MODETR (EXTERNAL FOR ARG. OF RTMI) MAIN 117
C FNMOD2 FUNC MODETR (EXTERNAL FOR ARG. OF RTMI) MAIN 118
C LGTHN SUB TABLE MAIN 119
C MMM SUB MAMPDE,RRRR MAIN 120
C MODETR SUB ALLMOD MAIN 121
C MODLST SUB MAIN MAIN 122
C MPOUT SUB TABLE MAIN 123
C MAMPDE SUB MAMPDE MAIN 124
C NEWPLY SUB THPT (M.I.T. CALCOMP ROUTINE) MAIN 125
C NMODF SUB FNMOD1,FNMOD2,LGTHN,MPOUT,WIDEN MAIN 126
C NUMBRI SUB THPT (M.I.T. CALCOMP ROUTINE) MAIN 127
C NXMODE SUB ALLMOD MAIN 128
C NXPNT SUB MODETR MAIN 129
C PAMPDE SUB MAIN MAIN 130
C PHASE SUB SOURCE MAIN 131
C PLOT1 SUB THPT (M.I.T. CALCOMP ROUTINE) MAIN 132
C PPAMP SUB MAIN MAIN 133
C PRATMO SUB MAIN MAIN 134
C RRRR SUB NMODF MAIN 135
C RTMI SUB MODETR (IBM SCIENTIFIC SUBROUTINE) MAIN 136
C SAI FUNC BRRB,MMM MAIN 137
C SCLGPH SUB THPT (M.I.T. CALCOMP ROUTINE) MAIN 138

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PROGRAM
 MAIN
 PAGE
 2

C	SOURCE	SUB	PPAMP	MAIN	141
C	SUSPECT	SUB	TABLE	MAIN	142
C	SYMBOLS	SUB	TMPT (N.I.T. CALCOMP ROUTINE)	MAIN	143
C	TABLE	SUB	MAIN	MAIN	144
C	TABPRT	SUB	MAIN	MAIN	145
C	TMPT	SUB	MAIN	MAIN	146
C	TOTINT	SUB	NAMPDE	MAIN	147
C	UPINT	SUB	TOTINT	MAIN	148
C	USEAS	SUB	TOTINT	MAIN	149
C	WIDEN	SUB	TABLE	MAIN	150
C				MAIN	151
C			----INPUTS THROUGH NAMELIST READ STATEMENTS----	MAIN	152
C				MAIN	153
C	NAM1 -- NAMELIST GROUP 1			MAIN	154
C				MAIN	155
C	NSTART	=FLAG DENOTING POINT IN MAIN PROGRAM AT WHICH COMPUTA-		MAIN	156
C		TION BEGINS. POSSIBLE VALUES OF 1 THROUGH 5 CAUSE		MAIN	157
C		NAM2, NAM3, NAM5, NAM7, OR NAM9 TO BE READ. NSTART=6		MAIN	158
C		CAUSES TERMINATION OF PROGRAM EXECUTION.		MAIN	159
C	NPRNT	=FLAG FOR PRINTING OPTION. IF NPRNT .LE. 0, A MINIMAL		MAIN	160
C		AMOUNT OF PRINTOUT WILL BE RETURNED.		MAIN	161
C	NPNCN	=FLAG FOR PUNCHING OPTION. IF NPNCN .LE. 0, NO INFO		MAIN	162
C		WILL BE PUNCHED ON CARDS.		MAIN	163
C				MAIN	164
C	NAM2 -- NAMELIST GROUP 2			MAIN	165
C				MAIN	166
C	LANGLF	=INTEGER WHICH SPECIFIES WHICH TYPE OF ATMOSPHERIC DAT		MAIN	167
C		IS INPUT. IF LANGEL .LE. 0, THE WIND COMPONENTS IN		MAIN	168
C		KNOTS ARE SPECIFIED, WHILE IF LANGE .GT. 0, THE WIND		MAIN	169
C		MAGNITUDE AND DIRECTION ARE SPECIFIED FOR EACH LAYER.		MAIN	170
C	IMAX	=NUMBER OF LAYERS OF FINITE THICKNESS IN MULTILAYER		MAIN	171
C		ATMOSPHERE.		MAIN	172
C	T(I)	=TEMPERATURE IN DEGREES KELVIN IN THE I-TH LAYER.		MAIN	173
C	VKNTX(I)	=X (WEST TO EAST) COMPONENT OF WIND VELOCITY IN I-TH		MAIN	174
C		LAYER.		MAIN	175
C	VKNTY(I)	=Y (SOUTH TO NORTH) COMPONENT OF WIND VELOCITY IN I-TH		MAIN	176
C		LAYER.		MAIN	177
C	WINDY(I)	=WIND VELOCITY MAGNITUDE IN KNOTS IN I-TH LAYER.		MAIN	178
C	WANGLE(I)	=WIND VELOCITY DIRECTION IN DEGREES, RECKONED COUNTER		MAIN	179
C		CLOCKWISE FROM X-AXIS.		MAIN	180
C	Z(I)	=HEIGHT IN KILOMETERS OF THE TOP OF THE I-TH LAYER OF		MAIN	181
C		FINITE THICKNESS.		MAIN	182
C				MAIN	183
C	NAM3 -- NAMELIST GROUP 3			MAIN	184
C				MAIN	185
C	IMAX	=NUMBER OF LAYERS OF FINITE THICKNESS.		MAIN	186
C	CI(I)	=SOUND SPEED IN KM/SEC IN I-TH LAYER.		MAIN	187
C	VXI(I)	=X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC).		MAIN	188
C	VY(I)	=Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC).		MAIN	189
C	HI(I)	=THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS.		MAIN	190
C				MAIN	191
C	NAM4 -- NAMELIST GROUP 4			MAIN	192
C				MAIN	193
C	THETKO	=DIRECTION IN DEGREES TO OBSERVER, RECKONED COUNTER		MAIN	194
C		CLOCKWISE FROM X AXIS.		MAIN	195
C	V1	=LOWER BOUND IN KM/SEC OF PHASE VELOCITY INTERVAL CON-		MAIN	196
C		SIDERED FOR NORMAL MODE TABULATION		MAIN	197
C	V2	=UPPER BOUND IN KM/SEC OF PHASE VELOCITY INTERVAL CON-		MAIN	198
C		SIDERED FOR NORMAL MODE TABULATION		MAIN	199
C	OM1	=MINIMUM ANGULAR FREQUENCY IN RAD/SEC CONSIDERED FOR		MAIN	200
C		NORMAL MODE TABULATION.		MAIN	201
C	OM2	=MAXIMUM ANGULAR FREQUENCY IN RAD/SEC CONSIDERED FOR		MAIN	202
C		NORMAL MODE TABULATION.		MAIN	203
C	NOM1	=INITIAL NUMBER OF DISCRETE FREQUENCIES BETWEEN OM1		MAIN	204
C		AND OM2, INCLUSIVE, AT WHICH NORMAL MODE DISPERSION		MAIN	205
C		FUNCTION IS STUDIED.		MAIN	206
C	NVPI	=INITIAL NUMBER OF DISCRETE PHASE VELOCITIES BETWEEN		MAIN	207
C		V1 AND V2, INCLUSIVE, AT WHICH NORMAL MODE DISPERSION		MAIN	208
C		FUNCTION IS STUDIED.		MAIN	209
C	MAXMOD	=MAXIMUM NUMBER OF MODES TO BE TABULATED.		MAIN	210

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C		MAIN 211	
C	NAM5 -- NAMELIST GROUP 5	MAIN 212	
C		MAIN 213	
C	IMAX =NUMBER OF LAYERS OF FINITE THICKNESS	MAIN 214	
C	CI(I) =SOUND SPEED IN KM/SEC IN I-TH LAYER	MAIN 215	
C	VXI(I) =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)	MAIN 216	
C	VYI(I) =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)	MAIN 217	
C	HI(I) =THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS	MAIN 218	
C	THETKD =DIRECTION IN DEGREES TO OBSERVER, RECKONED COUNTER	MAIN 219	
C	CLOCKWISE FROM X AXIS	MAIN 220	
C	MODND =NUMBER OF NORMAL MODES FOUND	MAIN 221	
C	KST(N) =INDEX OF FIRST TABULATED POINT IN N-TH MODE	MAIN 222	
C	KFIN(N) =INDEX OF LAST TABULATED POINT IN N-TH MODE. IN	MAIN 223	
C	GENERAL, KFIN(N)=KST(N+1)-1.	MAIN 224	
C	OMMOD(N) =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF	MAIN 225	
C	POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE	MAIN 226	
C	FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).	MAIN 227	
C	VPMOD(N) =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF	MAIN 228	
C	POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE	MAIN 229	
C	FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).	MAIN 230	
C		MAIN 231	
C	NAM6 -- NAMELIST GROUP 6	MAIN 232	
C		MAIN 233	
C	ZSCRCF =HEIGHT IN KM OF BURST ABOVE GROUND	MAIN 234	
C	ZORS =HEIGHT IN KM OF OBSERVER ABOVE GROUND	MAIN 235	
C		MAIN 236	
C	NAM7 -- NAMELIST GROUP 7	MAIN 237	
C		MAIN 238	
C	OMMOD(N) =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF	MAIN 239	
C	POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE	MAIN 240	
C	FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).	MAIN 241	
C	VPMOD(N) =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF	MAIN 242	
C	POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE	MAIN 243	
C	FOR N BETWEEN KST(NMODE) AND KFIN(NMODE)	MAIN 244	
C	MODND =NUMBER OF NORMAL MODES FOUND	MAIN 245	
C	KST(N) =INDEX OF FIRST TABULATED POINT IN N-TH MODE	MAIN 246	
C	KFIN(N) =INDEX OF LAST TABULATED POINT IN N-TH MODE. IN	MAIN 247	
C	GENERAL, KFIN(N)=KST(N+1)-1.	MAIN 248	
C	AMP(J) =AMPLITUDE FACTOR FOR GUIDED WAVE EXCITED BY POINT	MAIN 249	
C	ENERGY SOURCE. UNITS ARE KM**(-1). THE J-TH ELEMENT	MAIN 250	
C	CORRESPONDS TO ANGULAR FREQUENCY OMMOD(J) AND PHASE	MAIN 251	
C	VELOCITY VPMOD(J). THE AMPLITUDE FACTOR IS APPROPRIA	MAIN 252	
C	TO THE NMODE-TH MODE IF J .GE. KST(NMODE) AND J .LE.	MAIN 253	
C	KFIN(NMODE). A DETAILED DEFINITION OF AMP(J) IS GIVE	MAIN 254	
C	IN THE LISTING OF SUBROUTINE NAMPDE.	MAIN 255	
C	ALAM =A SCALING FACTOR DEPENDENT ON HEIGHT OF BURST, EQUAL	MAIN 256	
C	TO CURF ROOT OF (PRESSURE AT GROUND)/(PRESSURE AT	MAIN 257	
C	BURST HEIGHT) TIMES (SOUND SPEED AT GROUND)/(SOUND	MAIN 258	
C	SPEED AT BURST HEIGHT). SEE SUBROUTINE NAMPDE.	MAIN 259	
C	FACT =A GENERAL AMPLITUDE FACTOR DEPENDENT ON BURST HEIGHT	MAIN 260	
C	AND OBSERVER HEIGHT. A PRECISE DEFINITION IS GIVEN	MAIN 261	
C	IN THE LISTING OF SUBROUTINE NAMPDE.	MAIN 262	
C		MAIN 263	
C	NAM8 -- NAMELIST GROUP 8	MAIN 264	
C		MAIN 265	
C	YIELD =ENERGY YIELD OF EXPLOSION IN EQUIVALENT KILOTONS (KT)	MAIN 266	
C	OF TNT. 1 KT = 4.2X(10)**19 ERGS.	MAIN 267	
C		MAIN 268	
C	NAM9 -- NAMELIST GROUP 9	MAIN 269	
C		MAIN 270	
C	MODND =NUMBER OF NORMAL MODES FOUND	MAIN 271	
C	KST(N) =INDEX OF FIRST TABULATED POINT IN N-TH MODE	MAIN 272	
C	KFIN(N) =INDEX OF LAST TABULATED POINT IN N-TH MODE. IN	MAIN 273	
C	GENERAL, KFIN(N)=KST(N+1)-1	MAIN 274	
C	OMMOD(N) =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF	MAIN 275	
C	POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE	MAIN 276	
C	FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).	MAIN 277	
C	VPMOD(N) =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF	MAIN 278	
C	POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE	MAIN 279	
C	FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).	MAIN 280	

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C	AMPLTD(N)	=AMPLITUDE FACTOR REPRESENTING TOTAL MAGNITUDE OF	MAIN 281
C		FOURIER TRANSFORM OF WAVEFORM CONTRIBUTION OF SINGLE	MAIN 282
C		GUIDED MODE AT FREQUENCY OMMOD(N). IT REPRESENTS THE	MAIN 283
C		AMPLITUDE OF HMODE-TH MODE IF N IS BETWEEN KST(NMODE)	MAIN 284
C		AND KFIN(NMODE), INCLUSIVE. FOR PRECISE DEFINITION,	MAIN 285
C		SEE SUBROUTINE PPAMP.	MAIN 286
C	PHASQ(N)	=PHASE LAG AT FREQUENCY OMMOD(N) FOR NMODE MODE WHEN	MAIN 287
C		N BETWEEN KST(NMODE) AND KFIN(NMODE), RESPECTIVELY.	MAIN 288
C		THE INTEGRAND IS UNDERSTOOD TO HAVE THE FORM	MAIN 289
C		AMPLTD * COS(OMMOD * (TIME-DISTANCE/VPMOD) + PHASQ). FOR A	MAIN 290
C		PRECISE DEFINITION OF PHASQ, SEE SUBROUTINES TMPT	MAIN 291
C		AND PPAMP.	MAIN 292
C			MAIN 293
C	NAM10 -- NAMELIST GROUP 10		MAIN 294
C			MAIN 295
C	TFIRST	=FIRST TIME RELATIVE TO TIME OF DETONATION FOR WHICH	MAIN 296
C		WAVEFORM IS COMPUTED. UNITS ARE IN SECONDS.	MAIN 297
C	TEND	=APPROXIMATE TIME VALUE CORRESPONDING TO LAST POINT	MAIN 298
C		TABULATED FOR WAVEFORM (RELATIVE TO TIME OF DETONATIO	MAIN 299
C		FOR PRECISE DEFINITION, SEE SUBROUTINE TMPT.	MAIN 300
C	DELT	=INCPMENT OF TIME VALUES IN SECONDS FOR WHICH SUCCE-	MAIN 301
C		SIVE WAVEFORM POINTS ARE TABULATED.	MAIN 302
C	ROBS	=MAGNITUDE OF HORIZONTAL DISTANCE IN KM BETWEEN SOURCE	MAIN 303
C		AND OBSERVER.	MAIN 304
C	IOPT	=INTEGER CONTROLLING WHICH MODES ARE INCLUDED IN THE	MAIN 305
C		COMPUTED WAVEFORM. FOR PRECISE DEFINITION, SEE	MAIN 306
C		SUBROUTINE TMPT.	MAIN 307
C			MAIN 308
C		----	MAIN 309
C		PROGRAM FOLLOWS BELOWS ----	MAIN 310
C			MAIN 311
C			MAIN 312
C	DIMENSION STATEMENTS		MAIN 313
C		DIMENSION CI(100),VXI(100),VYI(100),HI(100),AMP(1000),AMPLTD(1000)	MAIN 314
C		DIMENSION T(100),VKNTX(100),VKNTY(100),ZI(100),PHASQ(1000)	MAIN 315
C		DIMENSION WANGLE(100),WINDY(100)	MAIN 316
C		DIMENSION OM(100),VP(100),INMODE(10000)	MAIN 317
C		DIMENSION KST(10),KFIN(10),OMMOD(1000),VPMOD(1000)	MAIN 318
C			MAIN 319
C	C ALLOCATION OF VARIABLES TO COMMON STORAGE		MAIN 320
C		COMMON IMAX,CI,VXI,VYI,HI	MAIN 321
C			MAIN 322
C	C NAMELIST STATEMENTS		MAIN 323
C		NAMELIST /NAM1/ NSTART,NPRNT,NPNCH	MAIN 324
C		NAMELIST /NAM2/ LANGLE,IMAX,T,VKNTX,VKNTY,WINDY,WANGLE,ZI	MAIN 325
C		NAMELIST /NAM3/ IMAX,CI,VXI,VYI,HI	MAIN 326
C		NAMELIST /NAM4/ THETKD,V1,V2,OM1,OM2,NCMI,NVPI,MAXMOD	MAIN 327
C		NAMELIST /NAM5/ IMAX,CI,VXI,VYI,HI,THETKD,MDFND,KST,KFIN,OMMOD,	MAIN 328
C		1 VPMOD	MAIN 329
C		NAMELIST /NAM6/ ZSRCCE,ZOBS	MAIN 330
C		NAMELIST /NAM7/ OMMOD,VPMOD,MDFND,KST,KFIN,AMP,ALAM,FACT	MAIN 331
C		NAMELIST /NAM8/ YIELD	MAIN 332
C		NAMELIST /NAM9/ MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASQ	MAIN 333
C		NAMELIST /NAM10/ TFIRST,TEND,DELT,ROBS,IOPT	MAIN 334
C			MAIN 335
C			MAIN 336
C	C BEFORE ANY DATA IS READ IN, ALL NAMELIST VALUES ARE PRESET TO ZERO.		MAIN 337
C	C THIS IS DONE SIMPLY TO MAKE NAMELIST PRINTOUT EASIER TO READ.		MAIN 338
C		NSTART=0	MAIN 339
C		NPRNT=0	MAIN 340
C		NPNCH=0	MAIN 341
C		LANGLE=0	MAIN 342
C		IMAX=0	MAIN 343
C		THETKD=0.0	MAIN 344
C		V1=0.0	MAIN 345
C		V2=0.0	MAIN 346
C		OM1=0.0	MAIN 347
C		OM2=0.0	MAIN 348
C		NCMI=0	MAIN 349
C		NVPI=0	MAIN 350

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MAXMOD=0	MAIN 351
MODND=0	MAIN 352
ZSCRCF=0.0	MAIN 353
ZQRS=0.0	MAIN 354
ALAM=0.0	MAIN 355
FACT=0.0	MAIN 356
VIFLD=0.0	MAIN 357
TFIRST=0.0	MAIN 358
TFND=0.0	MAIN 359
DFLTT=0.0	MAIN 360
RQRS=0.0	MAIN 361
INPT=0	MAIN 362
DO 21 IPR=1,100	MAIN 363
CI(IPR)=0.0	MAIN 364
VXI(IPR)=0.0	MAIN 365
VYI(IPR)=0.0	MAIN 366
HI(IPR)=0.0	MAIN 367
TI(IPR)=0.0	MAIN 368
VKNTX(IPR)=0.0	MAIN 369
VKNTY(IPR)=0.0	MAIN 370
ZI(IPR)=0.0	MAIN 371
WANGLE(IPR)=0.0	MAIN 372
WINDY(IPR)=0.0	MAIN 373
OM(IPR)=0.0	MAIN 374
21 VP(IPR)=0.0	MAIN 375
DO 31 IPR=1,10	MAIN 376
KST(IPR)=0	MAIN 377
31 KFIN(IPR)=0	MAIN 378
DO 41 IPR=1,1000	MAIN 379
AMP(IPR)=0.0	MAIN 380
AMPLTD(IPR)=0.0	MAIN 381
PHASO(IPR)=0.0	MAIN 382
OMMOD(IPR)=0.0	MAIN 383
41 VPMOD(IPR)=0.0	MAIN 384
C	MAIN 385
C	MAIN 386
C START OF EXECUTABLE PORTION OF PROGRAM	MAIN 387
C	MAIN 388
C NEWPLT IS A CALCOMP SUBROUTINE WHICH INITIATES THE CALCOMP PLOTTER	MAIN 389
C TAPE FILE. 4640 IS THE N.I.Y. COMPUTATION CENTER PROBLEM NO. 5923 IS	MAIN 390
C THE PROGRAMMER NO. GRAPH PAPER WITH BLACK INK IS REQUESTED.	MAIN 391
C CALL NEWPLT('M5640','5923','WHITE ','BLACK ')	MAIN 392
C	MAIN 393
C 1 READ (5,NAM1)	MAIN 394
C	MAIN 395
C IT IS CONSIDERED GOOD PRACTICE TO HAVE INPUT DATA PRINTED ON OUTPUT	MAIN 396
C WRITE (6,37)	MAIN 397
C FORMAT(1H ///// 27HINAM1 HAS JUST BEEN READ IN)	MAIN 398
C WRITE (6,NAM1)	MAIN 399
C	MAIN 400
C CURRENT VALUE OF NSTART CONTROLS THE STAGE AT WHICH COMPUTATION BEGINS	MAIN 401
C SINCE COMPUTED GO TO STATEMENTS SOMETIMES DO NOT COMPILE CORRECTLY IF	MAIN 402
C INDEX IS NOT EXPLICITLY DEFINED, WE PLAY IT SAFE WITH REDUNDANT	MAIN 403
C STATEMENT.	MAIN 404
C NSTART=NSTART	MAIN 405
C	MAIN 406
C GO TO (200,300,400,500,600,999),NSTART	MAIN 407
C	MAIN 408
C WE ARRIVE HERE IF NSTART=1	MAIN 409
C 200 READ (5,NAM2)	MAIN 410
C	MAIN 411
C WRITE (6,237)	MAIN 412
C 237 FORMAT(1H ///// 27H NAM2 HAS JUST BEEN READ IN)	MAIN 413
C WRITE (6,NAM2)	MAIN 414
C	MAIN 415
C CONVERT ATMOSPHERIC DATA TO STANDARD FORM	MAIN 416
C CALL ATMOS(T,VKNTX,VKNTY,ZI,WANGLE,WINDY,LANGLE)	MAIN 417
C IF (NPRNT .LE. 0) GO TO 270	MAIN 418
C	MAIN 419
C PRINT ATMOSPHERIC PROFILE IF NPRNT .GT. 0	MAIN 420

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CALL PRATMO	MAIN 421
C	MAIN 422
270 IF(NPNCH .LE. 0) GO TO 305	MAIN 423
C	MAIN 424
C PUNCH NAME DATA IF NPNCH .GT. 0	MAIN 425
WRITE (7,271)	MAIN 426
271 FORMAT (7H 8NAME3)	MAIN 427
IUMS = IMAX + 1	MAIN 428
WRITE (7,272) IMAX,(CI(I),I=1,IUMS)	MAIN 429
272 FORMAT (10H IMAX = ,I3,1H, / 8H CI = /	MAIN 430
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,)	MAIN 431
WRITE(7,274) (VXI(I),I=1,IUMS)	MAIN 432
274 FORMAT(9H VXI = /	MAIN 433
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,)	MAIN 434
WRITE(7,276) (VVI(I),I=1,IUMS)	MAIN 435
276 FORMAT(9H VVI = /	MAIN 436
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,)	MAIN 437
WRITE(7,278) (HII(I),I=1,IUMS)	MAIN 438
278 FORMAT (8H HII = /	MAIN 439
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,)	MAIN 440
WRITE (7,279)	MAIN 441
279 FORMAT (6H 8END)	MAIN 442
WRITE (6,583)	MAIN 443
WRITE (6,271)	MAIN 444
WRITE (6,272) IMAX,(CI(I),I=1,IUMS)	MAIN 445
WRITE(6,274) (VXI(I),I=1,IUMS)	MAIN 446
WRITE(6,276) (VVI(I),I=1,IUMS)	MAIN 447
WRITE(6,278) (HII(I),I=1,IUMS)	MAIN 448
WRITE (6,279)	MAIN 449
280 GO TO 305	MAIN 450
C	MAIN 451
C WE ARRIVE HERE IF NSTART=2	MAIN 452
300 READ (5,NAM3)	MAIN 453
WRITE (6,302)	MAIN 454
302 FORMAT(1H ///// 27H NAM3 HAS JUST BEEN READ IN)	MAIN 455
WRITE (6,NAM3)	MAIN 456
IF(NPRNT .LE. 0) GO TO 305	MAIN 457
C PRINT ATMOSPHERIC PROFILE IF NPRNT .GT. 0	MAIN 458
CALL PRATMO	MAIN 459
C	MAIN 460
C CONTINUING FROM 270, 280, 302, OR 303	MAIN 461
305 READ (5,NAM4)	MAIN 462
WRITE (6,307)	MAIN 463
307 FORMAT(1H ///// 27H NAM4 HAS JUST BEEN READ IN)	MAIN 464
WRITE (6,NAM4)	MAIN 465
C	MAIN 466
C CONVERT THETKD FROM DEGREES TO RADIANS	MAIN 467
THETK = (3.14159) * THETKD / 180.0	MAIN 468
NOM = NOMI	MAIN 469
VP = NVPI	MAIN 470
C	MAIN 471
C CONSTRUCT TABLE OF INMODE VALUES	MAIN 472
CALL TABLE(NM1,NM2,V1,V2,NOM,NVP,THETK,OM,VP,INMODE,NPRNT)	MAIN 473
C	MAIN 474
C COMPUTE DISPERSION CURVES OF GUIDED MODES	MAIN 475
CALL ALLMOD(NVP,NOM,MAXMOD,MOFNO,OM,VP,KST,KFIN,OMMOD,VPMOD,	MAIN 476
1 INMODE,THETK,KWOP)	MAIN 477
C	MAIN 478
C CHECK TO SEE IF ANY MODES WERE FOUND	MAIN 479
IF(KWOP .GE. 0) GO TO 320	MAIN 480
C	MAIN 481
C EXIT IF KWOP .LT. 0	MAIN 482
WRITE (6,311) KWOP	MAIN 483
311 FORMAT(1H , 5H KWOP = , I3)	MAIN 484
CALL EXIT	MAIN 485
C	MAIN 486
C CONTINUING WITH KWOP .GE. 0 FROM 308	MAIN 487
320 IF(NPRNT .LE. 0) GO TO 350	MAIN 488
C	MAIN 489
C PRINT NORMAL MODE DISPERSION CURVES	MAIN 490

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CALL MODLST(MDFND,OMMOD,VPMOD,KST,KFIN)	MAIN 491
C CONTINUING FROM 320 OR 321	MAIN 492
350 IF(NPNCH .LE. 0) GO TO 360	MAIN 493
C	MAIN 494
C PUNCH NAME5 DATA IF NPNCH .GT. 0	MAIN 495
WRITE (7,351)	MAIN 496
351 FORMAT (7H NAME5)	MAIN 497
IUMS = IMAX + 1	MAIN 498
WRITE (7,272) IMAX, (CI(I), I=1, IUMS)	MAIN 499
WRITE(7,274) (VXI(I), I=1, IUMS)	MAIN 500
WRITE(7,276) (VYI(I), I=1, IUMS)	MAIN 501
WRITE(7,278) (HI(I), I=1, IUMS)	MAIN 502
WRITE (7,352) THETKD,MDFND,(KST(I), I=1,MDFND)	MAIN 503
352 FORMAT (11H THETKD =,G16.8,1H,/10H MDFND =,13,1H,/9H KST =/	MAIN 504
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 505
WRITE(7,355) (KFIN(I), I=1,MDFND)	MAIN 506
355 FORMAT (10H KFIN = /	MAIN 507
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 508
KLAST = KFIN(MDFND)	MAIN 509
WRITE (7,357) (OMMOD(I), I=1,KLAST)	MAIN 510
357 FORMAT (11H OMMOD = /	MAIN 511
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 512
WRITE(7,359) (VPMOD(I), I=1,KLAST)	MAIN 513
359 FORMAT (11H VPMOD = /	MAIN 514
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 515
WRITE (7,279)	MAIN 516
WRITE (6,583)	MAIN 517
WRITE (6,351)	MAIN 518
WRITE (4,272) IMAX, (CI(I), I=1, IUMS)	MAIN 519
WRITE(4,274) (VXI(I), I=1, IUMS)	MAIN 520
WRITE(4,276) (VYI(I), I=1, IUMS)	MAIN 521
WRITE(4,278) (HI(I), I=1, IUMS)	MAIN 522
WRITE (6,352) THETKD,MDFND,(KST(I), I=1,MDFND)	MAIN 523
WRITE(6,355) (KFIN(I), I=1,MDFND)	MAIN 524
WRITE (6,357) (OMMOD(I), I=1,KLAST)	MAIN 525
WRITE(6,359) (VPMOD(I), I=1,KLAST)	MAIN 526
WRITE (6,279)	MAIN 527
C	MAIN 528
C CONTINUING FROM 350 OR 351	MAIN 529
360 GO TO 415	MAIN 530
C	MAIN 531
C	MAIN 532
C WE ARRIVE HERE IF NSTART=3	MAIN 533
400 READ (5,NAME5)	MAIN 534
WRITE (6,403)	MAIN 535
403 FORMAT(1H ///// 27H NAME5 HAS JUST BEEN READ IN)	MAIN 536
WRITE (6,NAME5)	MAIN 537
C	MAIN 538
C CONVERT THETKD FROM DEGREES TO RADIANS	MAIN 539
THETK = (3.14159) * THETKD / 180.0	MAIN 540
C	MAIN 541
C CONTINUING FROM 360 OR 402	MAIN 542
415 READ (5,NAME6)	MAIN 543
WRITE (6,417)	MAIN 544
417 FORMAT(1H ///// 27H NAME6 HAS JUST BEEN READ IN)	MAIN 545
WRITE (6,NAME6)	MAIN 546
C	MAIN 547
C COMPUTE YIELD INDEPENDENT AMPLITUDE FACTORS FOR GUIDED MODES	MAIN 548
CALL PAMPDE(ZSRCCE,ZOBS,MDFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,FACT,	MAIN 549
1 THETK, NPRNT)	MAIN 550
C	MAIN 551
450 IF(NPNCH .LE. 0) GO TO 460	MAIN 552
C	MAIN 553
C PUNCH NAME7 DATA IF NPNCH .GT. 0	MAIN 554
KLAST = KFIN(MDFND)	MAIN 555
WRITE (7,451)(AMP(I), I=1,KLAST)	MAIN 556
451 FORMAT (7H NAME7 / 9H AMP = /	MAIN 557
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 558
WRITE (7,452) ALAM,FACT	MAIN 559
	MAIN 560

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452 FORMAT (10H ALAM = ,G16.8,1H, / 10H FACT = ,G16.8,1H,)	MAIN 561
WRITE (7,455) MDFND,(KST(I),I=1,MDFND)	MAIN 562
455 FORMAT (10H MDFND =,13,1H,/8H KST =/	MAIN 563
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 564
WRITE(7,355) (KFIN(I),I=1,MDFND)	MAIN 565
WRITE (7,357) (OMMOD(I),I=1,KLAST)	MAIN 566
WRITE(7,359) (VPMOD(I),I=1,KLAST)	MAIN 567
WRITE (7,279)	MAIN 568
WRITE (6,583)	MAIN 569
WRITE (6,451)(AMP(I),I=1,KLAST)	MAIN 570
WRITE (6,452) ALAM,FACT	MAIN 571
WRITE (6,455) MDFND,(KST(I),I=1,MDFND)	MAIN 572
WRITE(6,355) (KFIN(I),I=1,MDFND)	MAIN 573
WRITE (6,357) (OMMOD(I),I=1,KLAST)	MAIN 574
WRITE(6,359) (VPMOD(I),I=1,KLAST)	MAIN 575
459 WRITE (6,279)	MAIN 576
C	MAIN 577
C CONTINUING FROM 450 OR 459	MAIN 578
460 GO TO 515	MAIN 579
C	MAIN 580
C	MAIN 581
C WE ARRIVE HERE IF NSTART=4	MAIN 582
500 READ (5,NAM7)	MAIN 583
WRITE (6,501)	MAIN 584
501 FORMAT(1H ///// 27H NAM7 HAS JUST BEEN READ IN)	MAIN 585
502 WRITE (6,NAM7)	MAIN 586
C	MAIN 587
C CONTINUING FROM 460 OR 502	MAIN 588
515 READ (5,NAM8)	MAIN 589
WRITE (6,516)	MAIN 590
516 FORMAT(1H ///// 27H NAM8 HAS JUST BEEN READ IN)	MAIN 591
517 WRITE (6,NAM8)	MAIN 592
C	MAIN 593
C COMPUTE YIELD DEPENDENT AMPLITUDES AND PHASE TERMS OF GUIDED MODES	MAIN 594
CALL PPAMP(YIELD,MDFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,FACT,AMPLTD,	MAIN 595
: PHASQ)	MAIN 596
518 IF(NPRNT .LE. 0) GO TO 580	MAIN 597
C THE RESULTS OF CALLING PPAMP ARE PRINTED OUT BY CALLING TABPRT	MAIN 598
520 CALL TABPRT(YIELD,MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASQ)	MAIN 599
C	MAIN 600
C CONTINUING FROM 518 OR 520	MAIN 601
580 IF(NPNCH .LE. 0) GO TO 590	MAIN 602
C	MAIN 603
C PUNCH NAM9 DATA IF NPNCH .GT. 0	MAIN 604
KLAST = KFIN(MDFND)	MAIN 605
WRITE (7,581) (AMPLTD(I),I=1,KLAST)	MAIN 606
581 FORMAT (7H &NAM9 / 12H AMPLTD = /	MAIN 607
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 608
WRITE (7,582) (PHASQ(I),I=1,KLAST)	MAIN 609
582 FORMAT (11H PHASQ = /	MAIN 610
1 (6X,G15.8,1H,,G15.8,1H,,G15.8,1H,,G15.8,1H,))	MAIN 611
WRITE (7,455) MDFND,(KST(I),I=1,MDFND)	MAIN 612
WRITE(7,355) (KFIN(I),I=1,MDFND)	MAIN 613
WRITE (7,357) (OMMOD(I),I=1,KLAST)	MAIN 614
WRITE(7,359) (VPMOD(I),I=1,KLAST)	MAIN 615
WRITE (7,279)	MAIN 616
WRITE (6,583)	MAIN 617
583 FORMAT(1H ///// 41H THE FOLLOWING DATA HAS JUST BEEN PUNCHED)	MAIN 618
WRITE (6,581) (AMPLTD(I),I=1,KLAST)	MAIN 619
WRITE (6,582) (PHASQ(I),I=1,KLAST)	MAIN 620
WRITE (6,455) MDFND,(KST(I),I=1,MDFND)	MAIN 621
WRITE(6,355) (KFIN(I),I=1,MDFND)	MAIN 622
WRITE (6,357) (OMMOD(I),I=1,KLAST)	MAIN 623
WRITE(6,359) (VPMOD(I),I=1,KLAST)	MAIN 624
584 WRITE (6,279)	MAIN 625
C	MAIN 626
C CONTINUING FROM 580 OR 584	MAIN 627
590 GO TO 615	MAIN 628
C	MAIN 629
C	MAIN 630

PROGRAM
MAIN
PAGE
9

C WE ARRIVE HERE IF NSTART=5	MAIN 631
600 READ (5,NAM9)	MAIN 632
WRITE (6,601)	MAIN 633
601 FORMAT(1H ///// 27H NAM9 HAS JUST BEEN READ IN)	MAIN 634
602 WRITE (6,NAM9)	MAIN 635
C	MAIN 636
C CONTINUING FROM 590 OR 602	MAIN 637
615 READ (5,NAM10)	MAIN 638
WRITE (6,616)	MAIN 639
616 FORMAT(1H ///// 29H NAM10 HAS JUST BEEN READ IN)	MAIN 640
WRITE (6,NAM10)	MAIN 641
C	MAIN 642
C COMPUTATION OF WAVEFORM	MAIN 643
CALL TMPT(TFIRST,TEND,DELT,POBS,MDFND,KST,KFIN,OMMOD,VPMOD,	MAIN 644
1 AMPLTD,PHASQ,INPT)	MAIN 645
C	MAIN 646
C REPEAT FOR NEXT WAVEFORM	MAIN 647
GO TO 1	MAIN 648
C	MAIN 649
C WE ARRIVE HERE IF NSTART = 6.	MAIN 650
C ENDPLT TERMINATES THE CALCOMP TAPE FILE.	MAIN 651
999 CALL ENDPLT	MAIN 652
CALL EXIT	MAIN 653
END	MAIN 654

PROGRAM
MAIN

PAGE
10

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C      AAAA (SUBROUTINE)              7/25/68
C
C      ----ABSTRACT----
C
C  TITLE - AAAA
C      THIS SUBROUTINE COMPUTES THE 2-BY-2 MATRIX A OF COEFFICIENTS
C      IN THE RESIDUAL EQUATIONS
C
C      D(PHI1)/DZ = (A11)*PHI1 + (A12)*PHI2
C
C      D(PHI2)/DZ = (A21)*PHI1 + (A22)*PHI2
C
C      DERIVED BY A. PIERCE, J. COMP. PHYS., VOL. 1, NO. 3, 343,~366,
C      1967. (SEE EQN. (10) OF THE PAPER.) THE EXPLICIT EXPRESSIONS
C      FOR THE A(I,J) ARE
C
C      A(1,1) = G*(K/BOM)**2 - GAMMA*G/(2*C**2)
C      A(1,2) = 1 - (C*K/BOM)**2
C      A(2,1) = (G*K)/(BOM*C)**2 - (BOM/C)**2
C      A(2,2) = -A(1,1)
C
C      WHERE GAMMA=1.4 IS THE SPECIFIC HEAT RATIO, G=.0098 KM/SEC**2
C      IS THE ACCELERATION OF GRAVITY, C IS THE SOUND SPEED, K IS THE
C      HORIZONTAL WAVE NUMBER AND BOM IS THE DOPPLER SHIFTED ANGULAR
C      FREQUENCY
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C  AUTHOR   - A.D. PIERCE, M.I.T., JULY, 1968
C
C      ----CALLING SEQUENCE----
C
C  SUBROUTINE ELINT, MMMM, NAMPDE, NMDFN
C      DIMENSION A(2,2)
C      CALL AAAA(OMEGA, AKX, AKY, C, VX, VY, A)
C
C  NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C      ----ARGUMENT LIST----
C
C      OMEGA      R*4      NO      INP
C      AKX        R*4      NO      INP
C      AKY        R*4      NO      INP
C      C          R*4      NO      INP
C      VX         R*4      NO      INP
C      VY         R*4      NO      INP
C      A          R*4      2-BY-2 OUT
C
C  NO COMMON STORAGE IS USED
C
C      ----INPUTS----
C
C      OMEGA      =ANGULAR FREQUENCY IN RAD/SEC
C      AKX        =X COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C      AKY        =Y COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C      C          =SOUND SPEED IN KM/SEC
C      VX         =X COMPONENT OF WIND VELOCITY IN KM/SEC
C      VY         =Y COMPONENT OF WIND VELOCITY IN KM/SEC
C
C      ----OUTPUTS----
C
C      A(I,J)     =(I,J)-TH ELEMENT OF MATRIX A OF COEFFICIENTS IN THE
C                  RESIDUAL EQUATIONS AS DEFINED IN THE ABSTRACT.
C
C      ----PROGRAM FOLLOWS BELOW
C
C      SUBROUTINE AAAA(OMEGA, AKX, AKY, C, VX, VY, A)
C
C      DIMENSION A(2,2)
C      BMSQ=(OMEGA-AXX*VX-AXY*VY)**2
C      CSQ=C*C

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AAAA 1
AAAA 2
AAAA 3
AAAA 4
AAAA 5
AAAA 6
AAAA 7
AAAA 8
AAAA 9
AAAA 10
AAAA 11
AAAA 12
AAAA 13
AAAA 14
AAAA 15
AAAA 16
AAAA 17
AAAA 18
AAAA 19
AAAA 20
AAAA 21
AAAA 22
AAAA 23
AAAA 24
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AAAA 26
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AAAA 28
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AAAA 44
AAAA 45
AAAA 46
AAAA 47
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AAAA 50
AAAA 51
AAAA 52
AAAA 53
AAAA 54
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AAAA 56
AAAA 57
AAAA 58
AAAA 59
AAAA 60
AAAA 61
AAAA 62
AAAA 63
AAAA 64
AAAA 65
AAAA 66
AAAA 67
AAAA 68
AAAA 69
AAAA 70

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PROGRAM
AAAA
PAUSE
11

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T=(AKX**2+AKY**2)/ROMSO
A(1,1)=.0098*T-.00686/CSO
C GAMMA*G/2 IS .00686
A(1,2)=1.0-CSO*T
A(2,1)=((96.04E-6)*T-ROMSO)/CSO
C G**2 IS 96.04E-6 KM**2/SEC**4
A(2,2)=-A(1,1)
RETURN
END

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AAAA 71
AAAA 72
AAAA 73
AAAA 74
AAAA 75
AAAA 76
AAAA 77
AAAA 78
AAAA 79

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PROGRAM
AAAA

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C	AKI (SUBROUTINE)	8/15/68	AKI	1
C			AKI	2
C			AKI	3
C	-----ABSTRACT-----		AKI	4
C			AKI	5
C	TITLE - AKI		AKI	6
C	EVALUATION OF INTEGRAL OF A(OMEGA)*COS(PHI(OMEGA)) FROM OM1 TO		AKI	7
C	OM2		AKI	8
C			AKI	9
C	A(OMEGA) AND PHI(OMEGA) ARE ASSUMED TO BE LINEAR BETWEEN		AKI	10
C	OM1 AND OM2, FOLLOWING THE METHOD OF AKI (J. GEOPHYS.		AKI	11
C	RES., VOL. 65 (1960), PP. 729-740). THE INTEGRAL IS		AKI	12
C	READILY EVALUATED AS		AKI	13
C			AKI	14
C	(PHI')**(-1) * (A1 + A'*(OM2-OM1)) * SIN(PHI1+X)		AKI	15
C	+ PHI'*(-2) * A' * COS(PHI1 + X)		AKI	16
C	- PHI'*(-1) * (A1 - A' * (OM2 - OM1)) * SIN(PHI1-X)		AKI	17
C	- PHI'*(-2) * A' * COS(PHI1 - X)		AKI	18
C			AKI	19
C	WHERE		AKI	20
C			AKI	21
C	A1 = AVERAGE VALUE OF A IN INTERVAL		AKI	22
C	PHI1 = AVERAGE VALUE OF PHI IN INTERVAL		AKI	23
C	A' = D(A) / D(OMEGA)		AKI	24
C	PHI' = D(PHI) / D(OMEGA)		AKI	25
C	X = PHI' * (OM2 - OM1) / 2		AKI	26
C			AKI	27
C	A SOMEWHAT MORE CONVENIENT FORMULA OBTAINABLE BY TRIGONO-		AKI	28
C	METRIC IDENTITIES IS		AKI	29
C			AKI	30
C	AKIINT = 2 * PHI'*(-1) * A1 * SIN(X) * COS(PHI1)		AKI	31
C	+ 2 * PHI'*(-2) * A' * (X * COS(X) - SIN(X))		AKI	32
C	* SIN(PHI1) .		AKI	33
C			AKI	34
C	WHENEVER X IS SMALL, SIN(X)/X AND COS(X) ARE EVALUATED BY		AKI	35
C	USING THEIR POWER SERIES REPRESENTATIONS.		AKI	36
C			AKI	37
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		AKI	38
C			AKI	39
C	AUTHORS - A.D.PIERCE AND J.POSFY, M.I.T., AUGUST,1968		AKI	40
C			AKI	41
C			AKI	42
C			AKI	43
C			AKI	44
C			AKI	45
C			AKI	46
C	-----USAGE-----		AKI	47
C			AKI	48
C	NO SUBROUTINES ARE CALLED		AKI	49
C			AKI	50
C	FORTRAN USAGE		AKI	51
C			AKI	52
C	CALL AKI(OM1,OM2,A1,A2,CTRIG1,STRIG1,CTRIG2,STRIG2,		AKI	53
C	1 DELPH,AKIINT)		AKI	54
C			AKI	55
C	INPUTS		AKI	56
C			AKI	57
C	OM1 LOWER LIMIT OF INTEGRATION OVER ANGULAR FREQUENCY		AKI	58
C	R*4 (RADIAN)		AKI	59
C			AKI	60
C	OM2 UPPER LIMIT OF INTEGRATION (RADIAN)		AKI	61
C	R*4		AKI	62
C			AKI	63
C	A1 VALUE OF A AT OMEGA = OM1		AKI	64
C	R*4		AKI	65
C			AKI	66
C	A2 VALUE OF A AT OMEGA = OM2		AKI	67
C	R*4		AKI	68
C			AKI	69
C			AKI	70
			PROGRAM	
			AKI	
			PAGE	
			13	

C	CTRIG1	COS(PHI) WHERE OMEGA = OM1	AKI	71
C	R#4		AKI	72
C			AKI	73
C	STRIG1	SIN(PHI) WHERE OMEGA = OM1	AKI	74
C	R#4		AKI	75
C			AKI	76
C	DELPH	CHANGE IN PHI OVER THE INTERVAL (PHI(OM2) - PHI(OM1))	AKI	77
C	R#4	(RADIANS)	AKI	78
C			AKI	79
C	OUTPUTS		AKI	80
C			AKI	81
C	CTRIG2	COS(PHI) WHERE OMEGA = OM2	AKI	82
C	R#4		AKI	83
C			AKI	84
C	STRIG2	SIN(PHI) WHERE OMEGA = OM2	AKI	85
C	R#4		AKI	86
C			AKI	87
C	AKIINT	VALUE OF INTEGRAL DEFINED IN ABSTRACT IN UNITS OF A*OMEGA	AKI	88
C	R#4		AKI	89
C			AKI	90
C			AKI	91
C		-----PROGRAM FOLLOWS BELOW-----	AKI	92
C			AKI	93
C			AKI	94
C	SUBROUTINE AKI(OM1,OM2,A1,A2,CTRIG1,STRIG1,CTRIG2,		AKI	95
C	1 STRIG2,DELPH,AKIINT)		AKI	96
C	DELPH=OM2-OM1		AKI	97
C	DELA=A2-A1		AKI	98
C			AKI	99
C	A1=(A2+A1)/2.0		AKI	100
C	X=DELPH/2.0		AKI	101
C	CTRX=COS(X)		AKI	102
C	STRX=SIN(X)		AKI	103
C	CTRIG1=CTRIG1*CTRX-STRIG1*STRX		AKI	104
C	STRIG1=STRIG1*CTRX+CTRIG1*STRX		AKI	105
C	CTRIG2=CTRIG1*CTRX-STRIG1*STRX		AKI	106
C	STRIG2=STRIG1*CTRX+CTRIG1*STRX		AKI	107
C	IF(ABS(X)-1.0F-2) 20,20,10		AKI	108
C	10 S1=STRX/X		AKI	109
C	S2=(S1-CTRX)/X**2		AKI	110
C	GO TO 30		AKI	111
C	20 S1=1.0-(1.0/6.0)*X**2+(1.0/120.0)*X**4		AKI	112
C	S2=(1.0/3.0)-(1.0/30.0)*X**2+(1.0/840.0)*X**4		AKI	113
C	30 AKIINT=(A1*S1*CTRIG1-DELA*DELPH*0.25*S2*STRIG1)*DELPH		AKI	114
C	RETURN		AKI	115
C	END		AKI	116

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AKI

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C	ALLMOD (SURRCUTINF)	6/25/68	ALLM	1
C			ALLM	2
C			ALLM	3
C	TITLE - ALLMOD		ALLM	4
C	PROGRAM TO TABULATE DISPERSION CURVES OF UP TO MAXMOD GUIDED		ALLM	5
C	MODES. ONLY PORTIONS OF CURVES WITH OMEGA BETWEEN OM(1) AND		ALLM	6
C	OM(NCOL) AND WITH PHASE VELOCITY BETWEEN VP(NCOL) AND VP(1)		ALLM	7
C	ARE TABULATED. THE ANGULAR DEVIATION OF GROUP VELOCITY DIREC-		ALLM	8
C	TION FROM PHASE VELOCITY DIRECTION THETK IS NEGLECTED.		ALLM	9
C	SUCCESSIVE MODES NUMBERED FROM 1 TO MODND ARE EACH TABULATED BY		ALLM	10
C	CALLING SUBROUTINE MODETR. STARTING POINTS FOR EACH MODE ARE		ALLM	11
C	FOUND BY CALLING SUBROUTINE NXMODE. THE NORMAL MODE DISPERSION		ALLM	12
C	FUNCTION (NMDF) SHOULD BE NEARLY ZERO FOR EVERY TABULATED POINT		ALLM	13
C	ON EACH DISPERSION CURVE. THE COMPUTATIONAL METHOD IS BASED		ALLM	14
C	ON THE PREVIOUSLY COMPUTED VALUES OF THE NMDF SIGN		ALLM	15
C	INMODE(IJ-1)*NROW+1) AT POINTS (I,J) IN A RECTANGULAR ARRAY OF		ALLM	16
C	NROW ROWS AND NCOL COLUMNS. DIFFERENT COLUMNS (J) CORRESPOND		ALLM	17
C	TO DIFFERENT ANGULAR FREQUENCIES OM(I) WHILE DIFFERENT ROWS (I)		ALLM	18
C	CORRESPOND TO DIFFERENT PHASE VELOCITIES VP(I). IT IS ASSUMED		ALLM	19
C	THAT VP(1) .GT. VP(2) .GT. VP(2), ETC. DISPERSION CURVES		ALLM	20
C	OF VARIOUS MODES APPEAR ON THIS ARRAY AS LINES OF DEMARCATION		ALLM	21
C	BETWEEN ADJACENT REGIONS WITH OPPOSITE INMODES. IT IS ASSUMED		ALLM	22
C	THAT DISPERSION CURVES SLOPE DOWNWARDS. MODES ARE NUMBERED		ALLM	23
C	STARTING FROM LOWER LEFT OF INMODE ARRAY.		ALLM	24
C			ALLM	25
C	PROGRAM NOTES		ALLM	26
C			ALLM	27
C	THE ARRAYS OMMOD AND VPMOD ARE USED TO STORE DISPERSION		ALLM	28
C	CURVES FOR ALL THE MODES TO CONSERVE STORAGE. FOR THE		ALLM	29
C	NMODE-TH MODE, VPMOD(KST(NMODE)+K-1) IS THE PHASE VELOCITY		ALLM	30
C	CORRESPONDING TO ANGULAR FREQUENCY OF OMMOD(KST(NMODE)+		ALLM	31
C	K-1). THE PAIR OF VALUES CORRESPONDS TO THE K-TH TABULAT		ALLM	32
C	POINT FOR THE MODE. THE LAST TABULATED POINT FOR THE		ALLM	33
C	NMODE-TH MODE IS LABELED BY THE PAIR VPMOD(KFIN(NMODE)),		ALLM	34
C	OMMOD(KFIN(NMODE)). THUS OMMOD(K), VPMOD(K) FOR		ALLM	35
C	K .GE. KST(NMODE) AND K .LT. KFIN(NMODE) DESCRIBE THE		ALLM	36
C	NMODE-TH MODE-S DISPERSION CURVE.		ALLM	37
C			ALLM	38
C	THE FLAG KWOP IS NORMALLY RETURNED AS 1. HOWEVER, IF		ALLM	39
C	NO DISPERSION CURVES ARE TABULATED, KWOP IS RETURNED AS		ALLM	40
C	-1.		ALLM	41
C			ALLM	42
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		ALLM	43
C	AUTHOR - A.D.PIERCE, M.I.T., JUNE, 1968		ALLM	44
C			ALLM	45
C	-----CALLING SEQUENCE-----		ALLM	46
C			ALLM	47
C	SFF MAIN PROGRAM		ALLM	48
C	DIMENSION OM(100),VP(100),KST(10),KFIN(10),OMMOD(1000),VPMOD(1000)		ALLM	49
C	DIMENSION INMODE(1000)		ALLM	50
C	DIMENSION CI(100),VXI(100),VYI(100),HI(100)		ALLM	51
C	THE SUBROUTINE USES VARIABLE DIMENSIONING. THE ASSIGNMENTS ABOVE ARE		ALLM	52
C	THOSE GIVEN BY MAIN PROGRAM		ALLM	53
C	COMMON IMAX,CI,VXI,VYI,HI		ALLM	54
C	ATMOSPHERIC VARIABLES MUST BE IN COMMON BEFORE ALLMOD IS CALLED.		ALLM	55
C	CALL ALLMOD(NROW,NCOL,MAXMOD,MODND,OM,VP,KST,KFIN,OMMOD,VP+OD,		ALLM	56
C	1 INMODE,THETK,KWCP		ALLM	57
C	IF(KWOP .NE. 1) GO SOMEWHERE		ALLM	58
C			ALLM	59
C	-----EXTERNAL SUBROUTINES REQUIRED-----		ALLM	60
C			ALLM	61
C	NXMODE,MODETR,NXTPNT,RTMI,FNMOD1,FNMOD2,NMDFN,AAAA,RRRR,MMMM,CAI,S		ALLM	62
C			ALLM	63
C	NXMODF AND MODETR ARE EXPLICITLY CALLED. THE REST ARE		ALLM	64
C	IMPLICITLY CALLED BY CALLING MODETR. FOR FURTHER INFORMATION		ALLM	65
C	ON IBM SCIENTIFIC SUBROUTINE PACKAGE ROUTINE RTMI, SEE DOCU-		ALLM	66
C	MENTATION OF MODETR.		ALLM	67
C			ALLM	68
C	-----ARGUMENT LIST-----		ALLM	69
C			ALLM	70

PROGRAM
ALLMOD
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C	NROW	I*4	NO	INP	ALLM	71
C	NCOL	I*4	NO	INP	ALLM	72
C	MAXMOD	I*4	NO	INP	ALLM	73
C	MODFND	I*4	NO	OUT	ALLM	74
C	OM	R*4	VAR	INP	ALLM	75
C	VP	R*4	VAR	INP	ALLM	76
C	KST	I*4	VAR	OUT	ALLM	77
C	KFIN	I*4	VAR	OUT	ALLM	78
C	OMMOD	R*4	VAR	OUT	ALLM	79
C	VPMOD	R*4	VAR	OUT	ALLM	80
C	INMODE	I*4	VAR	INP	ALLM	81
C	THETK	R*4	NO	INP	ALLM	82
C	KWOP	I*4	NO	OUT	ALLM	83
C	COMMON STORAGE USED				ALLM	84
C	COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETKP				ALLM	85
C					ALLM	86
C	IMAX	I*4	NO	INP	ALLM	87
C	CI	P*4	100	INP	ALLM	88
C	VXI	R*4	100	INP	ALLM	89
C	VYI	R*4	100	INP	ALLM	90
C	HI	R*4	100	INP	ALLM	91
C	OMEGAC	R*4	NO	OUT (USED INTERNALLY)	ALLM	92
C	VPHSEC	R*4	NO	OUT (USED INTERNALLY)	ALLM	93
C	THETKP	R*4	NO	OUT (USED INTERNALLY)	ALLM	94
C					ALLM	95
C	----INPUTS----				ALLM	96
C					ALLM	97
C	NROW	=NUMBER OF ROWS IN INMODE ARRAY. MAXIMUM INDEX OF VP(N).			ALLM	98
C	NCOL	=NUMBER OF COLUMNS IN INMODE ARRAY. MAXIMUM INDEX OF OM(N).			ALLM	99
C	MAXMOD	=MAXIMUM NUMBER OF MODES TO BE TABULATED			ALLM	100
C	OM(N)	=ANGULAR FREQUENCY OF N-TH COLUMN IN INMODE ARRAY			ALLM	101
C	VP(N)	=PHASE VELOCITY OF N-TH ROW IN INMODE ARRAY			ALLM	102
C	INMODE	=1,-1, OR 5 DEPENDING ON WHETHER SIGN OF NORMAL MODE DISPERSION FUNCTION IS + OR -, 5 IF NMDF DOESNT EXIST			ALLM	103
C		THE (J-1)*NROW+I-TH ELEMENT CORRESPONDS TO NMDF WHEN			ALLM	104
C		OMEGA=OM(J), PHASE VELOCITY=VP(I).			ALLM	105
C	THETK	=PHASE VELOCITY DIRECTION IN RADIAN RECKONED COUNTER-			ALLM	106
C		CLOCKWISE WITH RESPECT TO X AXIS.			ALLM	107
C	IMAX	=NUMBER OF ATMOSPHERIC LAYERS OF FINITE THICKNESS			ALLM	108
C	CI(I)	=SOUND SPEED IN I-TH LAYER			ALLM	109
C	VXI(I)	=X COMPONENT OF WIND VELOCITY IN I-TH LAYER			ALLM	110
C	VYI(I)	=Y COMPONENT OF WIND VELOCITY IN I-TH LAYER			ALLM	111
C	HI(I)	=THICKNESS OF I-TH LAYER			ALLM	112
C					ALLM	113
C	----OUTPUTS----				ALLM	114
C					ALLM	115
C	MODFND	=NUMBER OF MODES FOUND			ALLM	116
C	KST(N)	=INDEX OF FIRST TABULATED POINT IN N-TH MODE			ALLM	117
C	KFIN(N)	=INDEX OF LAST TABULATED POINT IN N-TH MODE. IN			ALLM	118
C		GENERAL, KFIN(N)=KST(N+1)-1.			ALLM	119
C	OMMOD(N)	=ARRAY STORING ANGULAR FREQUENCY ORDINATE OF POINTS			ALLM	120
C		ON DISPERSION CURVES. THE NMODE MODE IS STORED FOR			ALLM	121
C		N BETWEEN KST(NMODE) AND KFIN(NMODE).			ALLM	122
C	VPMOD(N)	=ARRAY STORING PHASE VELOCITY ORDINATE OF POINTS ON			ALLM	123
C		DISPERSION CURVES. THE NMODE-TH MODE IS STORED FOR			ALLM	124
C		N BETWEEN KST(NMODE) AND KFIN(NMODE).			ALLM	125
C	KWOP	=-1 IF NO MODES ARE TABULATED. OTHERWISE IT IS 1.			ALLM	126
C	OMEGAC	=INTERNALLY USED FREQUENCY TRANSMITTED AMONG SUBROUTIN			ALLM	127
C		THROUGH COMMON			ALLM	128
C	VPHSEC	=INTERNALLY USED PHASE VELOCITY TRANSMITTED AMONG			ALLM	129
C		SUBROUTINES THROUGH COMMON			ALLM	130
C	THETKP	=SAME AS THETK			ALLM	131
C					ALLM	132
C	----EXAMPLE----				ALLM	133
C					ALLM	134
C	SUPPOSE THE TABLE OF INMODE VALUES IS AS SHOWN BELOW WITH				ALLM	135
C					ALLM	136
C					ALLM	137
C					ALLM	138
C					ALLM	139
C					ALLM	140

PROGRAM
ALLMOD
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C	+++---++	NROW=6, NCOL=10	ALLM 141
C	+++---++		ALLM 142
C	+++---++	IF MAXMOD=10, YOU SHOULD FIND MDEFND=6.	ALLM 143
C	5---++		ALLM 144
C	5++-++++	KST(1)=1 KFIN(1)=4 OMMOD(1-36) SHOULD BE	ALLM 145
C	5++-++++	KST(2)=5 KFIN(2)=10 VPMOD(1-36) TABULATED	ALLM 146
C		KST(3)=11 KFIN(3)=21	ALLM 147
C		KST(4)=22 KFIN(4)=29	ALLM 148
C		KST(5)=30 KFIN(5)=34	ALLM 149
C		KST(6)=35 KFIN(6)=36	ALLM 150
C			ALLM 151
C			ALLM 152
C		----PROGRAM FOLLOWS BELOW----	ALLM 153
C			ALLM 154
C		SUBROUTINE ALLMOD(NROW,NCOL,MAXMOD,MDEFND,OM,VP,KST,KFIN,OMMOD,	ALLM 155
C		1 VPMOD,INMODE,THETK,KWOP)	ALLM 156
C			ALLM 157
C		DIMENSION CI(100),VXI(100),VYI(100),HI(100)	ALLM 158
C		DIMENSION OM(1),VP(1),KST(1),KFIN(1),OMMOD(1),VP OD(1),INMODE(1)	ALLM 159
C		COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETKP	ALLM 160
C			ALLM 161
C		STORE THETK IN COMMON	ALLM 162
C		THETKP=THETK	ALLM 163
C			ALLM 164
C		AT THIS POINT, WE HAVEN'T FOUND ANY MODES	ALLM 165
C		MDEFND=0	ALLM 166
C			ALLM 167
C		WE START SEARCH FOR FIRST MODE IN LOWER LEFT CORNER OF INMODE ARRAY.	ALLM 168
C		WE SEEK A POINT WITH INMODE .NE. 5 WHERE THE NMDF EXISTS.	ALLM 169
C		NMODE=1	ALLM 170
C		KST(NMODE)=1	ALLM 171
C		IST=NROW	ALLM 172
C			ALLM 173
C		THE SEARCH GOES TO THE RIGHT. IF WE DON'T FIND A POINT IN THE BOTTOM	ALLM 174
C		ROW, WE TRY THE (NROW-1)-TH ROW, ETC. AT STATEMENT 2 WE ARE STARTING	ALLM 175
C		AT THE LEFT OF A GIVEN ROW.	ALLM 176
C		2 JST=1	ALLM 177
C		3 IO=INMODE((JST-1)*NROW+IST)	ALLM 178
C		IF(IO .NE. 5) GO TO 10	ALLM 179
C			ALLM 180
C		IF JST IS NOT NCOL WE GO TO THE RIGHT.	ALLM 181
C		IF(JST .EQ. NCOL) GO TO 5	ALLM 182
C		JST=JST+1	ALLM 183
C		GO TO 3	ALLM 184
C			ALLM 185
C		AT THIS POINT WE HAVE EXHAUSTED AN ENTIRE ROW. WE GO TO THE NEXT	ALLM 186
C		HIGHER ROW PROVIDED IST .NE. 1. IF IST IS 1, THE ENTIRE SET OF	ALLM 187
C		INMODES ARE 0.	ALLM 188
C		5 IF(IST .EQ. 1) GO TO 7	ALLM 189
C		IST=IST-1	ALLM 190
C		GO TO 2	ALLM 191
C			ALLM 192
C		7 WRITE (6,*)	ALLM 193
C		8 FORMAT(1H0,51HTHE NORMAL MODE DISPERSION FUNCTION DOES NOT EXIST ,	ALLM 194
C		1 26HFOR ANY POINT IN THE ARRAY / 1H ,22HALLMOD RETURNS KWOP=-1)	ALLM 195
C		9 KWOP=-1	ALLM 196
C		RETURN	ALLM 197
C			ALLM 198
C		STATEMENT 10 IS START OF LOOP. EACH PASSAGE THROUGH LOOP CORRESPONDS	ALLM 199
C		TO A GIVEN MODE.	ALLM 200
C		10 CALL NXMODE(IST,JST,NCOL,NROW,INMODE,IFND,JFND,KEX)	ALLM 201
C			ALLM 202
C		IF YOU CANNOT FIND THE FIRST MODE YOU ARE IN TROUBLE	ALLM 203
C		IF(NMODE .NE. 1) GO TO 15	ALLM 204
C		IF(KEX .EQ. 1) GO TO 15	ALLM 205
C		WRITE (6,11)	ALLM 206
C		11 FORMAT(1H0,36HNXMODE COULD NOT FIND THE FIRST MODE/ 1H ,	ALLM 207
C		12HALLMOD RETURNS KWOP=-1)	ALLM 208
C		GO TO 9	ALLM 209
C			ALLM 210

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C	IF THE MODE SOUGHT IS NOT THE FIRST AND YOU CANNOT FIND IT, THEN THE	ALLM 211
C	RETURN IS CONSIDERED SATISFACTORY.	ALLM 212
	15 IF(KEX .EQ. -1) GO TO 50	ALLM 213
C	WE NOW TABULATE THE NMODE-TH MODE	ALLM 214
	CALL MCODETR(JFND,JFND,NMODE,KST,KFIN,OMMCD,VPMOD,NROW,NCOL,INMODE,	ALLM 215
	1 OM,VP,KRUD)	ALLM 216
C	IT IS DOUBTFUL THAT KRUD COULD BE -1. HOWEVER, IF IT DID HAPPEN, WE	ALLM 217
C	WOULD LIKE TO KNOW THAT IT DID.	ALLM 218
	IF(KRUD .EQ. 1) GO TO 30	ALLM 219
	WRITE (6,21) NMODE,IFND,JFND	ALLM 220
	21 FORMAT(1H0,23HMODETR RETURNS KRUD=-1.,2X,25HCURRENT VALUE OF NMODE	ALLM 221
	1 IS, 14, 3H, , 5HIFND=, 14,3H, , 5HJFND=, 14/ 1H,27HSEE DOCUME	ALLM 222
	2NTATION OF ALLMOD)	ALLM 223
C	WE KEEP NMODE THE SAME AND TRANSFER CONTROL TO STATEMENT 35	ALLM 224
	GO TO 35	ALLM 225
	30 MCFND=MCFND+1	ALLM 226
C	THIS IS THE CURRENT NUMBER OF MODES FOUND.	ALLM 227
C	WE NOW CHECK IF THIS IS MAXMOD. IF IT IS, THE RETURN IS WITH KWOP=1.	ALLM 228
	IF(MCFND .EQ. MAXMOD) GO TO 50	ALLM 229
	NMODE=NMODE+1	ALLM 230
	KST(NMODE)=KFIN(NMODE-1)+1	ALLM 231
C	WE SEEK NEW IST AND JST BEFORE CALLING NXMODE.	ALLM 232
	35 IO=INMODE((JFND-1)*NROW+IFND)	ALLM 233
	IF(IFND .EQ. 1) GO TO 40	ALLM 234
C	WE CHECK INMODE OF POINT ABOVE	ALLM 235
	IUP=INMODE((JFND-1)*NROW+IFND-1)	ALLM 236
C	IF THIS IS -10, THE POINT ABOVE IS THE ONE DESIRED	ALLM 237
	IF(IUP .NE. -10) GO TO 40	ALLM 238
	IST=IFND-1	ALLM 239
	JST=JFND	ALLM 240
	GO TO 10	ALLM 241
C	WE CHECK INMODE OF POINT TO RIGHT. THERE IS NO PLACE TO GO IF JFND=	ALLM 242
C	NCOL. THIS IS INTERPRETED AS SUCCESS PROVIDING MCFND .NE. 0.	ALLM 243
	40 IF(JFND .NE. NCOL) GO TO 43	ALLM 244
	GO TO 50	ALLM 245
C	IRT IS INMODE OF POINT TO RIGHT	ALLM 246
	43 IRT=INMODE((JFND)*NROW+IFND)	ALLM 247
	IF(IRT .NE. -10) GO TO 50	ALLM 248
	IST=IFND	ALLM 249
	JST=JFND+1	ALLM 250
	GO TO 10.	ALLM 251
C	THE SEARCH HAS TERMINATED. IF MCFND=0, WE HAVE BEEN UNSUCCESSFUL.	ALLM 252
	50 IF(MCFND .EQ. 0) GO TO 9	ALLM 253
	KWOP=1	ALLM 254
	RETURN	ALLM 255
	END	ALLM 256
		ALLM 257
		ALLM 258
		ALLM 259
		ALLM 260
		ALLM 261
		ALLM 262
		ALLM 263
		ALLM 264
		ALLM 265
		ALLM 266
		ALLM 267

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ALLMOD

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C      AMRNT (SUBROUTINE)              7/27/68
C
C      ----ABSTRACT----
C
C  TITLE - AMRNT
C      THIS SUBROUTINE COMPUTES THE AMBIENT PRESSURE IN DYNES/CM**2
C      AT A GIVEN ALTITUDE Z KM BY USE OF THE EQUATION
C
C      PRESUR = (1.E6)*EXP(-INTEGRAL FROM 0 TO Z OF GAMMA*G/C**2)
C
C      WHERE 1.E6 DYNES/CM**2 IS THE AMBIENT PRESSURE AT THE GROUND,
C      GAMMA=1.4 IS THE SPECIFIC HEAT RATIO FOR AIR, G=.0098 KM/SEC**2
C      IS THE ACCELERATION OF GRAVITY, AND C IS THE ALTITUDE DEPENDENT
C      SOUND SPEED IN KM/SEC. THE ABOVE EQUATION FOLLOWS FROM THE
C      HYDROSTATIC EQUATION  $D\rho/DZ = -\rho g$  AND THE IDEAL GAS LAW
C       $C^2 = \gamma p / \rho$ .
C
C      THE SOUND SPEED PROFILE IS THAT OF A MULTILAYER ATMOSPHERE AND
C      IS PRESUMED TO BE STORED IN COMMON BEFORE EXECUTION. THE
C      PROGRAM ALSO RETURNS THE INDEX I OF THE LAYER IN WHICH Z LIES.
C
C  PROGRAM NOTES
C
C      IN THE EVENT THAT THE INPUT VALUE OF Z SHOULD BE NEGATIVE
C      THE FIRST LAYER IS ASSUMED TO HOLD FOR Z .LT. 0 WITH THE
C      AMBIENT PRESSURE STILL EQUAL TO 1.E6 AT Z=0. THE PROGRAM
C      RETURNS PRESUR .GT. 1.E6 AND I=1.
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C2A-6515-4)
C  AUTHOR - A.D. PIERCE, M.I.T., JULY, 1968
C
C      ----CALLING SEQUENCE----
C
C  SUBROUTINE PAMPDE
C      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C      COMMON IMAX,CI,VXI,VYI,HI (THESE MUST BE STORED IN COMMON)
C      CALL AMRNT(Z,PRESUR,I)
C
C      ----EXTERNAL SUBROUTINES REQUIRED----
C
C      NO EXTERNAL SUBROUTINES ARE REQUIRED.
C
C      ----ARGUMENT LIST----
C
C      Z          R*4    NO    INP
C      PRESUR     R*4    NO    OUT
C      I          I*4    NO    OUT
C
C  COMMON STORAGE USED
C      COMMON IMAX,CI,VXI,VYI,HI
C
C      IMAX       I*4    NO    INP
C      CI         R*4    100   INP
C      VXI        R*4    100   INP (NOT USED BY THIS SUBROUTINE)
C      VYI        R*4    100   INP (NOT USED BY THIS SUBROUTINE)
C      HI         R*4    100   INP
C
C      ----INPUTS----
C
C      Z          =HEIGHT IN KM
C      IMAX       =NUMBER OF ATMOSPHERIC LAYERS WITH FINITE THICKNESS
C      CI(I)      =SOUND SPEED (KM/SEC) IN I-TH LAYER
C      VXI(I)     =X COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C      VYI(I)     =Y COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C      HI(I)      =THICKNESS IN KM OF I-TH LAYER
C
C      ----OUTPUTS----
C
C      PRESUR     =AMBIENT PRESSURE IN DYNES/CM**2 AT ALTITUDE Z
C      I          =INDEX OF LAYER IN WHICH Z LIES

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AMRN 1
AMRN 2
AMRN 3
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AMRN 7
AMRN 8
AMRN 9
AMRN 10
AMRN 11
AMRN 12
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AMRN 14
AMRN 15
AMRN 16
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C		AMBN 71
C	-----PROGRAM FOLLOWS BELOW-----	AMBN 72
C		AMBN 73
C	SUBROUTINE AMBNT(Z,PRESUR,I)	AMBN 74
C		AMBN 75
C	DIMENSION AND COMMON STATEMENTS	AMBN 76
C	DIMENSION CI(100),VXI(100),VYI(100),HI(100)	AMBN 77
C	COMMON IMAX,CI,VXI,VYI,HI	AMBN 78
C		AMBN 79
C	THE FINAL VALUE OF ENPON WILL BE THE INTEGRAL FROM 0 TO Z OF	AMBN 80
C	$-GAMMA*G/C**2$. THE RUNNING VALUE WILL BE THE SUBTOTAL.	AMBN 81
C	ENPON=0.0	AMBN 82
C		AMBN 83
C	THE RUNNING VALUE OF I WILL BE THE LAYER BEING CONSIDERED	AMBN 84
C	I=1	AMBN 85
C	Z LIES IN LAYER 1 IF IMAX=0.	AMBN 86
C	ZT=0.0	AMBN 87
C	IF(IMAX .EQ. 0) GO TO 30	AMBN 88
C		AMBN 89
C	TOP OF FIRST LAYER	AMBN 90
C	ZT=HI(1)	AMBN 91
C		AMBN 92
C	THE START OF A LOOP. THE CURRENT ZT DENOTES THE TOP OF THE I-TH LAYER	AMBN 93
C	10 IF(Z .GT. ZT) GO TO 20	AMBN 94
C		AMBN 95
C	Z LIES IN I-TH LAYER	AMBN 96
C	ZT-HI(I) IS HEIGHT OF BOTTOM OF I-TH LAYER	AMBN 97
C	Z-ZT+HI(I) IS DISTANCE OF Z ABOVE BOTTOM OF I-TH LAYER	AMBN 98
C	ENPON=ENPON-1.4*(.0098/CI(I)**2)*(Z-ZT+HI(I))	AMBN 99
C	12 GO TO 40	AMBN 100
C		AMBN 101
C	Z LIES ABOVE TOP OF I-TH LAYER	AMBN 102
C	20 ENPON=ENPON-1.4*(.0098/CI(I)**2)*HI(I)	AMBN 103
C	THE CURRENT ENPON IS THE INTEGRAL OF $-1.4*G/C**2$ UP TO THE TOP	AMBN 104
C	OF THE I-TH LAYER	AMBN 105
C	I=I+1	AMBN 106
C	IF(I .GT. IMAX) GO TO 30	AMBN 107
C	ZT=ZT+HI(I)	AMBN 108
C	ZT IS THE TOP OF THE NEW I-TH LAYER	AMBN 109
C	GO TO 10	AMBN 110
C	END OF LOOP	AMBN 111
C		AMBN 112
C	Z LIES IN UPPER HALFSpace	AMBN 113
C	30 ENPON=ENPON-1.4*(.0098/CI(I)**2)*(Z-ZT)	AMBN 114
C		AMBN 115
C	CONTINUING FROM 12 OR 30	AMBN 116
C	40 PRESUR=1.E6*EXP(ENPON)	AMBN 117
C	RETURN	AMBN 118
C	END	AMBN 119

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C	ATMOS (SUBROUTINE)	6/19/68	ATMO	1
C			ATMO	2
C			ATMO	3
C	-----ABSTRACT-----		ATMO	4
C			ATMO	5
C			ATMO	6
C	TITLE - ATMOS		ATMO	7
C	TABULATION OF WIND VELOCITY COMPONENTS AND SPEED OF SOUND FOR		ATMO	8
C	ALL LAYERS OF MODEL ATMOSPHERES		ATMO	9
C			ATMO	10
C	THE MODEL ATMOSPHERE CONSISTS OF UP TO 100 ISOTHERMAL		ATMO	11
C	LAYERS (THE TOP LAYER BEING INFINITE). EACH LAYER MAY		ATMO	12
C	HAVE A UNIQUE TEMPERATURE, THICKNESS AND WIND VELOCITY.		ATMO	13
C	SUBROUTINE ATMOS CONVERTS AN INPUT DESCRIPTION OF THE		ATMO	14
C	ATMOSPHERE'S PROPERTIES INTO ONE MORE APPROPRIATE FOR THE		ATMO	15
C	CALCULATIONS TO FOLLOW (SUCH AS EVALUATION OF THE NORMAL		ATMO	16
C	MODE DISPERSION FUNCTION IN NMDFN, DESCRIBED ELSEWHERE IN		ATMO	17
C	THIS SERIES).		ATMO	18
C			ATMO	19
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		ATMO	20
C			ATMO	21
C	AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., JUNE, 1968		ATMO	22
C			ATMO	23
C	-----USAGE-----		ATMO	24
C			ATMO	25
C	IMAX MUST BE STORED AS THE FIRST VARIABLE IN UNLABELED COMMON WHEN		ATMO	26
C	ATMOS IS CALLED.		ATMO	27
C			ATMO	28
C	NO FORTRAN SUBROUTINES ARE CALLED.		ATMO	29
C			ATMO	30
C	FORTRAN USAGE		ATMO	31
C			ATMO	32
C	CALL ATMOS(T,VKNTX,VKNTY,ZI,WANGLE,WINDY,LANGLE)		ATMO	33
C			ATMO	34
C	INPUTS		ATMO	35
C			ATMO	36
C	IMAX NUMBER OF LAYERS OF FINITE THICKNESS IN THE MODEL ATMOS-		ATMO	37
C	I*4 PHERE. (1.LE.IMAX.LE.99)		ATMO	38
C			ATMO	39
C	T T(I) IS TEMPERATURE OF LAYER I IN MODEL ATMOSPHERE.		ATMO	40
C	R*4(D) (DEGREES KELVIN)		ATMO	41
C			ATMO	42
C	VKNTX VKNTX(I) IS WIND VELOCITY COMPONENT IN X-DIRECTION (WEST		ATMO	43
C	R*4(D) TO EAST) FOR LAYER I. (KNOTS)		ATMO	44
C			ATMO	45
C	VKNTY VKNTY(I) IS WIND VELOCITY COMPONENT IN Y-DIRECTION (SOUTH		ATMO	46
C	R*4(D) TO NORTH) FOR LAYER I. (KNOTS)		ATMO	47
C			ATMO	48
C	ZI ZI(I) IS THE HEIGHT ABOVE THE GROUND OF THE TOP OF LAYER		ATMO	49
C	R*4(D) I. (KM)		ATMO	50
C			ATMO	51
C	WANGLE WANGLE(I) IS WIND VELOCITY DIRECTION FOR LAYER I, RECKONE		ATMO	52
C	R*4(D) COUNTER CLOCKWISE FROM THE X-AXIS. (DEGREES)		ATMO	53
C			ATMO	54
C	WINDY WINDY(I) IS MAGNITUDE OF WIND VELOCITY IN LAYER I.		ATMO	55
C	R*4(D) (KNOTS)		ATMO	56
C			ATMO	57
C	LANGLE SPECIFIES WHICH SORT OF WIND DATA IS INPUT.		ATMO	58
C	I*4 IF LANGLE.LE.0, VKNTX AND VKNTY ARE INPUT.		ATMO	59
C			ATMO	60
C	IF LANGLE.GT.0, WANGLE AND WINDY ARE INPUT.		ATMO	61
C			ATMO	62
C	OUTPUTS		ATMO	63
C			ATMO	64
C	THE OUTPUTS ARE STORED IN UNLABELED COMMON IN THE FOLLOWING		ATMO	65
C	ORDER, BEGINNING IN POSITION 2,		ATMO	66
C	C(100),VXI(100),VYI(100),HI(100)		ATMO	67
C			ATMO	68
C	CI CI(I) IS THE SPEED OF SOUND IN LAYER I OF THE MODEL ATMOS		ATMO	69
C	R*4(D) PHEPF. (KM/SEC)		ATMO	70
C				

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C	VXI	VXI(I) IS WIND VELOCITY COMPONENT IN X-DIRECTION (WEST TO	ATMO 71
C	R*4(D)	EAST) FOR LAYER I. (KM/SEC)	ATMO 72
C			ATMO 73
C	VYI	VYI(I) IS WIND VELOCITY COMPONENT IN Y-DIRECTION (SOUTH	ATMO 74
C	R*4(D)	TO NORTH) FOR LAYER I. (KM/SEC)	ATMO 75
C			ATMO 76
C	HI	HI(I) IS THE THICKNESS OF LAYER I. (KM)	ATMO 77
C	R*4(D)		ATMO 78
C			ATMO 79
C			ATMO 80
C		-----PROGRAM FOLLOWS BELOW-----	ATMO 81
C			ATMO 82
C			ATMO 83
C		SUBROUTINE ATMOS(T,VKNTX,VKNTY,ZI,WANGLE,WINDY,LANGLE)	ATMO 84
C			ATMO 85
C		DIMENSION CI(100),VXI(100),VYI(100),HI(100)	ATMO 86
C		DIMENSION T(100),VKNTX(100),VKNTY(100),ZI(100)	ATMO 87
C		DIMENSION WANGLE(100),WINDY(100)	ATMO 88
C		COMMON IMAX,CI,VXI,VYI,HI	ATMO 89
C			ATMO 90
C	JET	IS TOTAL NUMBER OF LAYERS.	ATMO 91
C		JET = IMAX + 1	ATMO 92
C		IMAX = JET - 1	ATMO 93
C		IF (LANGLE .LE. C) GO TO 20	ATMO 94
C		D3 = 3.1415927 / 180.0	ATMO 95
C	D3	IS THE NUMBER OF RADIAN IN A DEGREE	ATMO 96
C			ATMO 97
C		IF VKNTX AND VKNTY WERE NOT INPUT, THEY ARE NOW DETERMINED FROM WINDY	ATMO 98
C	AND WANGLE.		ATMO 99
C		DO 5 I=1,JET	ATMO 100
C		VKNTX(I) = WINDY(I) * COS(D3*WANGLE(I))	ATMO 101
C		5 VKNTY(I) = WINDY(I) * SIN(D3*WANGLE(I))	ATMO 102
C		20 D1 = 1.4 * R.3144 * 0.001 / 29.0	ATMO 103
C	D2	IS THE NUMBER OF KM/SEC PER KNOT.	ATMO 104
C		D2 = 0.0005148	ATMO 105
C			ATMO 106
C		DO 30 I = 1,JET	ATMO 107
C			ATMO 108
C		C THE SPEED OF SOUND = (GAMMA * P / RHO) FOR PERFECT GAS, AND (P/RHO	ATMO 109
C		= (R * T)	ATMO 110
C	R	IS THE (UNIVERSAL GAS CONSTANT)/(MOLECULAR WEIGHT)	ATMO 111
C		CI(I) = SQRT(D1*T(I))	ATMO 112
C			ATMO 113
C		(D2 * V(KNOTS)) = V(KM/SEC)	ATMO 114
C		VXI(I) = D2 * VKNTX(I)	ATMO 115
C	30	VYI(I) = D2 * VKNTY(I)	ATMO 116
C		IF(IMAX .EQ. 0) RETURN	ATMO 117
C		HI(I) = ZI(I)	ATMO 118
C		IF(IM .EQ. 1) RETURN	ATMO 119
C		DO 40 I=2,IMAX	ATMO 120
C	40	HI(I) = ZI(I) - ZI(I-1)	ATMO 121
C		RETURN	ATMO 122
C		END	ATMO 123

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C      BARR (SUBROUTINE)                      7/25/68                      BARR 1
C
C      ----ABSTRACT----                      BARR 2
C
C      TITLE - BARR                          BARR 3
C      THIS SUBROUTINE COMPUTES THREE FUNCTIONS R1,R2,R3 OF A VARIABLE BARR 4
C      X. THESE ARE DEFINED FOR X .GE. 0 BY THE FORMULAS BARR 5
C
C      R1= 1.0 +SINH(2Y)/(2Y) BARR 6
C
C      R2= (SINH(2Y)/2Y - 1.0)/Y**2 BARR 7
C
C      R3= (COSH(2Y)-1.0)/Y**2 BARR 8
C
C      WHERE Y= SQRT(X). FORMULAS FOR NEGATIVE X MAY BE OBTAINED BY BARR 9
C      ANALYTIC CONTINUATION. FOR SMALL VALUES OF X, THE FUNCTIONS BARR 10
C      ARE COMPUTABLE BY THE POWER SERIES BARR 11
C
C      R1= 2 + 4X/(3FACT) + (4X)**2/(5FACT) + (4X)**3/(7FACT) +... BARR 12
C
C      R2= 4/(3FACT) + 4*(4X)/(5FACT) + 4*(4X)**2/(7FACT) +... BARR 13
C
C      R3= 4/(2FACT) + 4*(4X)/(4FACT) + 4*(4X)**2/(6FACT) +... BARR 14
C
C      THE MANNER IN WHICH THESE PARTICULAR FUNCTIONS ARISE IN THE BARR 15
C      THEORY COMES FROM INTEGRATIONS OVER VARIOUS PRODUCTS OF CAI(X) BARR 16
C      AND SAI(X). IN PARTICULAR, FOR X POSITIVE, BARR 17
C
C      R1= (2/Y)(INTEGRAL ON Y FROM 0 TO Y OF (COSH(Y))**2) BARR 18
C
C      R2= (2/Y**3)(INTEGRAL ON Y FROM 0 TO Y OF (SINH(Y))**2) BARR 19
C
C      R3= (4/Y**2)(INTEGRAL ON Y FROM 0 TO Y OF SINH(Y)*COSH(Y)) BARR 20
C
C      WITH Y=SQRT(X). THE CORRESPONDING FORMULAS FOR X NEGATIVE CAN BARR 21
C      BE OBTAINED BY REPLACING SINH AND COSH BY SIN AND COS, RESPEC- BARR 22
C      TIVELY, AND BY REINTERPRETING Y AS SQRT(-X). BARR 23
C
C      LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4) BARR 24
C      AUTHOR - A.D.PIRCE, M.I.T., JULY, 1968 BARR 25
C
C      ----CALLING SEQUENCE---- BARR 26
C
C      SUBROUTINE FLINT BARR 27
C      X= BARR 28
C      CALL BARR(X,R1,R2,R3) BARR 29
C
C      ----EXTERNAL SUBROUTINES REQUIRED---- BARR 30
C
C      CAI, SAI BARR 31
C
C      ----ARGUMENT LIST---- BARR 32
C
C      X      R*4      NO      INP BARR 33
C      R1      P*4      NO      OUT BARR 34
C      R2      R*4      NO      OUT BARR 35
C      R3      R*4      NO      OUT BARR 36
C
C      NO COMMON STORAGE IS USED BARR 37
C
C      ----PROGRAM FOLLOWS BELOW---- BARR 38
C
C      SUBROUTINE BARR(X,R1,R2,R3) BARR 39
C      S=SAI(4.0*X) BARR 40
C      IF(ABS(X) .GT. 1.E-2) GO TO 3 BARR 41
C
C      COMPUTATION FOR SMALL X BARR 42
C      R2=2.0/3.0*(2.0/15.0)*X+(4.0/315.0)*X**2+(2.0/9.0)*X**3/315.0 BARR 43
C      R3=2.0+2.0*X/3.0+4.0*X**2/45.0+2.0*X**3/315.0 BARR 44
C      GO TO 4 BARR 45
C
C      PROGRAM BARR 46
C      PAGE BARR 47
C      23 BARR 48

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C
C COMPUTATION FOR X NOT NEAR ZERO
3 R2=(S-1.0)/X
  R3=(CAI(4.0*X)-1.0)/X
C
C COMPUTATION OF R1 FOR ARBITRARY X
4 R1=1.0+S
  RETURN
  END

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BBBB 71
BBBB 72
BBBB 73
BBBB 74
BBBB 75
BBBB 76
BBBB 77
BBBB 78
BBBB 79

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PROGRAM
BBBB

PAGE
24

C	CAT (FUNCTION)	7/25/69	CAT	1
C			CAT	2
C	-----ABSTRACT-----		CAT	3
C			CAT	4
C	TITLE - CAT		CAT	5
C	PROGRAM TO EVALUATE FUNCTION CAT(X) FOR GIVEN VARIABLE X.		CAT	6
C	IF X IS NEGATIVE, CAT(X)= COS(SORT(-X)). IF X IS POSITIVE,		CAT	7
C	CAT(X)= COSH(SORT(+X)). THE FUNCTION IS ALSO REPRESENTABLE		CAT	8
C	BY THE POWER SERIES		CAT	9
C			CAT	10
C	CAT(X)= 1 + X/(2FACT) + X**2/(4FACT) + X**3/(6FACT) + ...		CAT	11
C			CAT	12
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C2A-5515-4)		CAT	13
C			CAT	14
C	AUTHOR - A.D. PIERCE, M.I.T., JULY, 1968		CAT	15
C	-----CALLING SEQUENCE-----		CAT	16
C			CAT	17
C	CAT(ANY R*4 ARGUMENT) MAY BE USED IN ARITHMETIC EXPRESSIONS		CAT	18
C			CAT	19
C	-----EXTERNAL SUBROUTINES REQUIRED-----		CAT	20
C			CAT	21
C	NO EXTERNAL SUBROUTINES ARE REQUIRED		CAT	22
C			CAT	23
C	-----ARGUMENT LIST-----		CAT	24
C			CAT	25
C	X R*4 NO INP		CAT	26
C	CAT R*4 NO OUT		CAT	27
C			CAT	28
C	NO COMMON STORAGE IS USED		CAT	29
C			CAT	30
C	-----PROGRAM FOLLOWS BELOW-----		CAT	31
C			CAT	32
C	FUNCTION CAT(X)		CAT	33
C			CAT	34
C	IF(X .GE. 0.0) GO TO 11		CAT	35
C			CAT	36
C	X IS LESS THAN 0		CAT	37
C	10 CAT=COS(SORT(-X))		CAT	38
C	RETURN		CAT	39
C			CAT	40
C	X IS GREATER OR EQUAL TO 0		CAT	41
C	11 F=EXP(SORT(X))		CAT	42
C	THE HYPERBOLIC COSINE IS COMPUTED		CAT	43
C	CAT=0.5*(E+1./F)		CAT	44
C	RETURN		CAT	45
C	END		CAT	46

PROGRAM
CAT

PAGE
25

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C      ELINT (SUBROUTINE)              7/25/68
C
C      -----ABSTRACT-----
C
C      TITLE - ELINT
C      THIS SUBROUTINE COMPUTES THE INTEGRAL
C
C      AINT = INTEGRAL OVER Z FROM 0 TO H OF
C
C      (A1*F1(Z) + A2*F2(Z))*Z
C
C      (1)
C
C      THE FUNCTIONS F1(Z) AND F2(Z) ARE THE SOLUTIONS OF THE COUPLED
C      ORDINARY DIFFERENTIAL EQUATIONS
C
C      DF1/DZ = A11*F1 + A12*F2
C      DF2/DZ = A21*F1 + A22*F2
C
C      (2A)
C      (2B)
C
C      WHERE THE ELEMENTS OF THE MATRIX A ARE INDEPENDENT OF Z.
C      FOR GIVEN SOUND SPEED C, WIND VELOCITY COMPONENTS VX AND VY,
C      ANGULAR FREQUENCY OMEGA, AND WAVE NUMBER COMPONENTS AKX AND AKY
C      THE A(I,J) ARE COMPUTED BY CALLING AAAA. THE SOLUTION TO THE
C      DIFFERENTIAL EQUATIONS IS FIXED BY SPECIFICATION OF F1 AND F2
C      AT Z=H.
C
C      PROGRAM NOTES
C
C      THE GENERAL SOLUTION OF EONS. (2) IS
C
C      F1(Z) = CAI(X)*F1(H)-(H-Z)*SAI(X)*(A11*F1(H)+A12*F2(H))
C      F2(Z) = CAI(X)*F2(H)-(H-Z)*SAI(X)*(A21*F1(H)+A22*F2(H))
C
C      WITH X=(A11**2+A12*A21)*(H-Z)**2 SINCE A22=-A11. WE LET
C
C      R1=(INTEGRAL OF (CAI(X)**2)*(Z/H))
C      R2=(INTEGRAL OF ((H-Z)*SAI(X)**2)*(Z/H**3))
C      R3=(INTEGRAL OF ((H-Z)*SAI(X)*CAI(X))*(4/H**2))
C
C      WHERE IN EACH CASE THE INTEGRATION IS OVER Z FROM 0 TO H.
C      THE QUANTITIES R1,R2,R3 ARE COMPUTED BY CALLING BBBB.
C      THEN
C
C      AINT=(H/2)*(FP1)**2*R1+(H**3/2)*(FP2)**2*R2
C      -(H**2/2)*(FP1)*(FP2)*R3
C
C      WITH
C      FP1= A1*F1(H)+A2*F2(H)
C      FP2= A1*(A11*F1(H)+A12*F2(H))+A2*(A21*F1(H)+A22*F2(H))
C
C      IF LATTER TWO QUANTITIES REPRESENT THE COEFFICIENTS OF
C      CAI(X) AND (H-Z)*SAI(X) IN A1*F1+A2*F2.
C
C      LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C20-6515-4)
C
C      AUTHOR - A.D. PIERCE, M.I.T., JULY, 1968
C
C      -----CALLING SEQUENCE-----
C
C      SEE SUBROUTINE TOTINT
C      NO DIMENSION STATEMENTS REQUIRED
C      CALL ELINT(OMEGA,AKX,AKY,C,VX,VY,H,F1H,F2H,A1,A2,AINT)
C
C      -----EXTERNAL SUBROUTINES REQUIRED-----
C
C      AAAA, BBBB
C
C      -----ARGUMENT LIST-----
C
C      OMEGA      R*4      ND      INP
C      AKX        R*4      ND      INP
C      AKY        R*4      ND      INP

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FLIN  1
ELIN  2
FLIN  3
ELIN  4
FLIN  5
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PROGRAM
ELINT
PAGE
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C	C	R**4	NO	INP	ELIN	71
C	VX	R**4	NO	INP	ELIN	72
C	VY	R**4	NO	INP	ELIN	73
C	H	R**4	NO	INP	ELIN	74
C	F1H	R**4	NO	INP	ELIN	75
C	F2H	R**4	NO	INP	ELIN	76
C	A1	R**4	NO	INP	ELIN	77
C	A2	R**4	NO	INP	ELIN	78
C	AINT	R**4	NO	OUT	ELIN	79
C					ELIN	80
C	NO COMMON STORAGE USED				ELIN	81
C					ELIN	82
C	-----INPUTS-----				ELIN	83
C					ELIN	84
C	OMEGA	=ANGULAR FREQUENCY IN RAD/SEC			ELIN	85
C	AKX	=X COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)			ELIN	86
C	AKY	=Y COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)			ELIN	87
C	C	=SOUND SPEED IN KM/SEC			ELIN	88
C	VX	=X COMPONENT OF WIND VELOCITY IN KM/SEC			ELIN	89
C	VY	=Y COMPONENT OF WIND VELOCITY IN KM/SEC			ELIN	90
C	H	=INTEGRATION INTERVAL (LAYER THICKNESS) IN KM			ELIN	91
C	F1H	=VALUE OF F1(Z) AT UPPER LIMIT OF INTEGRAL			ELIN	92
C	F2H	=VALUE OF F2(Z) AT UPPER LIMIT OF INTEGRAL			ELIN	93
C	A1	=COEFFICIENT OF F1(Z) IN INTEGRAND			ELIN	94
C	A2	=COEFFICIENT OF F2(Z) IN INTEGRAND			ELIN	95
C					ELIN	96
C	-----OUTPUTS-----				ELIN	97
C					ELIN	98
C	AINT	=INTEGRAL OVER HEIGHT WITH RANGE H OF THE QUANTITY			ELIN	99
C		(A1*F1(Z)+A2*F2(Z))*2 WHERE F1(Z) AND F2(Z) ARE			ELIN	100
C		EQUAL TO F1H AND F2H, RESPECTIVELY, AT THE UPPER			ELIN	101
C		LIMIT AND SATISFY THE RESIDUAL DIFFERENTIAL EQUATIONS			ELIN	102
C					ELIN	103
C	-----PROGRAM FOLLOWS BELOW-----				ELIN	104
C					ELIN	105
C	SUBROUTINE ELINT(OMEGA,AKX,AKY,C,VX,VY,H,F1H,F2H,A1,A2,AINT)				ELIN	106
C	DIMENSION A(2,2)				ELIN	107
C	CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)				ELIN	108
C					ELIN	109
C	COMPUTATION OF FP1 AND FP2				ELIN	110
C	FP1=A1*F1H+A2*F2H				ELIN	111
C	FP2=A1*(A(1,1)*F1H+A(1,2)*F2H)+A2*(A(2,1)*F1H+A(2,2)*F2H)				ELIN	112
C					ELIN	113
C	COMPUTATION OF COEFFICIENTS OF R1,R2,R3				ELIN	114
C	S1=0.5*H*FP1**2				ELIN	115
C	S2=0.5*(H**3)*FP2**2				ELIN	116
C	S3=-0.5*(H**2)*FP1*FP2				ELIN	117
C					ELIN	118
C	COMPUTATION OF R1,R2,R3				ELIN	119
C	X=(A(1,1)*S2+A(1,2)*A(2,1))*H**2				ELIN	120
C	CALL RRRR(X,R1,R2,R3)				ELIN	121
C					ELIN	122
C	COMPUTATION OF AINT				ELIN	123
C	AINT=S1*R1+S2*R2+S3*R3				ELIN	124
C	RETURN				ELIN	125
C	END				ELIN	126

PROGRAM
ELINT

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C	FNM001 (FUNCTION)	6/19/68	FNM1	1
C			FNM1	2
C			FNM1	3
C	-----ABSTRACT-----		FNM1	4
C			FNM1	5
C	TITLE - FNM001		FNM1	6
C	EVALUATION OF NORMAL MODE DISPERSION FUNCTION AS FUNCTION OF		FNM1	7
C	PHASE VELOCITY V		FNM1	8
C			FNM1	9
C	THE NORMAL MODE DISPERSION FUNCTION DEPENDS ON THREE VARI		FNM1	10
C	ABLES. ANGULAR FREQUENCY Ω , PHASE VELOCITY V, AND		FNM1	11
C	DIRECTION OF PROPAGATION THETK. FNM001 OBTAINS V THROUGH		FNM1	12
C	ITS ARGUMENT, Ω AND THETK FROM COMMON. SUBROUTINE		FNM1	13
C	NMDFN IS THEN CALLED TO EVALUATE THE FUNCTION. (SEE		FNM1	14
C	PIERCE, J.COMP.PHYSICS, FEB.,1967, P.343-366 FOR DEFINI-		FNM1	15
C	TION OF NORMAL MODE DISPERSION FUNCTION.)		FNM1	16
C			FNM1	17
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C2A-6515-4)		FNM1	18
C			FNM1	19
C	AUTHORS - A.D.PIERCE AND J.POSEY, M.I.T., JUNE,1968		FNM1	20
C			FNM1	21
C			FNM1	22
C	-----USAGE-----		FNM1	23
C			FNM1	24
C	Ω MUST BE STORED IN WORD POSITION 402 OF UNLABELED COMMON, AND		FNM1	25
C	THETK MUST BE IN POSITION 404.		FNM1	26
C			FNM1	27
C	FNM001 CALLS SUBROUTINE NMDFN WHICH CALLS AAAA AND RRRR. RRRR		FNM1	28
C	CALLS AAAA AND MMMM. ALL THESE SUBROUTINES ARE DESCRIBED ELSE-		FNM1	29
C	WHERE IN THIS SERIES.		FNM1	30
C			FNM1	31
C	CALLING SEQUENCE		FNM1	32
C			FNM1	33
C	COMMON CM1(401), Ω ,CM2,THETK		FNM1	34
C	Ω = XXX		FNM1	35
C	THETK = XXX		FNM1	36
C	V = XXX		FNM1	37
C	FUNCTN = FNM001(V)		FNM1	38
C			FNM1	39
C	INPUTS		FNM1	40
C			FNM1	41
C	V PHASE VELOCITY (KM/SEC).		FNM1	42
C	R=4		FNM1	43
C			FNM1	44
C	Ω ANGULAR FREQUENCY (RAD/SEC).		FNM1	45
C	R=4		FNM1	46
C			FNM1	47
C	THETK PHASE VELOCITY DIRECTION MEASURED COUNTER-CLOCKWISE FROM		FNM1	48
C	R=4 X-AXIS.		FNM1	49
C			FNM1	50
C	OUTPUTS		FNM1	51
C			FNM1	52
C	THE ONLY OUTPUT IS THE VALUE OF THE NORMAL MODE DISPERSION FUNCTION		FNM1	53
C	FOR THE VALUES OF V, Ω , AND THETK WHICH HAVE BEEN INPUT.		FNM1	54
C			FNM1	55
C			FNM1	56
C	-----PROGRAM FOLLOWS BELOW-----		FNM1	57
C			FNM1	58
C			FNM1	59
C	FUNCTION FNM001(V)		FNM1	60
C			FNM1	61
C	DIMENSION C(100),VX(100),VY(100),H(100)		FNM1	62
C	COMMON IMAX,C,VX,VY,H,CMGAC,VPMSEC,THETK		FNM1	63
C			FNM1	64
C	Ω AND THETK OBTAINED FROM COMMON		FNM1	65
C	Ω = Ω GAC		FNM1	66
C	CALL NMDFN(Ω ,V,THETK,L,FPP,K)		FNM1	67
C	FNM001=FPP		FNM1	68
C	RETURN		FNM1	69
C	END		FNM1	70

PROGRAM
FNM001

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C	FNM02 (FUNCTION)	6/19/68	FNM2	1
C			FNM2	2
C			FNM2	3
C	-----ABSTRACT-----		FNM2	4
C			FNM2	5
C	TITLE - FNM02		FNM2	6
C	EVALUATION OF NORMAL MODE DISPERSION FUNCTION AS FUNCTION OF		FNM2	7
C	ANGULAR FREQUENCY OMEGA		FNM2	8
C			FNM2	9
C	THE NORMAL MODE DISPERSION FUNCTION DEPENDS ON THREE VARI		FNM2	10
C	ABLES, ANGULAR FREQUENCY OMEGA, PHASE VELOCITY V, AND		FNM2	11
C	DIRECTION OF PROPAGATION THETK. FNM02 OBTAINS OMEGA		FNM2	12
C	THROUGH ITS ARGUMENT, V AND THETK FROM COMMON. SUBROUTINE		FNM2	13
C	NMDFN IS THEN CALLED TO EVALUATE THE FUNCTION. (SEE		FNM2	14
C	PIERCE, J. COMP. PHYSICS, FEB., 1967, P. 343-366 FOR DEFINI-		FNM2	15
C	TION OF NORMAL MODE DISPERSION FUNCTION.)		FNM2	16
C			FNM2	17
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		FNM2	18
C			FNM2	19
C	AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., JUNE, 1968		FNM2	20
C			FNM2	21
C	-----USAGE-----		FNM2	22
C			FNM2	23
C	V MUST BE STORED IN WORD POSITION 403 OF UNLABELED COMMON, AND		FNM2	24
C	THETK MUST BE IN POSITION 404.		FNM2	25
C			FNM2	26
C	FNM02 CALLS SUBROUTINE NMDFN WHICH CALLS AAAA AND RRRR. RRRR		FNM2	27
C	CALLS AAAA AND MMMM. ALL THESE SUBROUTINES ARE DESCRIBED ELSE-		FNM2	28
C	WHERE IN THIS SERIES.		FNM2	29
C			FNM2	30
C	CALLING SEQUENCE		FNM2	31
C			FNM2	32
C	COMMON CM(402), V, THETK		FNM2	33
C	OMEGA = XXX		FNM2	34
C	V = XXX		FNM2	35
C	THETK = XXX		FNM2	36
C	FUNCTN = FNM02(OMEGA)		FNM2	37
C			FNM2	38
C	INPUTS		FNM2	39
C			FNM2	40
C	V PHASE VELOCITY (KM/SEC).		FNM2	41
C	R*4		FNM2	42
C			FNM2	43
C	OMEGA ANGULAR FREQUENCY (RADIAN/SEC).		FNM2	44
C	R*4		FNM2	45
C			FNM2	46
C	THETK PHASE VELOCITY DIRECTION MEASURED COUNTER-CLOCKWISE FROM		FNM2	47
C	R*4 X-AXIS.		FNM2	48
C			FNM2	49
C	OUTPUTS		FNM2	50
C			FNM2	51
C	THE ONLY OUTPUT IS THE VALUE OF THE NORMAL MODE DISPERSION FUNCTION		FNM2	52
C	FOR THE VALUES OF V, OMEGA, AND THETK WHICH HAVE BEEN INPUT.		FNM2	53
C			FNM2	54
C			FNM2	55
C	-----PROGRAM FOLLOWS BELOW-----		FNM2	56
C			FNM2	57
C			FNM2	58
C	FUNCTION FNM02(OMEGA)		FNM2	59
C			FNM2	60
C	DIMENSION CI(100), VXI(100), VYI(100), HI(100)		FNM2	61
C	COMMON /MAX, CI, VXI, VYI, HI, OMEGAC, VPHSEC, THETK		FNM2	62
C			FNM2	63
C	V AND THETK OBTAINED FROM COMMON		FNM2	64
C	V = VPHSEC		FNM2	65
C	CALL NMDFN(OMEGA, V, THETK, L, FPP, K)		FNM2	66
C	FNM02 = FPP		FNM2	67
C	RETURN		FNM2	68
C	END		FNM2	69
			FNM2	70

PROGRAM
FNM02
PAGE
29

C	LNKTHN (SUBROUTINE)	7/19/68	LNKT	1	
C			LNKT	2	
C			LNKT	3	
C	-----ABSTRACT-----		LNKT	4	
C			LNKT	5	
C	TITLE - LNKTHN		LNKT	6	
C	LENGTHEN THE MATRIX INMODE BY ADDING KL ROWS BETWEEN THE N1 AND		LNKT	7	
C	N1+1 ROWS		LNKT	8	
C			LNKT	9	
C	LNKTHN ADDS KL ELEMENTS TO THE VECTOR OF PHASE VELOCITIES		LNKT	10	
C	V, DIVIDING THE INTERVAL BETWEEN V(N1) AND V(N1+1) INTO		LNKT	11	
C	KL+1 EQUAL PARTS. FOR EACH NEW PHASE VELOCITY, A NEW ROW		LNKT	12	
C	IS ADDED TO THE INMODE MATRIX (DEFINED IN SUBROUTINE		LNKT	13	
C	MPOUT). INMODE IS STORED COLUMN BY COLUMN IN VECTOR FORM		LNKT	14	
C			LNKT	15	
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		LNKT	16	
C	AUTHOR - J.W. POSEY, M.I.T., JUNE, 1968		LNKT	17	
C			LNKT	18	
C			LNKT	19	
C	-----USAGE-----		LNKT	20	
C			LNKT	21	
C	OM, V, INMODE MUST BE DIMENSIONED IN THE CALLING PROGRAM		LNKT	22	
C	NMDFN IS ONLY SUBROUTINE CALLED		LNKT	23	
C			LNKT	24	
C	FORTRAN USAGE		LNKT	25	
C	CALL LNKTHN(OM, V, INMODE, NOM, NVP, NVPP, N1, KL, THETK)		LNKT	26	
C			LNKT	27	
C	INPUTS		LNKT	28	
C			LNKT	29	
C	OM VECTOR WHOSE ELEMENTS ARE THE VALUES OF ANGULAR FRE-		LNKT	30	
C	R*4(D) QUENCY CORRESPONDING TO THE COLUMNS OF THE INMODE MATRIX.		LNKT	31	
C			LNKT	32	
C	V VECTOR WHOSE ELEMENTS ARE THE VALUES OF PHASE VELOCITY		LNKT	33	
C	R*4(D) CORRESPONDING TO THE ROWS OF THE INMODE MATRIX.		LNKT	34	
C			LNKT	35	
C	INMODE EACH ELEMENT OF THIS MATRIX CORRESPONDS TO A POINT IN THE		LNKT	36	
C	I*4(D) FREQUENCY (OM) - PHASE VELOCITY (V) PLANE. IF THE NORMAL		LNKT	37	
C	MODE DISPERSION FUNCTION (FPP) IS POSITIVE AT THAT POINT,		LNKT	38	
C	THE ELEMENT IS +1. IF FPP IS NEGATIVE, THE ELEMENT IS		LNKT	39	
C	-1. IF FPP DOES NOT EXIST, THE ELEMENT IS 5. INMODE HAS		LNKT	40	
C	NVP ROWS (INCREASED TO NVPP) AND NOM COLUMNS. MATRIX IS		LNKT	41	
C	STORED IN VECTOR FORM COLUMN AFTER COLUMN.		LNKT	42	
C	NOM THE NUMBER OF ELEMENTS IN OM.		LNKT	43	
C	I*4		LNKT	44	
C	NVP THE NUMBER OF ELEMENTS IN V (WHEN LNKTHN IS CALLED).		LNKT	45	
C	I*4		LNKT	46	
C	N1 NUMBER OF INMODE ROW IMMEDIATELY ABOVE SPACE IN WHICH NEW		LNKT	47	
C	I*4 ROWS ARE TO BE ADDED		LNKT	48	
C			LNKT	49	
C	KL NUMBER OF ROWS TO BE ADDED		LNKT	50	
C	I*4		LNKT	51	
C	THETK PHASE VELOCITY DIRECTION (RADIAN)		LNKT	52	
C	R*4		LNKT	53	
C			LNKT	54	
C	OUTPUTS		LNKT	55	
C			LNKT	56	
C	THE OUTPUTS ARE NVPP (= NVP + KL) AND REVISED VERSIONS OF V AND		LNKT	57	
C	INMODE.		LNKT	58	
C			LNKT	59	
C			LNKT	60	
C	-----EXAMPLE-----		LNKT	61	
C			LNKT	62	
C	VALUES OF INMODE NOT VALID -- FOR ILLUSTRATION PURPOSES ONLY		LNKT	63	
C			LNKT	64	
C	V=1.0, 2.0		LNKT	65	
C	OM=1.0, 2.0		LNKT	66	PROGRAM
C	INMODE=1, -1, -1.1		LNKT	67	LNKTHN
C	CALL LNKTHN(OM, V, INMODE, 2, 2, NVPP, 1, 3, THETK)		LNKT	68	
C			LNKT	69	PAGE
C	UPON RETURN TO CALLING PROGRAM THE VALUES OF V AND NVPP ARE		LNKT	70	30

C	V=1.0,1.25,1.5,1.75,2.0	LNGT 71
C	NVPP=5	LNGT 72
C	INMODE WILL BE OF THE FORM	LNGT 73
C	INMODE=1,Y,Y,Y,-1,-1,Y,Y,1	LNGT 74
C	WHERE THE Y'S ARE NEW ELEMENTS, EACH OF WHICH MAY BE -1, 1,	LNGT 75
C	OR 5	LNGT 76
C		LNGT 77
C	ORIGINAL MATRIX	LNGT 78
C		LNGT 79
C	EXPANDED MATRIX	LNGT 80
C		LNGT 81
C		LNGT 82
C		LNGT 83
C		LNGT 84
C		LNGT 85
C		LNGT 86
C	----PROGRAM FOLLOWS BELOW----	LNGT 87
C		LNGT 88
C		LNGT 89
C	SUBROUTINE LNGETHNM,V,INMODE,NOM,NVP,NVPP,N1,KL,THETK)	LNGT 90
C		LNGT 91
C	VARIABLE DIMENSIONING	LNGT 92
C	DIMENSION OM(1),V(1),INMODE(1)	LNGT 93
C	COMMON IMAX,C(100),VXI(100),VYI(100),HI(100)	LNGT 94
C	DELVP = (V(N1+1)-V(N1)) / (KL+1)	LNGT 95
C	DELPP IS THE INTERVAL OF PHAS VELOCITIES FOR THE ADDED ROWS.	LNGT 96
C	NVPP = NVP + KL	LNGT 97
C	NVPP IS THE NEW NUMBER OF ROWS IN THE TOTAL MATRIX.	LNGT 98
C		LNGT 99
C	N2 IS NEW NUMBER OF OLD ROW NO. (N1+1)	LNGT 100
C	N2 = N1 + KL + 1	LNGT 101
C		LNGT 102
C	SHIFT OLD VALUES OF V(I) IN LOWER ROWS TO I+KL SPOTS. ONE HAS TO	LNGT 103
C	SHIFT THE NVP ELEMENT FIRST. NOTE THAT I RANGES FROM NVPP TO N2	LNGT 104
C	DOWNWARD WHILE I-KL RANGES FROM NVP TO N1+1.	LNGT 105
C	DO 71 IP=N2,NVPP	LNGT 106
C	I = NVPP - (IP-N2)	LNGT 107
C	71 V(I) = V(I-KL)	LNGT 108
C		LNGT 109
C	NEW VALUES OF VP ARE INSERTED INTO V	LNGT 110
C	DO 72 IP=1,KL	LNGT 111
C	I = N1 + IP	LNGT 112
C	72 V(I) = V(N1) + IP*DELVP	LNGT 113
C		LNGT 114
C	BEGINNING AT THE RIGHT INMODE IS LENGTHENED COLUMN BY COLUMN	LNGT 115
C	DO 90 JP=1,NOM	LNGT 116
C	J = NOM - (JP-1)	LNGT 117
C	DO 90 IP=1,NVPP	LNGT 118
C	I = NVPP - (IP-1)	LNGT 119
C		LNGT 120
C	THE IJ ELEMENT IN THE INMODE VECTOR IS THE J ELEMENT IN THE I ROW OF	LNGT 121
C	THE NEW INMODE MATRIX	LNGT 122
C	IJ = (J-1)*NVPP + I	LNGT 123
C		LNGT 124
C	IF I CORRESPONDS TO A NEW ROW INMODE(IJ) MUST BE DETERMINED FROM NMDFN	LNGT 125
C	IF (I.GT.N1.AND.I.LT.N2) GO TO 9	LNGT 126
C		LNGT 127
C	IJOLD IS NO. OF ELEMENT IN OLD INMODE VECTOR WHICH IS TO BE MOVED INTO	LNGT 128
C	IJ POSITION OF NEW VECTOR	LNGT 129
C	IJOLD = (J-1)*NVP + I	LNGT 130
C	NOTE THAT IOLD IS ALWAYS I IF I .LT. N1 BUT IOLD IS I-KL IF I .GE. N2.	LNGT 131
C	IJOLD IS COMPUTED ON THE BASIS OF NVP RATHER THAN NVPP ROWS.	LNGT 132
C	IF (I.GE.N2) IJOLD = IJOLD - KL	LNGT 133
C	INMODE(IJ) = INMODE(IJOLD)	LNGT 134
C	GO TO 80	LNGT 135
C		LNGT 136
C	9 CALL NMDFN(OM(I),V(I),THETK,L,FPP,K)	LNGT 137
C		LNGT 138
C	IF FPP EXISTS L = 1 AND INMODE(IJ) = (FPP/ABS(FPP))	LNGT 139
C	INMODE(IJ) = 1	LNGT 140

PROGRAM
LNGETHN
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      IF (L.FO.1.AND.FPP.(F.O.O) INMODE(IJ) = -1
C
C IF FPP DOES NOT EXIST L = -1
      IF (L.FO.-1) INMODE(IJ)=5
C
      GO CONTINUE
      GO CONTINUE
      RETURN
      END

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LNCT 141
LNCT 142
LNCT 143
LNCT 144
LNCT 145
LNCT 146
LNCT 147
LNCT 148
LNCT 149

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LNCTHN

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C      MMMM (SUBROUTINE)              7/25/68
C
C      ----ARSTRACT----
C
C  TITLE - MMMM
C      THIS SUBROUTINE COMPUTES THE 2-BY-2 TRANSFER MATRIX EM WHICH
C      CONNECTS THE SOLUTIONS OF THE RESIDUAL EQUATIONS AT THE TOP
C      OF A LAYER TO THOSE AT THE BOTTOM OF THE LAYER BY THE RELATIONS
C
C      PHI1(ZB)= EM(1,1)*PHI1(ZB+H)+ EM(1,2)*PHI2(ZB+H)
C
C      PHI2(ZB)= EM(2,1)*PHI1(ZB+H)+ EM(2,2)*PHI2(ZB+H)
C
C      WHERE ZB DENOTES THE HEIGHT OF THE BOTTOM OF AN ISOTHERMAL
C      LAYER (THICKNESS H) WITH CONSTANT WINDS. THE QUANTITIES
C      PHI1(Z) AND PHI2(Z) SATISFY THE RESIDUAL EQUATIONS.
C
C       $\partial(\text{PHI1})/\partial Z = A(1,1)*\text{PHI1}(Z) + A(1,2)*\text{PHI2}(Z)$ 
C
C       $\partial(\text{PHI2})/\partial Z = A(2,1)*\text{PHI1}(Z) + A(2,2)*\text{PHI2}(Z)$ 
C
C      WHERE THE A(I,J) ARE CONSTANT OVER THE LAYER AND WHERE
C      A(2,2)=-A(1,1). ON THIS BASIS, ONE CAN SHOW THAT
C
C       $\text{EM}(I,J) = \text{CAI}(X)*\text{KDELTA}(I,J) - H*\text{SAI}(X)*A(I,J)$ 
C
C      WHERE
C
C       $X = (A(1,1)**2 + A(1,2)*A(2,1))*H**2$ 
C
C      AND WHERE KDELTA(I,J) IS THE KRONECKER DELTA (1 IF INDICES
C      EQUAL, 0 OTHERWISE). THE FUNCTIONS CAI AND SAI ARE DEFINED IN
C      THE DESCRIPTIONS OF THE CORRESPONDING FUNCTION SUBPROGRAMS.
C
C      THE MATRIX A IS COMPUTED FOR GIVEN FREQUENCY, WAVE NUMBER, SOUND
C      SPEED, AND WIND VELOCITY BY CALLING SUBROUTINE AAAA.
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C  AUTHOR - A.D. PIERCE, M.I.T., JULY, 1968.
C
C      ----CALLING SEQUENCE----
C
C  SFF SUBROUTINES NAMPDE,RRRR
C      DIMENSION EM(2,2)
C      CALL MMMM(OMEGA,AKX,AKY,C,VX,VY,H,EM)
C
C      ----EXTERNAL SUBROUTINES REQUIRED----
C
C      AAAA,CAI,SAI
C
C      ----ARGUMENT LIST----
C
C      OMEGA      R*4      NO      INP
C      AKX        R*4      NO      INP
C      AKY        R*4      NO      INP
C      C          R*4      NO      INP
C      VX         R*4      NO      INP
C      VY         R*4      NO      INP
C      A          R*4      NO      INP
C      EM         R*4      2-BY-2 OUT
C
C  NO COMMON STORAGE IS USED
C
C      ----INPUTS----
C
C      OMEGA      =ANGULAR FREQUENCY IN RAD/SEC
C      AKX        =X COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C      AKY        =Y COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM
C      C          =SOUND SPEED IN KM/SEC
C      VX         =X COMPONENT OF WIND VELOCITY IN KM/SEC

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MMMM 1
MMMM 2
MMMM 3
MMMM 4
MMMM 5
MMMM 6
MMMM 7
MMMM 8
MMMM 9
MMMM 10
MMMM 11
MMMM 12
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MMMM 14
MMMM 15
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MMMM 70

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MMMM
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C	VY	=Y COMPONENT OF WIND VELOCITY IN KM/SEC	MMMM	71
C	H	=THICKNESS IN KM OF LAYER	MMMM	72
C			MMMM	73
C		----OUTPUTS----	MMMM	74
C			MMMM	75
C	FM	=2-BY-2 TRANSFER MATRIX WHICH RELATES THE SOLUTIONS OF	MMMM	76
C		THE RESIDUAL EQUATIONS AT THE TOP OF A LAYER TO THOSE	MMMM	77
C		AT THE BOTTOM OF THE LAYER	MMMM	78
C			MMMM	79
C		----PROGRAM FOLLOWS BELOW----	MMMM	80
C			MMMM	81
C		SUBROUTINE MMMM(OMEGA,AKX,AKY,C,VX,VY,H,FM)	MMMM	82
C			MMMM	83
C		DIMENSION A(2,2),FM(2,2)	MMMM	84
C			MMMM	85
C		COMPUTE A(I,J), CAI(X), AND SAI(X)	MMMM	86
C		CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)	MMMM	87
C		X=(A(1,1)**2+A(1,2)*A(2,1))*H**2	MMMM	88
C		CA=CAI(X)	MMMM	89
C		SA=SAI(X)	MMMM	90
C			MMMM	91
C		COMPUTE THE TERMS -H*SAI(X)*A(I,J)	MMMM	92
C		TA=H*SA	MMMM	93
C		DO 90 I=1,2	MMMM	94
C		DO 90 J=1,2	MMMM	95
C		90 FM(I,J)=-TA*A(I,J)	MMMM	96
C			MMMM	97
C		ADD IN CAI(X)*KDELTA(I,J) TERMS BY ADDING CA TO DIAGONAL ELEMENTS	MMMM	98
C		DO 190 I=1,2	MMMM	99
C		190 FM(I,I)=FM(I,I)+CA	MMMM	100
C			MMMM	101
C		RETURN	MMMM	102
C		END	MMMM	103

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MMMM

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C	MODETR (SUBROUTINE)	6/25/68	MDTR	1
C			MDTR	2
C			MDTR	3
C	-----ABSTRACT-----		MDTR	4
C			MDTR	5
C	TITLE - MODETR		MDTR	6
C	PROGRAM TO TABULATE A TABLE OF PHASE VELOCITY VERSUS FREQUENCY		MDTR	7
C	FOR A GIVEN GUIDED MODE. THE NORMAL MODE DISPERSION FUNCTION		MDTR	8
C	IS ZERO FOR EACH LISTING OF THE TABLE. THE COMPUTATIONAL		MDTR	9
C	METHOD IS BASED ON THE PREVIOUSLY COMPUTED VALUES OF THE NMDF		MDTR	10
C	SIGN INMODE((J-1)*NROW+1) AT POINTS (I,J) IN A RECTANGULAR		MDTR	11
C	ARRAY OF NROW ROWS AND NCOL COLUMNS. DIFFERENT COLUMNS (J)		MDTR	12
C	CORRESPOND TO DIFFERENT FREQUENCIES WHILE DIFFERENT ROWS (I)		MDTR	13
C	CORRESPOND TO DIFFERENT PHASE VELOCITIES. DISPERSION CURVES		MDTR	14
C	OF VARIOUS MODES APPEAR ON THIS ARRAY AS LINES OF DEMARCATION		MDTR	15
C	BETWEEN ADJACENT REGIONS WITH DIFFERENT INMODES. TWO ADJACENT		MDTR	16
C	POINTS WITH INMODES OF OPPOSITE SIGN BRACKET A POINT ON THE		MDTR	17
C	ACTUAL DISPERSION CURVE. IF THE POINTS CORRESPOND TO THE SAME		MDTR	18
C	FREQUENCY, THEN THE PHASE VELOCITY CORRESPONDING TO THAT OMEGA		MDTR	19
C	ON THE DISPERSION CURVE IS FOUND BY CALLING RTM1, A 360 PACKAGE		MDTR	20
C	ROUTINE FOR SOLVING NONLINEAR EQUATIONS, AND CONSIDERING THE		MDTR	21
C	NMDF AS A FUNCTION OF VPHSE WITH OMEGA FIXED. SIMILARLY, IF		MDTR	22
C	THE POINTS CORRESPOND TO THE SAME PHASE VELOCITY, THE APPROPRIA		MDTR	23
C	OMEGA CORRESPONDING TO THIS PHASE VELOCITY IS FOUND BY CALLING		MDTR	24
C	RTM1 WITH THE NMDF CONSIDERED AS A FUNCTION OF OMEGA WITH		MDTR	25
C	VPHSE FIXED.		MDTR	26
C			MDTR	27
C	THE PROGRAM SUCCESSIVELY CONSIDERS EACH PAIR OF ADJACENT POINTS		MDTR	28
C	WITH OPPOSITE INMODES BRACKETING A LINE OF DEMARCATION AND		MDTR	29
C	PROCEEDS IN THE DIRECTION OF INCREASING FREQUENCY UNDER THE		MDTR	30
C	ASSUMPTION THAT THE PHASE VELOCITY CURVE SLOPES DOWNWARDS.		MDTR	31
C			MDTR	32
C	PROGRAM NOTES		MDTR	33
C			MDTR	34
C	THE MODES ARE NUMBERED. THE INPUT INTEGER NMODE DESIGNATES		MDTR	35
C	WHICH MODE IS BEING TABULATED. THE PAIRS OF FREQUENCY		MDTR	36
C	AND PHASE VELOCITY VALUES ARE STORED AS OMMOD(KST(INMODE))		MDTR	37
C	OMMOD(KST(INMODE)+1),OMMOD(KST(INMODE)+2),.....		MDTR	38
C	OMMOD(KFIN(INMODE)),VPMOD(KST(INMODE)),VPMOD(KST(INMODE)+1),		MDTR	39
CVPMOD(KFIN(INMODE)). THE ARRAYS OMMOD AND VPMOD		MDTR	40
C	ARE USED TO STORE DISPERSION CURVES FOR ALL MODES.		MDTR	41
C			MDTR	42
C	KST(INMODE) IS INPUT WHILE KFIN(INMODE) IS DETERMINED DURING		MDTR	43
C	THE COMPUTATION. THE TOTAL NUMBER OF POINTS EXTRACTABLE		MDTR	44
C	FROM THE ARRAY OF INMODE VALUES DETERMINES KFIN-KST+1.		MDTR	45
C	IF A SINGLE POINT CANNOT BE CALCULATED, THE PROGRAM		MDTR	46
C	RETURNS KRUD=-1. OTHERWISE IT RETURNS KRUD=1.		MDTR	47
C			MDTR	48
C	1 SUBROUTINE RTM1 FOR SOLVING A NONLINEAR EQUATION		MDTR	49
C	IS ALLOWED A MAXIMUM OF TEN ITERATIONS TO FIND THE		MDTR	50
C	PHASE VELOCITY TO ACCURACY OF 1.E-5 KM/SEC OR THE		MDTR	51
C	FREQUENCY TO FOUR SIGNIFICANT FIGURES. IF THE SEARCH IS		MDTR	52
C	UNSUCCESSFUL A MESSAGE IS PRINTED AND THE POINT IS		MDTR	53
C	SKIPPED OVER.		MDTR	54
C			MDTR	55
C	THE INPUT PARAMETERS IST,JST ARE COORDINATES OF A POINT IN		MDTR	56
C	THE INMODE ARRAY. THIS POINT SHOULD BE THAT POINT FURTHER		MDTR	57
C	TO THE UPPER LEFT OF THOSE POINTS LYING BELOW THE LINE OF		MDTR	58
C	DEMARCATION FOR THE MODE CONSIDERED, PROVIDING THAT POINT		MDTR	59
C	DOES NOT HAVE INMODE=5.		MDTR	60
C			MDTR	61
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		MDTR	62
C	AUTHOR - A.D. PIERCE, M.I.T., JUNF, 1968		MDTR	63
C			MDTR	64
C	-----CALLING SEQUENCE-----		MDTR	65
C			MDTR	66
C	SEE SUBROUTINE ALLMOD		MDTR	67
C	DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),INMODE(1),OM(1),VP(1)		MDTR	68
C	(SUBROUTINE USES VARIABLE DIMENSIONING)		MDTR	69
C	CALL MODETR(IST,JST,NMODE,KST,KFIN,OMMOD,VPMOD,NROW,NCOL,INMODE,		MDTR	70

PROGRAM
 MCODETR
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C      I GM,VP,KRUD)
C      IF( KRUD ,FO. ) ) GO SOMEWHERE
C
C      ----EXTERNAL SUBROUTINES REQUIRED----
C
C      NKTPT, PTM, FNMOD1, FNMOD2, NMDFN, AAAA, PPPP, MMMM,CAI,SAI
C      (FNMOD1 AND FNMOD2 CALL NMDFN, WHICH IN TURN CALLS AAAA AND PPPP
C      PPPP CALLS AAAA AND MMMM. DESCRIPTIONS OF THESE PROGRAMS ARE
C      GIVEN ELSEWHERE IN THIS SERIES.)
C
C      PTM IS A SUBROUTINE CODED BY IBM TO DETERMINE A ROOT OF A GENERAL
C      NONLINEAR EQUATION F(X)=0 BY MEANS OF MUELLER-S ITERATION SCHEME
C      OF SUCCESSIVE BISECTION AND INVERSE PARABOLIC INTERPOLATION. A
C      COMPLETE DESCRIPTION AND DECK LISTING IS GIVEN ON PAGES 198-199 OF
C      DOCUMENT H20-0205-2, SYSTEM/360 SCIENTIFIC SUBROUTINE PACKAGE
C      (360A-CM-0XX) VERSION 11, PROGRAMMER-S MANUAL, IBM, TECHNICAL
C      PUBLICATIONS DEPARTMENT, 112 EAST POST ROAD, WHITE PLAINS, N.Y.
C      10601, PUBLISHED 1966, 1967.
C
C      ----ARGUMENT LIST----
C
C      IST      I*4      NO      INP
C      JST      I*4      NO      INP
C      NMDFN     I*4      NO      INP
C      KST      I*4      VAR      INP (ONLY KST(NMODF) NEEDED)
C      KFIN     I*4      VAR      OUT (ONLY KFIN(NMODF) COMPUTED)
C      NMDFN(N) R*4      VAR      OUT (COMPUTED FOR N .GF. KST(NMODF))
C      VPMDFN(N) R*4      VAR      OUT (COMPUTED FOR N .GF. KST(NMODF))
C      NRDFN     I*4      NO      INP
C      NRCL      I*4      NO      INP
C      INMODF     I*4      VAR      INP
C      OM        R*4      VAR      INP
C      VP        R*4      VAR      INP
C      KRUD      I*4      NO      OUT
C
C      COMMON STORAGE USED
C      COMMON IMAX,CI,VXI,VYI,HI,OMEGAC,VPHSEC,THETK
C
C      IMAX      I*4      NO      INP
C      CI        R*4      100     INP
C      VXI       R*4      100     INP
C      VYI       R*4      100     INP
C      HI        R*4      100     INP
C      OMEGAC    R*4      NO      OUT (USED INTERNALLY)
C      VPHSEC    R*4      NO      OUT (USED INTERNALLY)
C      THETK     R*4      NO      INP
C
C      ----INPUTS----
C
C      IST      =ROW INDEX OF START POINT, WHICH MUST LIE BELOW LINE
C                OF DEMARCATION
C      JST      =COLUMN INDEX OF START POINT
C      NMDFN     =NUMBER LABELLING MODE TO BE TABULATED
C      KST(NMODF) =INDEX OF NMDFN AND VPMDFN CORRESPONDING TO FIRST
C                POINT TABULATED.
C      NRDFN     =NUMBER OF ROWS IN INMODF ARRAY
C      NRCL      =NUMBER OF COLUMNS IN INMODF ARRAY
C      INMODF     =ARRAY WHOSE (J-1)*NRDFN+I-TH ELEMENT IS THE SIGN OF
C                THE NORMAL MODE DISPERSION FUNCTION WHEN OMEGA=OM(J),
C                VPHSF=VP(I).
C      OM        =VECTOR OF FREQUENCIES AT WHICH INMODF IS TABULATED.
C      VP        =VECTOR OF PHASE VELOCITIES AT WHICH INMODF IS
C                TABULATED.
C      IMAX      =NUMBER OF ATMOSPHERIC LAYERS OF FINITE THICKNESS.
C      CI(I)     =SOUND SPEED IN I-TH LAYER IN KM/SEC.
C      VX(I)     =X COMPONENT OF WIND VELOCITY IN I-TH LAYER IN KM/SEC
C      VY(I)     =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER IN KM/SEC
C      HI(I)     =THICKNESS IN KM OF I-TH LAYER
C      THETK     =PHASE VELOCITY DIRECTION IN RADIAN'S W.R.T. X AXIS

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MDTR 71
MDTR 72
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PROGRAM
MODETR
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C      ----OUTPUTS----
C      KFIN(NMODE) = INDEX OF OMMOD AND VPMOD CORRESPONDING TO LAST
C      POINT TABULATED.
C      OMMOD(N) = ANGULAR FREQUENCY OF POINTS ON DISPERSION CURVE.
C      N=KST(NMODE) UP TO KFIN(NMODE) CORRESPONDS TO NMODE-
C      MODE.
C      VPMOD(N) = PHASE VELOCITY OF POINTS ON DISPERSION CURVE.
C      N=KST(NMODE) UP TO KFIN(NMODE) CORRESPONDS TO NMODE-
C      MODE.
C      KRUD = FLAG INDICATING IF ANY POINTS ON DISPERSION CURVE
C      HAVE BEEN FOUND. 1 IF YES, -1 IF NO.
C      OMFGAC = INTERNALLY USED FREQUENCY TRANSMITTED AMONG SUB-
C      ROUTINES THROUGH COMMON.
C      VPHSEC = INTERNALLY USED PHASE VELOCITY TRANSMITTED AMONG
C      SUBROUTINES THROUGH COMMON.
C      ----EXAMPLE----
C      SUPPOSE THE TABLE OF INMODE VALUES IS AS SHOWN BELOW WITH
C      ++++++ NR0W=7, NC0L=14
C      ++++++
C      +----- OM=.1,.2,.3,.4,.5,.6,.7,.8,.9,1.0,1.1,1.2,1.3,
C      +----- 1.4
C      +-----
C      +----- VP=.5,.45,.40,.35,.30,.25,.20
C      +-----
C      NMODE=2, IST=3, JST=1, KST(1)=7
C      THEN ONE MIGHT FIND KRUD=1, KFIN(2)=23, AND
C      OMMOD(7)=.1 VPMOD(7)=.43 OMMOD(16)=.75 VPMOD(16)=.3
C      OMMOD(8)=.2 VPMOD(8)=.42 OMMOD(17)=.8 VPMOD(17)=.29
C      OMMOD(9)=.3 VPMOD(9)=.41 OMMOD(18)=.9 VPMOD(18)=.285
C      OMMOD(10)=.33 VPMOD(10)=.4 OMMOD(19)=1.0 VPMOD(19)=.29
C      OMMOD(11)=.36 VPMOD(11)=.35 OMMOD(20)=1.1 VPMOD(20)=.27
C      OMMOD(12)=.40 VPMOD(12)=.34 OMMOD(21)=1.2 VPMOD(21)=.265
C      OMMOD(13)=.50 VPMOD(13)=.33 OMMOD(22)=1.3 VPMOD(22)=.26
C      OMMOD(14)=.50 VPMOD(14)=.32 OMMOD(23)=1.4 VPMOD(23)=.255
C      OMMOD(15)=.70 VPMOD(15)=.31
C      ----PROGRAM FOLLOWS BELOW----
C      SUBROUTINE MCODETR(IST,JST,NMODE,KST,KFIN,OMMOD,VPMOD,NR0W,NC0L,
C      1 INM,OM,VP,KRUD)
C      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C      DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),INMODE(1),OM(1),VP(1)
C      COMMON /MAX,CI,VXI,VYI,HI,OMFGAC,VPHSEC,THEK
C      FUNCTIONS FNM0D1 AND FNM0D2 ARE USED AS ARGUMENTS OF RTM1
C      EXTERNAL FNM0D1,FNM0D2
C      INDEX OF FIRST POINT ON DISPERSION CURVE IS LABELLED AS K
C      K=KST(NMODE)
C      IO=INMODE((JST-1)*NR0W+IST)
C      WE CHECK TO SEE IF POINT ABOVE (IST,JST) HAS A DIFFERENT INMODE
C      2 IF(IST.EQ.1) GO TO 5
C      IUP=INMODE((JST-1)*NR0W+IST-1)
C      IF(IUP.EQ.-10) GO TO 10
C      IF IT DOESNT, WE CHECK THE POINT ON THE RIGHT. WE CAN ALSO ARRIVE AT
C      5 FROM 2 IF IST=1.
C      5 IF(JST.EQ.NC0L) GO TO 8
C      IST=INMODE((JST)*NR0W+IST)

```

PROGRAM
 MCODETR
 PAGE
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IF(IISID.EQ.-10) GO TO 15	MDTR 211	
C	MDTR 212	
C IF WE ARRIVE AT A, WE CANNOT FIND A POINT EITHER ABOVE OF TO THE RIGHT	MDTR 213	
C OF (IST,JST) WHICH HAS A INMODE OF OPPOSITE SIGN.	MDTR 214	
KRUOD=-1	MDTR 215	
RETURN	MDTR 216	
C	MDTR 217	
C WE ASSIGN A TYPE INDEX TO THE POINT (IST,JST). SEE DESCRIPTION OF	MDTR 218	
C NEXTPT FOR DEFINITION OF TYPE INDEX.	MDTR 219	
C	MDTR 220	
10 ITYP1=1	MDTR 221	
C OPPOSITE SIGN ABOVE	MDTR 222	
GO TO 20	MDTR 223	
C	MDTR 224	
15 ITYP1=2	MDTR 225	
C OPPOSITE SIGN TO RIGHT	MDTR 226	
C	MDTR 227	
C WE NOW CAN IDENTIFY OUR FIRST BRACKETING	MDTR 228	
20 I1=IST	MDTR 229	
J1=JST	MDTR 230	
C	MDTR 231	
C STATEMENT 25 IS START OF LOOP TERMINATING AT 10. EACH PASSAGE THROUGH	MDTR 232	
C LOOP GENERATES A NEW POINT ON THE DISPERSION CURVE.	MDTR 233	
25 IF(ITYP1.EQ.2) GO TO 50	MDTR 234	
C	MDTR 235	
C CALCULATION IF ITYP1=1. STORE FREQUENCY IN COMMON. FIND PHASE VFLD-	MDTR 236	
C CITY WITHIN BRACKETED INTERVAL.	MDTR 237	
OMEGAC=OM(J1)	MDTR 238	
VOWN=VP(I1)	MDTR 239	
VUP=VP(I1-1)	MDTR 240	
EPS=1.E-4	MDTR 241	
CALL RTM1(VA,F,FNMOD1,VOWN,VUP,EPS,4,IER)	MDTR 242	
OMMOD(K)=OMEGAC	MDTR 243	
VPMOD(K)=VA	MDTR 244	
GO TO 100	MDTR 245	
C	MDTR 246	
C CALCULATION IF ITYP1=2. STORE PHASE VELOCITY IN COMMON. FIND FREQUEN	MDTR 247	
C IN BRACKETED INTERVAL.	MDTR 248	
50 VPHSEC=VP(I1)	MDTR 249	
OMLEF=OM(J1)	MDTR 250	
OMRIT=OM(J1+1)	MDTR 251	
EPS=(1.E-4)*OMRIT	MDTR 252	
CALL RTM1(OMA,F,FNMOD2,OMLEF,OMRIT,EPS,4,IER)	MDTR 253	
OMMOD(K)=OMA	MDTR 254	
VPMOD(K)=VPHSEC	MDTR 255	
C	MDTR 256	
100 CONTINUE	MDTR 257	
C WE HAVE NOW FOUND THE K-TH POINT. WE DO NOT YET KNOW IF THIS IS THE	MDTR 258	
C FINAL POINT FOR THE NMODE-TH MODE. HOWEVER, WE SET KFIN(NMODE)=K	MDTR 259	
KFI NMODE)=K	MDTR 260	
C WHEN THE SUBROUTINE RETURNS, THE CURRENT STORED KFIN(NMODE) WILL BE	MDTR 261	
C THE CORRECT ONE.	MDTR 262	
C	MDTR 263	
C WE NOW PREPARE THE SEARCH FOR THE NEXT POINT.	MDTR 264	
K=K+1	MDTR 265	
170 CALL NEXTPT(I1,J1,ITYP1,I2,J2,ITYP2,NROW,NCOL,INMODE,KUDOS)	MDTR 266	
180 IF(KUDOS.EQ.-1) GO TO 200	MDTR 267	
I1=I2	MDTR 268	
J1=J2	MDTR 269	
ITYP1=ITYP2	MDTR 270	
190 GO TO 25	MDTR 271	
C	MDTR 272	
200 CONTINUE	MDTR 273	
C WE CONTINUE HERE AFTER AN UNSUCCESSFUL ATTEMPT TO FIND THE NEXT POINT.	MDTR 274	
C PROVIDING WE HAVE FOUND AT LEAST ONE POINT, WE CAN EXIT WITH KRUOD=1.	MDTR 275	
IF(K.LE.KST(NMODE)) GO TO 8	MDTR 276	PROGRAM
KRUOD=1	MDTR 277	MODETR
RETURN	MDTR 278	
C	MDTR 279	PAGE
END	MDTR 280	38

C	MODLST (SUBROUTINE)	6/19/68	MDLT	1
C			MDLT	2
C			MDLT	3
C	----ABSTRACT----		MDLT	4
C			MDLT	5
C			MDLT	6
C	TITLE - MODLST		MDLT	7
C	TABULATION OF SELECTED POINTS ON THE PHASE VELOCITY (VPHS) VS.		MDLT	8
C	ANGULAR FREQUENCY (OMEGA) CURVES OF SELECTED MODES		MDLT	9
C			MDLT	10
C	NO COMPUTATION OR CHANGING OF UNITS IS PERFORMED BY SUB-		MDLT	11
C	ROUTINE MODLST, IT MERELY PRINTS OUT THE INPUT IN LABELED		MDLT	12
C	AND ORDERED FASHION.		MDLT	13
C			MDLT	14
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL 024-4515-4)		MDLT	15
C			MDLT	16
C	AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., JUNE, 1968		MDLT	17
C			MDLT	18
C			MDLT	19
C	----USAGE----		MDLT	20
C			MDLT	21
C	NO SUBROUTINES ARE CALLED.		MDLT	22
C			MDLT	23
C	KFIN, OMMOD, VPMOD, KST WILL ASSUME THE DIMENSIONS SPECIFIED IN		MDLT	24
C	THE CALLING PROGRAM. (DIMENSION OF KST AND KFIN MUST BE .GE. NMOD)		MDLT	25
C			MDLT	26
C	FORTRAN USAGE		MDLT	27
C			MDLT	28
C	CALL MODLST (MDFND,OMMOD,VPMOD,KST,KFIN)		MDLT	29
C			MDLT	30
C	INPUTS		MDLT	31
C			MDLT	32
C	MDFND NUMBER OF MODES TO BE PRINTED OUT.		MDLT	33
C	I*4		MDLT	34
C			MDLT	35
C	OMMOD VECTOR STORING ANGULAR FREQUENCY COORDINATE OF POINTS ON		MDLT	36
C	I*4(I) DISPERSION CURVES. MODE M IS STORED FROM ELEMENT KST(M)		MDLT	37
C	THROUGH ELEMENT KFIN(M). (RAD/SEC)		MDLT	38
C			MDLT	39
C	VPMOD VECTOR STORING PHASE VELOCITY COORDINATE OF POINTS ON		MDLT	40
C	I*4(I) DISPERSION CURVES. MODE M IS STORED FROM ELEMENT KST(M)		MDLT	41
C	THROUGH ELEMENT KFIN(M). (KM/SEC)		MDLT	42
C			MDLT	43
C	KST SEE OMMOD AND VPMOD ABOVE.		MDLT	44
C	I*4(I)		MDLT	45
C			MDLT	46
C	KFIN SEE OMMOD AND VPMOD ABOVE.		MDLT	47
C	I*4(I)		MDLT	48
C			MDLT	49
C	OUTPUTS		MDLT	50
C			MDLT	51
C	THE OUTPUT IS AN ORDERED AND LABELED PRINT OUT OF THE INPUTS, EX-		MDLT	52
C	CLUDING KST AND KFIN. (SEE EXAMPLE BELOW.)		MDLT	53
C			MDLT	54
C			MDLT	55
C	----EXAMPLE----		MDLT	56
C			MDLT	57
C	CALLING PROGRAM		MDLT	58
C			MDLT	59
C	DIMENSION KST(2),KFIN(2),OMMOD(5),VPMOD(5)		MDLT	60
C	MDFND = 2		MDLT	61
C	KST = 1,3		MDLT	62
C	KFIN = 2,5		MDLT	63
C	OMMOD = 0.1,0.2,0.1,0.15,0.2		MDLT	64
C	VPMOD = 1.0,2.0,2.0,2.5,3.0		MDLT	65
C	CALL MODLST (MDFND,OMMOD,VPMOD,KST,KFIN)		MDLT	66
C			MDLT	67
C	PRINT OUT		MDLT	68
C			MDLT	69
C	TABULATION OF FIRST 2 MODES		MDLT	70
				30

```
C
C
C
C      MODE    1
C
C      OMEGA (RAD/SEC)          VPHSE (KM/SEC)
C
C           0.100000              1.000000
C           0.200000              2.000000
C
C
C      MODE    2
C
C      OMEGA (RAD/SEC)          VPHSE (KM/SEC)
C
C           0.100000              2.000000
C           0.150000              2.500000
C           0.200000              3.000000
C
C END OF EXAMPLE
C
C      ----PROGRAM FOLLOWS BELOW----
C
C      SUBROUTINE MODLST(MDFND,OMMOD,VPMOD,KST,KFIN)
C
C VARIABLE DIMENSIONING
C      DIMENSION KFIN(1), OMMOD(1),VPMOD(1),KST(1)
C      WRITE(6,11) MDFND
C 11 FORMAT(1H1,25X,19HTABULATION OF FIRST, 16.6M MODES)
C      DO 100 I=1,MDFND
C        WRITE (6,21) I
C 21 FORMAT(1H ///,1H ,35X, 5HMODEF ,13//, 1H ,12X,15HOMEGA (RAD/SEC),
C 1 10X,14HVPHSE (KM/SEC) //)
C        K1=KST(I)
C        K2 =KFIN(I)
C        DO 100 J=K1,K2
C          OMEGA=OMMOD(J)
C          VPHSE=VPMOD(J)
C 31 FORMAT (1H ,12X,F15.6,10X,F14.6)
C          WRITE (6,31) OMEGA,VPHSE
C 100 CONTINUE
C      RETURN
C      END
```

PROGRAM
MOOLST

PAGE
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C	MPROUT (SUBROUTINE)	7/19/68	MPROUT	1
C			MPROUT	2
C			MPROUT	3
C	-----ABSTRACT-----		MPROUT	4
C			MPROUT	5
C	TITLE - MPROUT		MPROUT	6
C	TABULATION OF NORMAL MODE DISPERSION FUNCTION SIGN AT POINTS		MPROUT	7
C	IN A RECTANGULAR REGION OF FREQUENCY - PHASE VELOCITY PLANE		MPROUT	8
C			MPROUT	9
C	THE VECTOR V OF PHASE VELOCITIES IS CONSTRUCTED BY TAKING		MPROUT	10
C	VALUES AT INTERVALS OF $(V2-V1)/(NVP-1)$ FROM $V2$ DOWN TO		MPROUT	11
C	$V1$. SIMILARLY, VECTOR OM OF ANGULAR FREQUENCIES IS CON-		MPROUT	12
C	STRUCTED BY TAKING VALUES AT INTERVALS OF $(OM2-OM1)/($		MPROUT	13
C	$NOM-1)$ FROM $OM1$ UP TO $OM2$. NEXT, MATRIX INMODE IS CON-		MPROUT	14
C	STRUCTED WITH NVP ROWS AND NOM COLUMNS. SINCE INMODE IS		MPROUT	15
C	STORED IN VECTOR FORM, COLUMN AFTER COLUMN, ELEMENT J IN		MPROUT	16
C	ROW I IS REPRESENTED AS $INMODE((J-1)*NVP + I)$. THE VALUE		MPROUT	17
C	OF THIS ELEMENT IS DETERMINED BY CALLING SUBROUTINE AMDFN		MPROUT	18
C	TO EVALUATE THE NORMAL MODE DISPERSION FUNCTION, FPP, FOR		MPROUT	19
C	FREQUENCY $OM(J)$ AND PHASE VELOCITY $V(I)$. IF FPP DOES NOT		MPROUT	20
C	EXIST, THE ELEMENT IS SET EQUAL TO 5. OTHERWISE THE ELE-		MPROUT	21
C	MENT WILL BE 1 TIMES THE SIGN OF FPP.		MPROUT	22
C			MPROUT	23
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		MPROUT	24
C	AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., JUNE, 1968		MPROUT	25
C			MPROUT	26
C			MPROUT	27
C	-----USAGE-----		MPROUT	28
C	VARIABLES OM, V, INMODE MUST BE DIMENSIONED IN CALLING PROGRAM		MPROUT	29
C	FORTRAN SUBROUTINE NMODFN (DESCRIBED ELSEWHERE IN THIS SERIES) IS		MPROUT	30
C	CALLED		MPROUT	31
C			MPROUT	32
C	FORTRAN USAGE		MPROUT	33
C	CALL MPROUT(OM1,OM2,V1,V2,NOM,NVP,INMODE,OM,V,THETK)		MPROUT	34
C			MPROUT	35
C	INPUTS		MPROUT	36
C			MPROUT	37
C	OM1 MINIMUM ANGULAR FREQUENCY TO BE CONSIDERED (RADIAN / SEC)		MPROUT	38
C	R*4		MPROUT	39
C	OM2 MAXIMUM ANGULAR FREQUENCY TO BE CONSIDERED (RADIAN / SEC)		MPROUT	40
C	R*4		MPROUT	41
C	V1 MINIMUM PHASE VELOCITY TO BE CONSIDERED (KM / SEC)		MPROUT	42
C	R*4		MPROUT	43
C	V2 MAXIMUM PHASE VELOCITY TO BE CONSIDERED (KM / SEC)		MPROUT	44
C	R*4		MPROUT	45
C	NOM NUMBER OF FREQUENCIES TO BE CONSIDERED (NO. OF ELEMENTS		MPROUT	46
C	IN ROW AND NO. OF COLUMNS IN INMODE)		MPROUT	47
C	I*4		MPROUT	48
C	NVP NUMBER OF PHASE VELOCITIES TO BE CONSIDERED (NO. OF ELE-		MPROUT	49
C	I*4 MENTS IN V AND NO. OF ROWS IN INMODE)		MPROUT	50
C			MPROUT	51
C	THETK DIRECTION OF PHASE VELOCITY MEASURED COUNTER CLOCKWISE		MPROUT	52
C	R*4 FROM X-AXIS (RADIAN)		MPROUT	53
C			MPROUT	54
C	OUTPUTS		MPROUT	55
C			MPROUT	56
C	INMODE MATRIX OF NORMAL MODE DISPERSION FUNCTION SIGNS (SEE		MPROUT	57
C	I*4(1) ABSTRACT ABOVE FOR EXPLANATION OF ELEMENT VALUES)		MPROUT	58
C			MPROUT	59
C	OM VECTOR OF NOM VALUES OF ANGULAR FREQUENCY AT EQUAL INTER-		MPROUT	60
C	R*4(1) VALS FROM OM1 TO OM2 INCLUSIVE (RADIAN / SEC)		MPROUT	61
C			MPROUT	62
C	V VECTOR OF NVP VALUES OF PHASE VELOCITY AT EQUAL INTERVALS		MPROUT	63
C	R*4(1) FROM V2 TO V1 INCLUSIVE (KM / SEC)		MPROUT	64
C			MPROUT	65
C			MPROUT	66
C			MPROUT	67
C			MPROUT	68
C	-----EXAMPLE-----		MPROUT	69
C			MPROUT	70

PROGRAM
I MPROUT
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[illegible]

END

PPCU 141

PROGRAM
T MPOUJ

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C	NAMPDE (SUBROUTINE)	6/27/68	NAMP	1
C			NAMP	2
C			NAMP	3
C	----	ABSTRACT----	NAMP	4
C			NAMP	5
C	TITLE - NAMPDE		NAMP	6
C	PROGRAM TO DETERMINE AN AMPLITUDE FACTOR AMPLTD OF A GUIDED		NAMP	7
C	MODE EXCITED BY A POINT ENERGY SOURCE IN THE ATMOSPHERE. THE		NAMP	8
C	SOURCE IS AT ALTITUDE ZSCRCE KM AND THE OBSERVER IS AT ALTITUDE		NAMP	9
C	ZORS IN KM. THE PARTICULAR AMPLTD COMPUTED CORRESPONDS TO AN		NAMP	10
C	ANGULAR FREQUENCY OMEGA (RAD/SEC), A PHASE VELOCITY VPHSE		NAMP	11
C	(KM/SEC), AND A PHASE VELOCITY DIRECTION THETK (RADIAN) REC-		NAMP	12
C	KONED COUNTER-CLOCKWISE FROM THE X AXIS. PARAMETERS DEFINING		NAMP	13
C	THE AMBIENT ATMOSPHERE ARE PRESUMED TO BE STORED IN COMMON.		NAMP	14
C	THE NORMAL MODE DISPERSION FUNCTION AMDF IS PRESUMED TO VANISH		NAMP	15
C	FOR ARGUMENTS OMEGA,VPHSE,THETK.		NAMP	16
C			NAMP	17
C	THE ACTUAL DEFINITION OF AMPLTD IS AS FOLLOWS. LET S1(Z) AND		NAMP	18
C	S2(Z) BE THE SOLUTIONS OF THE RESIDUAL EQUATIONS		NAMP	19
C			NAMP	20
C	$DIS1)/GZ = (A11)*S1 + (A12)*S2$	(1-)	NAMP	21
C	$DIS2)/GZ = (A21)*S1 + (A22)*S2$	(1-)	NAMP	22
C			NAMP	23
C	WHERE THE MATRIX A IS AS COMPUTED BY AAAA AND AS DEFINED BY		NAMP	24
C	A.O.PIERCE, J. COMP. PHYS., VOL. 1, NO. 3, FEB., 1967, PP. 343-		NAMP	25
C	368, EQS. 11. THE ELEMENTS OF A SHOULD BE CONSIDERED AS FUNC-		NAMP	26
C	TIONS OF ALTITUDE. WE DEFINE THE REDUCED PRESSURE ZFN(Z) AS		NAMP	27
C			NAMP	28
C	$ZFN(Z) = (G/C)*S1 - C*S2$	(2)	NAMP	29
C			NAMP	30
C	WHERE G IS ACCELERATION OF GRAVITY AND C IS SOUND SPEED. THEN		NAMP	31
C			NAMP	32
C			NAMP	33
C	$AMPLTD = (1/2) * \frac{S2(ZSCRCE)*ZFN(ZORS)}{BOM(ZSCRCE)*INTEGRAL}$	(3)	NAMP	34
C			NAMP	35
C	WHERE		NAMP	36
C			NAMP	37
C	$BOM(Z) = OMEGA - KX*VX(Z) - KY*VY(Z)$	(4)	NAMP	38
C			NAMP	39
C	IS THE DOPPLER SHIFTED ANGULAR FREQUENCY. THE INTEGRAL IS 1/2		NAMP	40
C	OF THE I-SUR1 DEFINED BY A.O.PIERCE, J. ACOUST. SOC. AMER.,		NAMP	41
C	VOL. 37, NO. 2, FEB., 1965, PP. 718-727, EQ. (51). SPECIFICALLY		NAMP	42
C			NAMP	43
C	INTEGRAL = (INTEGRAL OVER Z FROM 0 TO INFINITY) OF		NAMP	44
C			NAMP	45
C	$(BOM*((KX*VX+KY*VY)/K)*YFN(Z)**2$		NAMP	46
C	$+ (K*OMEGA/BOM**3)*ZFN(Z)**2$	(5)	NAMP	47
C			NAMP	48
C	WHERE K IS THE MAGNITUDE OF THE WAVE-NUMBER VECTOR (KX,KY) AND		NAMP	49
C			NAMP	50
C	$YFN(Z) = (1/C)*S1(Z)$	(6)	NAMP	51
C			NAMP	52
C	PROGRAM NOTES		NAMP	53
C			NAMP	54
C			NAMP	55
C	THE INTEGRAL IS COMPUTED BY SUBROUTINE TOTINT IN TWO PART		NAMP	56
C	AS X3*X7. THE FIRST IS OBTAINED BY CALLING TOTINT WITH		NAMP	57
C	IT=3, WHILE THE SECOND IS OBTAINED BY CALLING TOTINT WITH		NAMP	58
C	IT=7. THE IT PARAMETER GOVERNS THE CHOICE OF COEFFICIENT		NAMP	59
C	A1, A2, A3 RETURNED TO TOTINT BY SUBROUTINE USEAS. FOR		NAMP	60
C	FURTHER INFORMATION, SEE THE DOCUMENTATION ON TOTINT AND		NAMP	61
C	USEAS.		NAMP	62
C			NAMP	63
C			NAMP	64
C	THE NORMALIZATION OF S1 AND S2 CANNOT AFFECT AMPLTD.		NAMP	65
C	HOWEVER, TOTINT ADOPTS NORMALIZATION WHERE		NAMP	66
C	$S1 = -\text{SORT}(GG)*A12$		NAMP	67
C	$S2 = \text{SQRT}(GG)*(GG+A11)$		NAMP	68
C	AT THE BOTTOM OF THE UPPER HALF SPACE. THE NUMERATOR OF		NAMP	69
C	EQ. (3) IS ACCORDINGLY COMPUTED WITH SAME NORMALIZATION.		NAMP	70

PROGRAM
NAMPDE
PAGE
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C      HERE GG=SQRT(A11**2+A12**2).
C
C      THE ONLY BOUNDARY CONDITION EXPLICITLY USED IS THE UPPER
C      BOUNDARY CONDITION WHEREBY BOTH S1(2) AND S2(2) DECREASE
C      EXPONENTIALLY WITH INCREASING HEIGHT IN THE UPPER
C      HALFSpace. IF THIS CANNOT BE SATISFIED, THE PROGRAM
C      RETURNS AMPLTD=0. THIS WOULD IMPLY THAT THE POINT
C      CONSIDERED IS PRACTICALLY IDENTICAL TO ONE WHERE OMEGA
C      IS THE CUTOFF FREQUENCY FOR THE GUIDED MODE UNDER
C      CONSIDERATION.
C
C      WHEN NPRNT IS POSITIVE, SEVERAL PARAMETERS DESCRIBING THE
C      PROFILES OF S1 AND S2 ARE PRINTED UNDER THE HEADING PRO-
C      VIDED BY SUBROUTINE PAMPDE. SEE PAMPDE FOR THE DEFINI-
C      TIONS OF THESE PARAMETERS.
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C  AUTHOR - A.C. PIERCE, M.I.T., JUNE, 1968
C
C      ----CALLING SEQUENCE----
C
C  SFE SUBROUTINE PAMPDE
C      DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C      COMMON IMAX,CI,VXI,VYI,HI (THESE MUST BE IN COMMON)
C      CALL PAMPDE(ZSRCCE,ZOBS,OMEGA,VPHSE,THFTK,AMPLTD,NPRNT)
C
C      ----EXTERNAL SUBROUTINES REQUIRED----
C
C      TOTINT,MMMM,AAAA,USEAS,UPINT,ELINT,BPHH,CAI,SAI
C
C      (THE FIRST THREE ARE EXPLICITLY CALLED. THE REMAINING SUBROUTINES
C      ARE IMPLICITLY CALLED WHEN TOTINT IS CALLED.)
C
C      ----ARGUMENT LIST----
C
C      ZSRCCE      R*4      NO      INP
C      ZOBS        R*4      NO      INP
C      OMEGA        R*4      NO      INP
C      VPHSE        R*4      NO      INP
C      THFTK        R*4      NO      INP
C      AMPLTD       R*4      NO      OUT
C      NPRNT        I*4      NO      INP
C
C      COMMON STORAGE USED
C      COMMON IMAX,CI,VXI,VYI,HI
C
C      IMAX         I*4      NO      INP
C      CI           R*4      100     INP
C      VXI          R*4      100     INP
C      VYI          R*4      100     INP
C      HI           R*4      100     INP
C
C      ----INPUTS----
C
C      ZSRCCE      *HEIGHT OF SOURCE IN KM
C      ZOBS        *HEIGHT OF OBSERVER
C      OMEGA        *ANGULAR FREQUENCY IN RAD/SEC
C      VPHSE        *PHASE VELOCITY IN KM/SEC
C      THFTK        *PHASE VELOCITY DIRECTION (RAD/SEC) RECKONED
C      NPRNT        *PRINT OPTION INDICATOR (SEE NAME IN MAIN PROGRAM).
C                  *COUNTER-CLOCKWISE FROM X AXIS.
C      IMAX        *NUMBER OF ATMOSPHERIC LAYERS WITH FINITE THICKNESS
C      CI(1)        *SOUND SPEED (KM/SEC) IN I-TH LAYER
C      VXI(1)       *X COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C      VYI(1)       *Y COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER
C      HI(1)        *THICKNESS IN KM OF I-TH LAYER
C
C      ----OUTPUTS----

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NAMP 71
NAMP 72
NAMP 73
NAMP 74
NAMP 75
NAMP 76
NAMP 77
NAMP 78
NAMP 79
NAMP 80
NAMP 81
NAMP 82
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NAMP 84
NAMP 85
NAMP 86
NAMP 87
NAMP 88
NAMP 89
NAMP 90
NAMP 91
NAMP 92
NAMP 93
NAMP 94
NAMP 95
NAMP 96
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NAMP 139
NAMP 140

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PROGRAM
PAMPDE
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C	AMPLTD	=AMPLITUDE FACTOR FOR GUIDED WAVE EXCITED BY POINT	NAMP	141
C		ENERGY SOURCE. UNITS ARE KM**(-1).	NAMP	142
C			NAMP	143
C	----	EXAMPLE----	NAMP	144
C			NAMP	145
C	SUPPOSE THE ATMOSPHERE IS ISOTHERMAL AND THERE ARE NO WINDS. THEN		NAMP	146
C	THERE IS ONLY ONE MODE, FOR WHICH VPHSE=C. FURTHERMORE, YFN(1)		NAMP	147
C	AND S1(2) ARE BOTH ZERO. THE ZFN(2) DECREASES EXPONENTIALLY		NAMP	148
C	WITH HEIGHT AS EXP(-(0.3*G/C**2)). THE RESULTING AMPLTD		NAMP	149
C	SHOULD BE		NAMP	150
C	AMPLTD=(-(0.3*G/C**2)*EXP(-(0.3*(G/C**2)*(ZCRCE+ZSCRC)))		NAMP	151
C	REGARDLESS OF VALUES OF OMEGA AND THETK. IF C=1/3 KM/SEC,		NAMP	152
C	G=.01 KM/SEC**2, ZCRCE=0, ZSCRC=0, THEN AMPLTD=.027 KM**(-1).		NAMP	153
C			NAMP	154
C			NAMP	155
C	----	PROGRAM FOLLOWS BELOW----	NAMP	156
C			NAMP	157
C			NAMP	158
C	SUBROUTINE NAMPDE(ZSCRC,ZOBS,OMEGA,VPHSE,THETK,AMPLTD,NPRT)		NAMP	159
C			NAMP	160
C	DIMENSION C1(100),VXI(100),VYI(100),HI(100)		NAMP	161
C	DIMENSION A(2,2),EM(2,2)		NAMP	162
C	DIMENSION ZIJ(2),S1(2),S2(2),VXIJ(2),VYIJ(2),CIJ(2)		NAMP	163
C	DIMENSION STATEMENTS ADDED IN THE DEBUG PROCESS		NAMP	164
C	DIMENSION LAYJ(2),UFLT(2),RPP(2,2),EMP(100,2,2),DUMMY(2,2)		NAMP	165
C	DIMENSION PHI(100),PHI2(100)		NAMP	166
C	COMMON IMAX,C1,VXI,VYI,HI		NAMP	167
C			NAMP	168
C	COMPUTE WAVE NUMBER VECTOR COMPONENTS		NAMP	169
C	AKX=(OMEGA/VPHSE)*COS(THETK)		NAMP	170
C	AKY=(OMEGA/VPHSE)*SIN(THETK)		NAMP	171
C			NAMP	172
C	THE SOURCE AND OBSERVER LOCATIONS ARE NUMBERED ACCORDING TO HEIGHT		NAMP	173
C	IF(ZSCRC.GT.,ZOBS) GO TO 10		NAMP	174
C	ZIJ(1)=ZSCRC		NAMP	175
C	ZIJ(2)=ZOBS		NAMP	176
C	ASCRCE=1		NAMP	177
C	NOBS=2		NAMP	178
C	GO TO 20		NAMP	179
C	10 ZIJ(1)=ZOBS		NAMP	180
C	ZIJ(2)=ZSCRC		NAMP	181
C	NCRS=1		NAMP	182
C	NSCRCE=2		NAMP	183
C			NAMP	184
C	WE DENOTE S1 AND S2 AT BOTTOM OF UPPER HALFSACE BY F1 AND F2. THEIR		NAMP	185
C	COMPUTATION IS AS FOLLOWS.		NAMP	186
C	20 IMAX=2		NAMP	187
C	J=IMAX+1		NAMP	188
C	C=C1(J)		NAMP	189
C	VX=VXI(J)		NAMP	190
C	VY=VYI(J)		NAMP	191
C	CALL AXAA(OMEGA,AKX,AKY,C,VX,VY,A)		NAMP	192
C	X=A(1,1)**2+A(1,2)*A(2,1)		NAMP	193
C	IF(X.LE.,0.0) GO TO 200		NAMP	194
C	G=SQRT(X)		NAMP	195
C	GRT=SQRT(G)		NAMP	196
C	F1=-GRT*A(1,2)		NAMP	197
C	F2=GRT*(A(1,1)+G)		NAMP	198
C			NAMP	199
C	WE COMPUTE ZM REPRESENTING THE BOTTOM OF THE UPPER HALFS ACE		NAMP	200
C	ZM=0.0		NAMP	201
C	IF(IMAX.EQ.,0) GO TO 31		NAMP	202
C	DO 30 IC=1,IMAX		NAMP	203
C	30 ZM=ZM+HI(IC)		NAMP	204
C			NAMP	205
C	WE STORE F1P,F2P,ZMP		NAMP	206
C	31 F1P=F1		NAMP	207
C	F2P=F2		NAMP	208
C	ZMP=ZM		NAMP	209
C			NAMP	210

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C COMPUTATION OF LAYJZ(JZ) AND DELT(JZ)	NAMP 211
C LAYJZ(JZ) IS THE INDEX OF THE LAYER IN WHICH ZIJZ(JZ) LIES.	NAMP 212
C WHILE DELT(JZ) IS THE DISTANCE OF ZIJZ(JZ) ABOVE THE BOTTOM EDGE OF	NAMP 213
C THE LAYER	NAMP 214
DO 35 JZ=1,2	NAMP 215
LAYJZ(JZ)=IMAX+1	NAMP 216
32 DELT(JZ)=ZIJZ(JZ)-ZM	NAMP 217
IF(DELT(JZ) .GT. 0.0) GO TO 35	NAMP 218
IF(LAYJZ(JZ) .EQ. 1) GO TO 35	NAMP 219
LAYJZ(JZ)=LAYJZ(JZ)-1	NAMP 220
ZM=ZM-HI(LAYJZ(JZ))	NAMP 221
C AT THIS POINT ZM DENOTES THE BOTTOM OF THE LAYJZ(JZ) LAYER	NAMP 222
GO TO 32	NAMP 223
35 ZM=ZMP	NAMP 224
C	NAMP 225
C COMPUTATION OF EM MATRICES FOR ALL IMAX LAYERS OF FINITE THICKNESS	NAMP 226
C EM(IP,JP) FOR I-TH LAYER IS STORED AS EMP(I,IP,JP)	NAMP 227
DO 36 I=1,IMAX	NAMP 228
C=C1(I)	NAMP 229
VX=VX1(I)	NAMP 230
VY=VY1(I)	NAMP 231
H=HI(I)	NAMP 232
CALL MPM4(OMEGA,AKX,AKY,C,VX,VY,H,EM)	NAMP 233
DO 36 IP=1,2	NAMP 234
DO 36 JP=1,2	NAMP 235
36 EMP(I,IP,JP)=EM(IP,JP)	NAMP 236
C	NAMP 237
C COMPUTATION OF RPP MATRIX. THIS ACCOMPLISHES THE SAME AS CALLING	NAMP 238
C SUBROUTINE RRRR	NAMP 239
RPP(1,1)=1.0	NAMP 240
RPP(1,2)=0.0	NAMP 241
RPP(2,1)=0.0	NAMP 242
RPP(2,2)=1.0	NAMP 243
DO 36 I=1,IMAX	NAMP 244
JASA=IMAX+1-I	NAMP 245
DO 37 IP=1,2	NAMP 246
DO 37 JP=1,2	NAMP 247
37 DUMMY(IP,JP)=EMP(JASA,IP,1)*RPP(1,JP)+EMP(JASA,IP,2)*RPP(2,JP)	NAMP 248
DO 38 IP=1,2	NAMP 249
DO 38 JP=1,2	NAMP 250
38 RPP(IP,JP)=DUMMY(IP,JP)	NAMP 251
C	NAMP 252
QUOT = ABS(RPP(1,1))/(ABS(RPP(1,1))+ABS(RPP(1,2))+ABS(RPP(2,1))	NAMP 253
1 +ABS(RPP(2,2)))	NAMP 254
IF (QUOT .LT. 0.1) GO TO 120	NAMP 255
F2ROT=F2P/RPP(1,1)	NAMP 256
GO TO 150	NAMP 257
120 QUOT = ABS(RPP(1,2))/(ABS(RPP(1,1))+ABS(RPP(1,2))+ABS(RPP(2,1))	NAMP 258
1 +ABS(RPP(2,2)))	NAMP 259
IF (QUOT .LT. 0.1) GO TO 130	NAMP 260
F2BOT=-F1P/RPP(1,2)	NAMP 261
GO TO 150	NAMP 262
130 F2ROT=RPP(2,1)*F1P+RPP(2,2)*F2P	NAMP 263
150 F2ROT=F2BOT	NAMP 264
PHI1(1)=0.0	NAMP 265
PHI2(1)=F2BOT	NAMP 266
KTOUP=1	NAMP 267
K=IMAX+1	NAMP 268
PHI1(K)=F1P	NAMP 269
PHI2(K)=F2P	NAMP 270
331 T1=PHI1(K)	NAMP 271
T2=PHI2(K)	NAMP 272
K=K-1	NAMP 273
IF(K .EQ. 1) GO TO 400	NAMP 274
C=C1(K)	NAMP 275
VX=VX1(K)	NAMP 276
VY=VY1(K)	NAMP 277
CALL A7A(OMEGA,AKX,AKY,C,VX,VY,A)	NAMP 278
X=A(1,1)*2+A(1,2)*A(2,1)	NAMP 279
IF(X .GT. 0.0) GO TO 340	NAMP 280

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333	PHI1(K)=EMP(K,1,1)*T1+EMP(K,1,2)*T2	NAMP	281
	PHI2(K)=EMP(K,2,1)*T1+EMP(K,2,2)*T2	NAMP	282
	GO TO 331	NAMP	283
340	D1=A(1,1)*T1+A(1,2)*T2	NAMP	284
	D2=A(2,1)*T1+A(2,2)*T2	NAMP	285
	IF (D1 .LT. 0.0 .AND. T1 .LT. 0.0) GO TO 341	NAMP	286
	IF (D1 .GT. 0.0 .AND. T1 .GT. 0.0) GO TO 341	NAMP	287
	IF (D2 .LT. 0.0 .AND. T2 .LT. 0.0) GO TO 341	NAMP	288
	IF (D2 .GT. 0.0 .AND. T2 .GT. 0.0) GO TO 341	NAMP	289
	GO TO 333	NAMP	290
341	CONTINUE	NAMP	291
C	AT THIS POINT THE CURRENT VALUE OF K IS NOT ZERO OR ONE	NAMP	292
	KTCUP=K	NAMP	293
	DO 360 K=2,KTCUP	NAMP	294
	JET=K-1	NAMP	295
	T1=PHI1(JET)	NAMP	296
	T2=PHI2(JET)	NAMP	297
	PHI1(K)=EMP(JET,2,2)*T1-EMP(JET,1,2)*T2	NAMP	298
360	PHI2(K)=-EMP(JET,2,1)*T1+EMP(JET,1,1)*T2	NAMP	299
400	IF (NPRAT.LT.2) GO TO 415	NAMP	300
	NZC1 = 0	NAMP	301
	NZC2 = 0	NAMP	302
	IAP1MX = 1	NAMP	303
	IAP2MX = 1	NAMP	304
	AP1MX = ABS(PHI1(1))	NAMP	305
	AP2MX = ABS(PHI2(1))	NAMP	306
	DO 407 LNM1=1,IMAX	NAMP	307
	LN = LNM1 + 1	NAMP	308
	AP1P = ABS(PHI1(LN))	NAMP	309
	IF (AP1P.LE.AP1MX) GO TO 403	NAMP	310
	IAP1MX = LN	NAMP	311
	AP1P = AP1P	NAMP	312
403	AP2P = ABS(PHI2(LN))	NAMP	313
	IF (AP2P.LE.AP2MX) GO TO 405	NAMP	314
	AP2MX = AP2P	NAMP	315
	IAP2MX = LN	NAMP	316
405	IF ((PHI1(LNM1)*PHI1(LN)).LT.0.0) NZC1 = NZC1 + 1	NAMP	317
	IF ((PHI2(LNM1)*PHI2(LN)).LT.0.0) NZC2 = NZC2 + 1	NAMP	318
407	CONTINUE	NAMP	319
	R1 = PHI1(IAP1MX)/AP1MX	NAMP	320
	R2 = PHI2(IAP2MX)/AP2MX	NAMP	321
	R3 = PHI2(1)/AP2MX	NAMP	322
	WRITE (6,409) OMEGA,VPHSE,IAP1MX,R1,NZC1,IAP2MX,R2,NZC2,R3	NAMP	323
409	FORMAT (1H,2F12.5,9X,13,F12.5,9X,13,9X,13,F12.5,9X,13,F12.5)	NAMP	324
415	DO 450 JZ=1,2	NAMP	325
	IDA=LAYJZ(JZ)	NAMP	326
	C=C1(IDA)	NAMP	327
	VX=VX1(IDA)	NAMP	328
	VY=VY1(IDA)	NAMP	329
	C1JZ(JZ)=C1(IDA)	NAMP	330
	VX1JZ(JZ)=VX1(IDA)	NAMP	331
	VY1JZ(JZ)=VY1(IDA)	NAMP	332
	IF (IDA .EQ. IMAX+1) GO TO 420	NAMP	333
	IF (IDA .LE. KTCUP) GO TO 430	NAMP	334
	JET=IDA+1	NAMP	335
	H=HI(IDA)-DEL T(JZ)	NAMP	336
	CALL MMMG(OMEGA,AKX,AKY,C,VX,VY,H,EM)	NAMP	337
	S1(JZ)=EM(1,1)*PHI1(JET)+EM(1,2)*PHI2(JET)	NAMP	338
	S2(JZ)=EM(2,1)*PHI1(JET)+EM(2,2)*PHI2(JET)	NAMP	339
	GO TO 450	NAMP	340
420	EN=EXP(-G*DEL T(JZ))	NAMP	341
	S1(JZ)=F1P*EN	NAMP	342
	S2(JZ)=F2P*EN	NAMP	343
	GO TO 450	NAMP	344
430	H=DEL T(JZ)	NAMP	345
	CALL MMMG(OMEGA,AKX,AKY,C,VX,VY,H,EM)	NAMP	346
	S1(JZ)=EM(2,2)*PHI1(IDA)-EM(1,2)*PHI2(IDA)	NAMP	347
	S2(JZ)=-EM(2,1)*PHI1(IDA)+EM(1,1)*PHI2(IDA)	NAMP	348
450	CONTINUE	NAMP	349
C		NAMP	350

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C AT THIS POINT S1(JZ),S2(JZ),CIJZ(JZ), ETC. ARE STORED FOR JZ=1 AND 2.	NAMP 351
C WE COMPUTE THE DOPPLER SHIFTED ANGULAR FREQUENCY AT SOURCE ALTITUDE.	NAMP 352
100 BOM=OMEGA-AXX*VXIJJZ(NSCRCE)-AK'*VYIJZ(NSCRCE)	NAMP 353
C	NAMP 354
C WE COMPUTE ZFA AT OBSERVER ALTITUDE	NAMP 355
ZFN=(.0098/CIJZ(NOBS))*S1(NOBS)-CIJZ(NOBS)*S2(NOBS)	NAMP 356
C	NAMP 357
C HERE WE TAKE THE ACCELERATION OF GRAVITY TO BE .0098 KM/SEC**2.	NAMP 358
C	NAMP 359
C COMPUTATION OF INTEGRALS	NAMP 360
IT=3	NAMP 361
CALL TOTINT(OMEGA,AKX,AKY,IT,L,X3,PHI1,PHI2)	NAMP 362
IF(L.EQ.-1) GO TO 200	NAMP 363
IT=7	NAMP 364
CALL TOTINT(OMEGA,AKX,AKY,IT,L,X7,PHI1,PHI2)	NAMP 365
IF(L.EQ.-1) GO TO 200	NAMP 366
C	NAMP 367
C FINAL ANSWER	NAMP 368
AMPLTD= 0.5*S2(NSCRCE)*ZFN/((X3+X7)*ROM)	NAMP 369
RETURN	NAMP 370
C	NAMP 371
C IF YOU ARRIVE HERE, THE UPPER BOUNDARY CONDITION COULD NOT BE SATISFIE	NAMP 372
200 AMPLTD=0.0	NAMP 373
RETURN	NAMP 374
C	NAMP 375
END	NAMP 376

PROGRAM
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C      NMDFN (SUBROUTINE)              7/25/68
C
C      ----ABSTRACT----
C
C  TITLE - NMDFN
C      SUBROUTINE TO COMPUTE THE NORMAL MODE DISPERSION FUNCTION FPP
C      FOR GIVEN ANGULAR FREQUENCY OMEGA, PHASE VELOCITY MAGNITUDE
C      VPHSE AND PHASE VELOCITY DIRECTION THETK. FPP SHOULD VANISH
C      IF BOTH UPPER AND LOWER BOUNDARY CONDITIONS ARE SATISFIED FOR
C      THE SOLUTIONS OF THE RESIDUAL EQUATIONS
C
C       $D(PH11)/DZ = A(1,1)*PH11(Z) + A(1,2)*PH12(Z)$ 
C
C       $D(PH12)/DZ = A(2,1)*PH11(Z) + A(2,2)*PH12(Z)$ 
C
C      WHERE THE ELEMENTS OF THE MATRIX A VARY WITH HEIGHT Z, BUT ARE
C      CONSTANT IN EACH LAYER OF A MULTILAYER ATMOSPHERE. THE ELEMENT
C      OF A ARE FUNCTIONS OF OMEGA, AKX AND AKY AS DESCRIBED IN
C      SUBROUTINE AAAA WHERE
C
C       $AKX = OMEGA * COS(THETK) / VPHSE$ 
C
C       $AKY = OMEGA * SIN(THETK) / VPHSE$ 
C
C      THE FUNCTION FPP IS DEFINED AS THE VALUE OF PH11 AT THE GROUND
C      (Z=0) WHEN (1) THE UPPER BOUNDARY CONDITION OF PH11 AND PH12
C      DECREASING EXPONENTIALLY WITH HEIGHT IN THE UPPER HALFSpace
C      IS SATISFIED, AND (2) PH11 AND PH12 AT THE BOTTOM OF THE UPPER
C      HALFSpace ARE GIVEN BY  $A(1,2)$  AND  $-(G+A(1,1))$  WHERE
C       $G = \sqrt{A(1,1)^2 + A(1,2)*A(2,1)}$ . THE ELEMENTS OF A HERE ARE
C      THOSE APPROPRIATE TO THE UPPER HALFSpace. CONDITIONS (1) AND
C      (2) ARE NOT INDEPENDENT. CONDITION (1) IMPLIES THAT  $G \geq 0$ .
C      AND CONDITION (2) WITH  $G \geq 0$  POSITIVE IMPLIES (1). IF  $G \geq 0$  IS
C      NEGATIVE, FPP DOES NOT EXIST AND L=-1 IS RETURNED. OTHERWISE
C      L=1 IS RETURNED.
C
C  PROGRAM NOTES
C
C      THE PARAMETERS DEFINING THE MULTILAYER MODEL ATMOSPHERE
C      ARE PRESUMED TO BE STORED IN COMMON.
C
C      THE SUBROUTINE RRRR IS USED TO GENERATE THE MATRIX RPP
C      WHICH CONNECTS SOLUTIONS OF THE RESIDUAL EQUATIONS AT
C      THE BOTTOM OF THE UPPER HALFSpace TO SOLUTIONS AT THE
C      GROUND. IN TERMS OF THIS MATRIX, THE NMDF IS GIVEN BY
C
C       $FPP = RPP(1,1)*A(1,2) - RPP(1,2)*(G+A(1,1))$ 
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C  AUTHOR - A.C. PIERCE, M.I.T., AUGUST, 1968
C
C      ----CALLING SEQUENCE----
C
C  SEE SUBROUTINES LNGETH, WIDEN, MPOUT
C      DIMENSION C(100), VX(100), VY(100), HI(100)
C      COMMON IMAX, C, VX, VY, HI (THESE MUST BE STORED IN COMMON)
C      CALL NMDFN(OMEGA, VPHSE, THETK, L, FPP, K)
C
C      ----EXTERNAL SUBROUTINES REQUIRED----
C
C      RRRR, MMMM, AAAA, CAI, SAI
C
C      ----ARGUMENT LIST----
C
C      OMEGA      R=4      ND      INP
C      VPHSE      R=4      ND      INP
C      THETK      R=4      ND      INP
C      L          I=4      ND      OUT
C      FPP        R=4      ND      OUT

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NMDF 1
NMDF 2
NMDF 3
NMDF 4
NMDF 5
NMDF 6
NMDF 7
NMDF 8
NMDF 9
NMDF 10
NMDF 11
NMDF 12
NMDF 13
NMDF 14
NMDF 15
NMDF 16
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C	K	I*4	ND	OUT (ALWAYS RETURNED AS K=0)	NMDF	71	
C					NMDF	72	
C	COMMON STORAGE USED				NMDF	73	
C	COMMON IMAX,CI,VXI,VYI,HI				NMDF	74	
C					NMDF	75	
C	IMAX	I*4	ND	INP	NMDF	76	
C	CI	R*4	100	INP	NMDF	77	
C	VXI	R*4	100	INP	NMDF	78	
C	VYI	R*4	100	INP	NMDF	79	
C	HI	R*4	100	INP	NMDF	80	
C					NMDF	81	
C				----INPUTS----	NMDF	82	
C					NMDF	83	
C	OMEGA			=ANGULAR FREQUENCY IN RAD/SEC	NMDF	84	
C	VPHSE			=PHASE VELOCITY MAGNITUDE IN KM/SEC	NMDF	85	
C	THETK			=PHASE VELOCITY DIRECTION RECKONED COUNTER CLOCKWISE	NMDF	86	
C				FROM THE X AXIS IN RADIANS	NMDF	87	
C	IMAX			=NUMBER OF LAYERS OF FINITE THICKNESS	NMDF	88	
C	CI(I)			=SOUND SPEED IN KM/SEC IN I-TH LAYER	NMDF	89	
C	VXI(I)			=X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)	NMDF	90	
C	VYI(I)			=Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)	NMDF	91	
C	HI(I)			=THICKNESS IN KM OF I-TH LAYER OF I-TH THICKNESS	NMDF	92	
C					NMDF	93	
C				----OUTPUTS----	NMDF	94	
C					NMDF	95	
C	L			=1 IF NORMAL MODE DISPERSION FUNCTION EXISTS, -1 IF	NMDF	96	
C				IT DOES NOT.	NMDF	97	
C	FPP			=NORMAL MODE DISPERSION FUNCTION	NMDF	98	
C	K			=DUMMY PARAMETER ALWAYS RETURNED AS K=0	NMDF	99	
C					NMDF	100	
C				----PROGRAM FOLLOWS BELOW----	NMDF	101	
C					NMDF	102	
C	SUBROUTINE NMDFN(OMEGA,VPHSE,THETK,L,FPP,K)				NMDF	103	
C					NMDF	104	
C	DIMENSION AND COMMON STATEMENTS LOCATING PARAMETERS DEFINING MODEL				NMDF	105	
C	MULTILAYER ATMOSPHERE				NMDF	106	
C	DIMENSION CI(100),VXI(100),VYI(100),HI(100)				NMDF	107	
C	COMMON IMAX,CI,VXI,VYI,HI				NMDF	108	
C					NMDF	109	
C	DIMENSION A(2,2),RPP(2,2)				NMDF	110	
C					NMDF	111	
C	COMPUTATION OF AKX AND AKY				NMDF	112	
C	AKX=OMEGA*COS(THETK)/VPHSE				NMDF	113	
C	AKY=OMEGA*SIN(THETK)/VPHSE				NMDF	114	
C					NMDF	115	
C	COMPUTATION OF MATRIX A AND G**2 FOR UPPER HALFSpace				NMDF	116	
C	J=IMAX+1				NMDF	117	
C	C=C11J				NMDF	118	
C	VX=VXI(J)				NMDF	119	
C	VY=VYI(J)				NMDF	120	
C	CALL AAAA(OMEGA,AKX,AKY,C,VX,VY,A)				NMDF	121	
C	GUSQ=A(1,1)**2+A(1,2)*A(2,1)				NMDF	122	
C					NMDF	123	
C	IF(GUSQ.GT. 0.0) GO TO 11				NMDF	124	
C					NMDF	125	
C	GUSQ IS LESS THAN ZERO				NMDF	126	
C	L=-1				NMDF	127	
C	RETURN				NMDF	128	
C					NMDF	129	
C	GUSQ IS GREATER THAN ZERO				NMDF	130	
C	11 L=1				NMDF	131	
C	GU=SQRT(GUSQ)				NMDF	132	
C					NMDF	133	
C	COMPUTATION OF RPP MATRIX				NMDF	134	
C	CALL RRRR(OMEGA,AKX,AKY,RPP,K)				NMDF	135	
C					NMDF	136	PROGRAM
C	COMPUTATION OF FPP				NMDF	137	NMDFN
C	FPP=RPP(1,1)*A(1,2)-RPP(1,2)*(GU*A(1,1))				NMDF	138	
C					NMDF	139	PAGE
C	RETURN				NMDF	140	51

END

NMDF 141

PROGRAM
NMDFN

" PAGE
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C      NXMODE (SUBROUTINE)                6/24/68
C
C      ----ABSTRACT----
C
C  TITLE ~ NXMODE
C      PROGRAM TO FIND A POINT WITH COORDINATES I=IFND,J=JFND IN AN
C      ARRAY WITH NOM COLUMNS AND NVP ROWS. FOUND POINT CORRESPONDS
C      TO STARTING POSITION FOR CALCULATION OF PHASE VELOCITY VERSUS
C      FREQUENCY OF A PARTICULAR GUIDED MODE. A TABLE OF VALUES OF
C      THE SIGN OF THE NORMAL MODE DISPERSION FUNCTION IS PRESUMED
C      TO BE STORED AS INMODE(IJ-1)*NVP+1 FOR EACH POINT (I,J) IN THE
C      ARRAY. DIFFERENT COLUMNS (J) CORRESPOND TO DIFFERENT FREQUEN-
C      CIES WHILE DIFFERENT ROWS (I) CORRESPOND TO DIFFERENT PHASE
C      VELOCITIES. THE SEARCH PROCEEDS FROM AN INITIAL POINT (IST,JST
C      TO SUCCESSIVE ADJACENT POINTS HAVING THE SAME INMODE AS THE
C      STARTING POINT. THE DETERMINATION OF (IFND,JFND) IS SUBJECT TO
C      THE FOLLOWING RULES.
C
C      1. IT MUST LIE BELOW OR TO THE LEFT OF A POINT WITH
C      OPPOSITE INMODE
C
C      2. IF MUST BE THE HIGHEST POINT (LOWEST I) IN THE REGION
C      SATISFYING CONDITION 1
C
C      3. IF MORE THAN 1 POINT SATISFY 1 AND 2, THEN THE POINT
C      DETERMINED IS THAT FURTHEST TO THE LEFT.
C
C      4. ONLY POINTS IN THE RECTANGLE ARE CONSIDERED
C
C      THE COMPUTATION ASSUMES REGION OF SUCCESSIVELY ADJACENT POINTS
C      HAVING SAME INMODE IS SIMPLY CONNECTED AND THAT PHASE VELOCITY
C      CURVES BEND DOWNWARDS, I.E.,  $dVP/d\Omega < 0$ . (THIS CAN BE
C      THE CASE PROVIDING VP IS GREATER THAN THE MAXIMUM WIND
C      VELOCITY.) IF THE POINT IS FOUND, K=1, IF NOT FOUND, K=-1.
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C  AUTHOR   - A.D.PIERCE, M.I.T., JUNE, 1968
C
C      ----CALLING SEQUENCE----
C
C  SEE SUBROUTINE ALLMOD
C      DIMENSION INMODE(I) (VARIABLE DIMENSIONING)
C      CALL NXMODE(IST,JST,NOM,NVP,INMODE,IFND,JFND,K)
C
C  NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C      ----ARGUMENT LIST----
C
C      IST      I*4      ND      INP
C      JST      I*4      ND      INP
C      NOM      I*4      ND      INP
C      NVP      I*4      ND      INP
C      INMODE   I*4      VAR     INP
C      IFND     I*4      ND      OUT
C      JFND     I*4      ND      OUT
C      K        I*4      ND      OUT
C
C  NO COMMON STORAGE USED
C
C      ----INPUTS----
C
C      IST      =ROW INDEX OF START POINT
C      JST      =COLUMN INDEX OF START POINT
C      NOM      =NO. OF COLUMNS OF ARRAY
C      NVP      =NO. OF ROWS OF ARRAY
C      INMODE(I) =SIGN OF NORMAL MODE DISPERSION FUNCTION, 1 IF POS.,
C      -1 IF NEG., 5 IF IT DOESN'T EXIST. LET I=L MOD NVP,
C      J=(I-1)/NVP+1. INMODE(I) IS SIGN OF NMDF FOR
C       $\Omega = \Omega(I)$ , PHASE VEL. =  $VP(I)$ , WHERE  $\Omega(I) \geq \Omega(I)$ 

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NXMD 1
NXMD 2
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NXMD 4
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NXMD 7
NXMD 8
NXMD 9
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NXMD 11
NXMD 12
NXMD 13
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NXMD 70

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PROGRAM
NXMODE
PAGE
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C	AND VP(I) .LE. VP(I-1).	NXMD 71
C		NXMD 72
C	----OUTPUTS----	NXMD 73
C		NXMD 74
C	IFND =ROW INDEX OF FOUND POINT	NXMD 75
C	JFND =COLUMN INDEX OF FOUND POINT	NXMD 76
C	K = FLAG INDICATING IF POINT (IFND,JFND) IS FOUND, 1 IF	NXMD 77
C	YES, -1 IF NO.	NXMD 78
C		NXMD 79
C	----EXAMPLE----	NXMD 80
C		NXMD 81
C	SUPPOSE THE ARRAY OF INMODE VALUES IS AS SHOWN BELOW	NXMD 82
C		NXMD 83
C	++++++ NVP=8, NOM=11	NXMD 84
C	++++++	NXMD 85
C	5----- IF IST=8,JST=5 THEN IFND=3,JFND=2,K=1	NXMD 86
C	55----- IF IST=2,JST=5 THEN IFND=1,JFND=9,K=1	NXMD 87
C	55----- IF IST=3,JST=7 THEN IFND=3,JFND=2,K=-1	NXMD 88
C	55----- IF IST=8,JST=2 THEN K=-1	NXMD 89
C	55----- IF IST=2,JST=11 THEN K=-1	NXMD 90
C	55-----	NXMD 91
C		NXMD 92
C	----PROGRAM FOLLOWS BELOW----	NXMD 93
C		NXMD 94
C	SUBROUTINE NXMODE(IST,JST,NOM,NVP,INMODE,IFND,JFND,K)	NXMD 95
C		NXMD 96
C	DIMENSION INMODE(1)	NXMD 97
C	1 IF(IST .GT. NVP .OR. JST .GT. NOM) GO TO 100	NXMD 98
C	10 = INMODE((JST-1)*NVP+IST)	NXMD 99
C	3 IF(10 .NE. 1 .AND. 10 .NE. -1) GO TO 100	NXMD 100
C		NXMD 101
C	C THE POINT (IST,JST) LIES IN THE ARRAY AND THE NORMAL MODE DISPERSION	NXMD 102
C	C FUNCTION EXISTS AT THIS POINT WITH A SIGN IC. WE FIRST GO UP UNTIL	NXMD 103
C	C A DIFFERENT INMODE IS ENCOUNTERED OR UNTIL WE REACH I=1	NXMD 104
C	I=IST	NXMD 105
C	J=JST	NXMD 106
C	10 IF(I .EQ. 1) GO TO 30	NXMD 107
C	I=I-1	NXMD 108
C	ICLK=INMODE((J-1)*NVP+I)	NXMD 109
C	IF(ICLK .EQ. 10) GO TO 10	NXMD 110
C	I=I+1	NXMD 111
C		NXMD 112
C	C THE CURRENT I IS NOT 1. IF THE ICLK OF THE POINT ABOVE IS NOT 5, WE	NXMD 113
C	C MOVE TO THE LEFT.	NXMD 114
C	15 IF(ICLK .EQ. 5) GO TO 50	NXMD 115
C	IF(J .EQ. 1) GO TO 20	NXMD 116
C	J=J-1	NXMD 117
C	ICLK=INMODE((J-1)*NVP+I)	NXMD 118
C		NXMD 119
C	C IF THE ICLK OF THE CONSIDERED NEW POINT IS 10, WE TRY TO GO HIGHER	NXMD 120
C	C AGAIN.	NXMD 121
C	IF(ICLK .EQ. 10) GO TO 10	NXMD 122
C	J=J+1	NXMD 123
C		NXMD 124
C	C WE HAVE -10 ABOVE THE CURRENT POINT AND ARE EITHER ON THE FAR LEFT OF	NXMD 125
C	C THE TABLE OR ELSE HAVE A DIFFERENT SIGN AT THE POINT TO THE LEFT.	NXMD 126
C	C THIS IS INTERPRETED AS SUCCESS.	NXMD 127
C	20 K=1	NXMD 128
C	IFND=I	NXMD 129
C	JFND=J	NXMD 130
C	RETURN	NXMD 131
C		NXMD 132
C	C THE CONSIDERED NEW POINT IS ON THE FIRST ROW. WE GO TO THE RIGHT.	NXMD 133
C	30 IF(J .EQ. NOM) GO TO 60	NXMD 134
C	J=J+1	NXMD 135
C	ICLK=INMODE((J-1)*NVP+I)	NXMD 136
C	IF(ICLK .EQ. 10) GO TO 30	NXMD 137
C	J=J-1	NXMD 138
C		NXMD 139
C		NXMD 140

PROGRAM
NXMODE
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C		AXMD 141
C	IF THE POINT AT THE RIGHT OF CURRENT (I,J) IS -10, WE HAVE SUCCESS	AXMD 142
	IF(ICHK .EQ. -10) GO TO 20	AXMD 143
C		AXMD 144
C	IF IT IS NOT -10, WE ALLOW FOR POSSIBILITY OF INMODE=5 IN UPPER RIGHT	AXMD 145
C	HAND CORNER OF THE TABLE AND TRY TO SKIRT THESE FIVES BY MOVING EITHER	AXMD 146
C	DOWNWARDS OR TO THE RIGHT.	AXMD 147
	40 IF(I .EQ. NVP) GO TO 70	AXMD 148
	I=I+1	AXMD 149
	ICHK=INMODE((J-1)*NVP+I)	AXMD 150
C		AXMD 151
C	IF THIS ICHK IS +10 WE ARE IN A POSITION TO MAKE A TRY OF MOVING TO	AXMD 152
C	THE RIGHT.	AXMD 153
	44 IF(ICHK .NE. 10) GO TO 80	AXMD 154
C		AXMD 155
C	IF WE ARE ON THE RIGHT HAND SIDE OF THE TABLE THE DESIRED POINT CANNOT	AXMD 156
C	BE FOUND. WE RETURN WITH K=-1	AXMD 157
	45 IF(J .EQ. NOM) GO TO 100	AXMD 158
	J=J+1	AXMD 159
C		AXMD 160
C	IT IS TAKEN FOR GRANTED THAT THE INMODE OF POINT ABOVE CURRENT (I,J)	AXMD 161
C	IS 5 SINCE IT WAS FOUND TO BE 5 TO THE LEFT AND ABOVE. THE INMODE OF	AXMD 162
C	THE POINT TO THE LEFT IS 10. IF THE NEW INMODE IS +10, WE HAVE TO TRY	AXMD 163
C	TO MOVE FURTHER TO THE RIGHT.	AXMD 164
	ICHK=INMODE((J-1)*NVP+I)	AXMD 165
	IF(ICHK .EQ. 10) GO TO 45	AXMD 166
	J=J+1	AXMD 167
C		AXMD 168
C	IF THE CURRENT ICHK IS 5, WE TRY TO GO DOWN AGAIN. THE OTHER POSS-	AXMD 169
C	IBILITY, ICHK=-10 INDICATES SUCCESS	AXMD 170
	IF(ICHK .EQ. -10) GO TO 20	AXMD 171
	GO TO 40	AXMD 172
C		AXMD 173
C	WE CONTINUE HERE FROM 15. THE POINT ABOVE THE CURRENT (I,J) HAS	AXMD 174
C	ICHK .EQ. 5. THE SITUATION IS SUCH THAT WE CAN RESUME CALCULATION	AXMD 175
C	AT 45 AND TRY TO MOVE FURTHER TO THE RIGHT.	AXMD 176
	50 GO TO 45	AXMD 177
C		AXMD 178
C	WE CONTINUE HERE WITH I=1,J=NOM FROM STATEMENT 30. SINCE WE HAVE NO	AXMD 179
C	PLACE TO GO THE SEARCH IS UNSUCCESSFUL. WE RETURN WITH K=-1.	AXMD 180
	60 GO TO 100	AXMD 181
C		AXMD 182
C	WE CONTINUE HERE FROM STATEMENT 40 WITH I .EQ. NVP AND INMODE=5 TO THE	AXMD 183
C	RIGHT OF THE CURRENT (I,J). WE RETURN WITH K=-1.	AXMD 184
	70 GO TO 100	AXMD 185
C		AXMD 186
C	WE CONTINUE HERE FROM STATEMENT 44 WITH THE POINT BELOW HAVING	AXMD 187
C	ICHK .NE. 10. THE POINT AT THE RIGHT HAS ICHK .EQ. 5. WE CANNOT	AXMD 188
C	SKIRT THE FIVES AND HENCE WE RETURN WITH K=-1.	AXMD 189
	80 GO TO 100	AXMD 190
C		AXMD 191
C	WE CONTINUE HERE FROM 1,3,45,60,70,OR 80. THE SEARCH WAS UNSUCCESSFUL	AXMD 192
	100 K=-1	AXMD 193
	RETURN	AXMD 194
	END	AXMD 195

PR. GRAM
AXMD 195

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C      NROW      I=4      NO      INP      NXPT 71
C      NCOL      I=4      NO      INP      NXPT 72
C      INM       I=4      VAR     INP      NXPT 73
C      K         I=4      NO      OUT     NXPT 74
C                                     NXPT 75
C NC COMMON STORAGE USED             NXPT 76
C                                     NXPT 77
C      ----INPUTS----                 NXPT 78
C                                     NXPT 79
C      I1          =ROW INDEX OF START POINT      NXPT 80
C      J2          =COLUMN INDEX OF START POINT   NXPT 81
C      ITP1        =TYPE INDEX OF START POINT, 1 MEANS POINT ABOVE HAS NXPT 82
C                  DIFFERENT INM, 2 MEANS POINT TO RIGHT HAS DIFFERENT NXPT 83
C                  INM.                                     NXPT 84
C      NROW        =NUMBER OF ROWS IN ARRAY        NXPT 85
C      NCOL        =NUMBER OF COLUMNS IN ARRAY    NXPT 86
C      INM         =SIGN OF NORMAL MODE DISPERSION FUNCTION, 1 IF POS., NXPT 87
C                  -1 IF NEG., 5 IF IT DOESN'T EXIST. LET I=L MOD NVP, NXPT 88
C                  J=(I-1)/NVP+1. INMODE(I) IS SIGN OF NMDF FOR NXPT 89
C                  OMEGA=OM(J), PHASE VEL. =VP(I), WHERE OM(J) .GE. CM(J) NXPT 90
C                  AND VP(I) .LE. VP(I-1)           NXPT 91
C                                     NXPT 92
C      ----OUTPUTS----                 NXPT 93
C                                     NXPT 94
C      I2          =ROW INDEX OF FOUND POINT       NXPT 95
C      J2          =COLUMN INDEX OF FOUND POINT    NXPT 96
C      ITP2        =TYPE INDEX OF FOUND POINT      NXPT 97
C      K           =FLAG INDICATING IF POINT (I2,J2) IS FOUND, 1 IF YES, NXPT 98
C                  -1 IF NO                         NXPT 99
C                                     NXPT 100
C      ----EXAMPLE----                 NXPT 101
C                                     NXPT 102
C SUPPOSE THE ARRAY OF INM VALUES IS AS SHOWN BELOW NXPT 103
C                                     NXPT 104
C      ++++++---      NROW=8, NCOL=11             NXPT 105
C      ++++++---      NXPT 106
C      5-----+      IF I1=3,J1=4,ITP1=1 THEN I2=3,J2=5, NXPT 107
C      55-----+      ITP2=1,K=1                  NXPT 108
C      55-----+      NXPT 109
C      55-----+      IF I1=1,J1=9,ITP1=2 THEN I2=7,J2=10, NXPT 110
C      55-----+      ITP2=1,K=1                  NXPT 111
C      55-----+      NXPT 112
C                                     NXPT 113
C      IF I1=3,J1=7,ITP1=1 THEN I2=3,J2=7,         NXPT 114
C      ITP2=2,K=1                                     NXPT 115
C                                     NXPT 116
C      IF I1=3,J1=11,ITP1=1 THEN K=-1              NXPT 117
C                                     NXPT 118
C      ----PROGRAM FOLLOWS BELOW----              NXPT 119
C                                     NXPT 120
C      SUBROUTINE NXPNT(I1,J1,ITP1,I2,J2,ITP2,NROW,NCOL,INM,K) NXPT 121
C      DIMENSION INM(1)                             NXPT 122
C      IO=INM((J1-1)*NROW+1)                         NXPT 123
C      1 IF( IO .EQ. 5 .OR. I1 .GT. NROW .OR. J1 .GE. NCOL) GO TO 30 NXPT 124
C      IR IS INM OF POINT TO THE RIGHT. IO IS INM OF POINT (I1,J1). NXPT 125
C      5 IR=INM((J1)*NROW+1)                          NXPT 126
C      6 IF( IR .NE. IO ) GO TO 15                     NXPT 127
C      7 IF( I1 .EQ. 1 ) GO TO 30                     NXPT 128
C      IR HAS THE SAME SIGN AS IO. WE CHECK IRU REPRESENTING INM OF UPPER NXPT 129
C      RIGHT POINT. IF THIS IS -10, THE RIGHT POINT IS THE DESIRED POINT. NXPT 130
C      IF IT IS NOT -10, WE CANNOT FIND (I2,J2).     NXPT 131
C      10 IRU=INM((J1)*NROW+1)                       NXPT 132
C      11 IF( IRU .NE. -10 ) GO TO 30                 NXPT 133
C      ITP2=1                                           NXPT 134
C      I2=I1                                           NXPT 135
C      J2=J1+1                                         NXPT 136
C                                     NXPT 137
C                                     NXPT 138
C                                     NXPT 139
C                                     NXPT 140

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PROGRAM
 NXPNT
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K=1	NXPT 141
RETURN	NXPT 142
C	NXPT 143
C WE ARRIVE HERE FROM STATEMENT 6. THE POINT TO THE RIGHT HAS A	NXPT 144
C DIFFERENT INM. IF THIS IS -10 AND ITYP1=1, WE HAVE (I2,J2)=(I1,J1)	NXPT 145
C WITH ITYP2=2. IF THIS IS 5, WE CANNOT FIND (I2,J2).	NXPT 146
15 IF(IR .EQ. 5) GO TO 30	NXPT 147
C	NXPT 148
C IR=-10 AT THIS POINT	NXPT 149
IF(ITYP1 .NE. 1) GO TO 25	NXPT 150
I2=I1	NXPT 151
J2=J1	NXPT 152
ITYP2=2	NXPT 153
K=1	NXPT 154
RETURN	NXPT 155
C	NXPT 156
C IR=-10, ITYP1 IS 2. WE CONTINUE FROM STATEMENT 15. IF WE ARE ON THE	NXPT 157
C BOTTOM ROW, WE CANNOT FIND NEW POINT	NXPT 158
25 IF (I1.EQ.NROW) GO TO 30	NXPT 159
C	NXPT 160
C WE CONSIDER POINTS BELOW AND TO LOWER RIGHT	NXPT 161
ID=INM((J1-1)*NROW+I1+1)	NXPT 162
IDR=INM((J1)*NROW+I1+1)	NXPT 163
C	NXPT 164
C IF IDR IS 5, WE CANNOT FIND THE NEW POINT	NXPT 165
26 IF(IDR .EQ. -5) GO TO 30	NXPT 166
C	NXPT 167
C IF IDR IS 10, THE NEXT POINT IS THE DR POINT	NXPT 168
27 IF(IDR .NE. 10) GO TO 28	NXPT 169
I2=I1+1	NXPT 170
J2=J1+1	NXPT 171
ITYP2=1	NXPT 172
K=1	NXPT 173
RETURN	NXPT 174
C	NXPT 175
C IR=-10, ITYP1 IS 2, IDR IS -10. WE CONTINUE FROM STATEMENT 27.	NXPT 176
28 IF(ID .NE. 10) GO TO 30	NXPT 177
C	NXPT 178
C THE NEXT POINT IS THE DOWN POINT	NXPT 179
I2=I1+1	NXPT 180
J2=J1	NXPT 181
ITYP2=2	NXPT 182
K=1	NXPT 183
RETURN	NXPT 184
C	NXPT 185
C WE ARRIVE HERE FROM 1,7,11,15,25,26. THE NEXT POINT CANNOT BE FOUND	NXPT 186
30 K=-1	NXPT 187
RETURN	NXPT 188
END	NXPT 189

PROGRAM
NXTPT

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C	PAMPDE (SUBROUTINE)	7/30/68	PAMP	1	
C			PAMP	2	
C	-----ABSTRACT-----		PAMP	3	
C			PAMP	4	
C	TITLE - PAMPDE		PAMP	5	
C	PROGRAM TO COMPUTE AND STORE AMPLITUDE FACTORS AMP(J) AND FACT		PAMP	6	
C	AND SCALING FACTOR ALAM. THE QUANTITY AMP(J) IS THE QUANTITY		PAMP	7	
C	AMPLTD COMPUTED BY SUBROUTINE NAMPDE WHEN THE ANGULAR FREQUENCY		PAMP	8	
C	IS OMMOD(J) AND THE PHASE VELOCITY IS VPMOD(J). IT CORRESPONDS		PAMP	9	
C	TO THE NMODE-TH GUIDED MODE WHEN J IS BETWEEN KST(NMODE) AND		PAMP	10	
C	KFIN(NMODE), INCLUSIVE. THE QUANTITY FACT IS DEPENDENT ON		PAMP	11	
C	SOURCE ALTITUDE ZSCRCE AND OBSERVER ALTITUDE ZOBS AND IS GIVEN		PAMP	12	
C			PAMP	13	
C	FACT = CONST*CI(1)*UED*(PSCRCE/1.E6)**0.333333		PAMP	14	
C			PAMP	15	
C	WHERE CONST=4.0/SQRT(2*PI), CI(1) IS THE SOUND SPEED AT THE		PAMP	16	
C	GROUND, (PSCRCE/1.E6) IS THE AMBIENT PRESSURE AT ZSCRCE DIVIDED		PAMP	17	
C	BY THE AMBIENT PRESSURE AT THE GROUND. THE QUANTITY UED IS		PAMP	18	
C	THE SQUARE ROOT OF (AMBIENT DENSITY AT ZOBS)/(AMBIENT DENSITY A		PAMP	19	
C	ZSCRCE). THE SCALING FACTOR ALAM IS GIVEN BY		PAMP	20	
C			PAMP	21	
C	ALAM = (1.E6/PSCRCE)**(0.333333)*(CI(1)/CI(1SCR)		PAMP	22	
C			PAMP	23	
C	WHERE CI(1SCR) IS THE SOUND SPEED AT THE SOURCE ALTITUDE. THE		PAMP	24	
C	SIGNIFICANCE OF THESE QUANTITIES IS EXPLAINED IN SUBROUTINE		PAMP	25	
C	PPAMP.		PAMP	26	
C			PAMP	27	
C	PROGRAM NOTES		PAMP	28	
C			PAMP	29	
C	THE PARAMETERS IMAX,CI,VXI,VYI,HI DEFINING THE MULTILAYER		PAMP	30	
C	ATMOSPHERE ARE PRESUMED STORED IN COMMON. THE AMBIENT		PAMP	31	
C	PRESSURES ARE COMPUTED BY CALLING SUBROUTINE AMBNT WHICH		PAMP	32	
C	ALSO COMPUTES THE INDICES IOBS AND ISCR OF THE LAYERS		PAMP	33	
C	IN WHICH OBSERVER AND SOURCE, RESPECTIVELY, LIE.		PAMP	34	
C			PAMP	35	
C	IN COMPUTING AMBIENT DENSITIES, THE IDEAL GAS LAW		PAMP	36	
C	RHO= GAMMA*P/C**2 IS USED. THUS UED = (CI(1SCR)/CI(1ORS)		PAMP	37	
C	SQRT(POBS/PSCRCE).		PAMP	38	
C			PAMP	39	
C	IF NPRNT IS POSITIVE, A HEADING IS PRINTED FOR A TABLE		PAMP	40	
C	TO BE PRINTED BY SUBROUTINE NAMPDE. SEE FORMAT STATEMENT		PAMP	41	
C	19 FOR THE DEFINITIONS OF TERMS IN THE HEADING. PHI1		PAMP	42	
C	AND PHI2 SATISFY THE RESIDUAL EQUATIONS PRESENTED IN THE		PAMP	43	
C	ABSTRACT OF NAMPDE.		PAMP	44	
C			PAMP	45	
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		PAMP	46	
C			PAMP	47	
C	AUTHORS - A.D. PIERCE AND J. PUSEY, M.I.T., JULY, 1968		PAMP	48	
C			PAMP	49	
C	-----CALLING SEQUENCE-----		PAMP	50	
C			PAMP	51	
C	SEE THE MAIN PROGRAM		PAMP	52	
C	DIMENSION CI(100),VXI(100),VYI(100),HI(100)		PAMP	53	
C	DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMP(1)		PAMP	54	
C	THE PROGRAM USES VARIABLE DIMENSIONING FOR QUANTITIES IN ITS		PAMP	55	
C	ARGUMENT LIST.		PAMP	56	
C	COMMON IMAX,CI,VXI,VYI,HI THESE MUST BE STORED IN COMMON)		PAMP	57	
C	CALL PAMPDE(ZSCRCE,ZOBS,MDFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,		PAMP	58	
C	1 FACT,T,ETK,NPRNT)		PAMP	59	
C			PAMP	60	
C	-----EXTERNAL SUBROUTINES REQUIRED-----		PAMP	61	
C			PAMP	62	
C	AMBNT,NAMPDE,TOTINT,MMM,AAAA,USEAS,UPINT,ELINT,B0BB,CAI,SAI		PAMP	63	
C			PAMP	64	
C	-----ARGUMENT LIST-----		PAMP	65	
C			PAMP	66	PROGRAM
C	ZSCRCE R*4 NO INP		PAMP	67	PAMPDE
C	ZOBS R*4 NO INP		PAMP	68	
C	MDFND I*4 NO INP		PAMP	69	PAGE
C	KST I*4 VAR INP		PAMP	70	59

C	KFIN	I*4	VAR	INP	PAMP	71
C	OMMOD	R*4	VAR	INP	PAMP	72
C	VPMOD	R*4	VAR	INP	PAMP	73
C	AMP	R*4	VAR	OUT	PAMP	74
C	ALAM	R*4	NO	OUT	PAMP	75
C	FACT	R*4	NO	OUT	PAMP	76
C	THETK	R*4	NO	INP	PAMP	77
C	NPRNT	I*4	NO	INP	PAMP	78
C					PAMP	79
C	COMMON STORAGE USED				PAMP	80
C	COMMON IMAX,CI,VXI,VYI,HI				PAMP	81
C					PAMP	82
C	IMAX	I*4	NO	INP	PAMP	83
C	CI	R*4	100	INP	PAMP	84
C	VXI	R*4	100	INP	PAMP	85
C	VYI	R*4	100	INP	PAMP	86
C	HI	R*4	100	INP	PAMP	87
C					PAMP	88
C					PAMP	89
C					PAMP	90
C	ZSCRCE	=HEIGHT IN KM OF BURST ABOVE GROUND			PAMP	91
C	ZOBS	=HEIGHT IN KM OF OBSERVER ABOVE GROUND			PAMP	92
C	MOFND	=NUMBER OF GUIDED MODES FOUND			PAMP	93
C	KST(N)	=INDEX OF FIRST TABULATED POINT IN N-TH MODE			PAMP	94
C	KFIN(N)	=INDEX OF LAST TABULATED POINT IN N-TH MODE. IN GENERAL, KFIN(N)=KST(N+1)-1.			PAMP	95
C					PAMP	96
C	OMMOD(N)	=ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE FOR N BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE.			PAMP	97
C					PAMP	98
C	VPMOD(N)	=ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORE FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).			PAMP	99
C					PAMP	100
C					PAMP	101
C	THETK	=DIRECTION IN RADIAN TO OBSERVER FROM SOURCE, REFERENCE COUNTER CLOCKWISE FROM X AXIS.			PAMP	102
C					PAMP	103
C	NPRNT	=PRINT OPTION INDICATOR (SEE NAME IN MAIN PROGRAM).			PAMP	104
C					PAMP	105
C	CI(I)	=NUMBER OF LAYERS OF FINITE THICKNESS.			PAMP	106
C					PAMP	107
C	VXI(I)	=SOUND SPEED IN KM/SEC IN I-TH LAYER			PAMP	108
C					PAMP	109
C	VYI(I)	=X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)			PAMP	110
C					PAMP	111
C	HI(I)	=Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)			PAMP	112
C					PAMP	113
C					PAMP	114
C					PAMP	115
C	AMP(J)	=AMPLITUDE FACTOR FOR GUIDED WAVE EXCITED BY POINT ENERGY SOURCE. UNITS ARE KM**(-1). THE J-TH ELEMENT CORRESPONDS TO ANGULAR FREQUENCY OMMOD(J) AND PHASE VELOCITY VPMOD(J). THE AMPLITUDE FACTOR IS APPROPRIATE TO THE NMODE-TH MODE IF J.GE. KST(NMODE) AND J.LE. KFIN(NMODE). THE AMP(J) IS THE SAME AS AMPLTD COMPUT BY SUBROUTINE NAMPOE.			PAMP	116
C					PAMP	117
C					PAMP	118
C					PAMP	119
C					PAMP	120
C	ALAM	=A SCALING FACTOR DEPENDENT ON HEIGHT OF BURST, EQUAL TO CUBE ROOT OF (PRESSURE AT GROUND)/(PRESSURE AT BURST HEIGHT) TIMES (SOUND SPEED AT GROUND)/(SOUND SPEED AT BURST HEIGHT).			PAMP	121
C					PAMP	122
C					PAMP	123
C	FACT	=A GENERAL AMPLITUDE FACTOR DEPENDENT ON BURST HEIGHT AND OBSERVER HEIGHT. A PRECISE DEFINITION IS GIVEN IN THE ABSTRACT.			PAMP	124
C					PAMP	125
C					PAMP	126
C					PAMP	127
C					PAMP	128
C					PAMP	129
C					PAMP	130
C					PAMP	131
C					PAMP	132
C					PAMP	133
C					PAMP	134
C					PAMP	135
C					PAMP	136
C					PAMP	137
C					PAMP	138
C					PAMP	139
C					PAMP	140
C					PAMP	140

SUBROUTINE NAMPOE(ZSCRCE,ZOBS,MOFND,KST,KFIN,OMMOD,VPMOD,AMP,ALAM,
 1 FACT,THETK,NPRNT)

DIMENSION CI(100),VXI(100),VYI(100),HI(100)
 DIMENSION KST(10),KFIN(10),OMMOD(100),VPMOD(100),AMP(1000)
 COMMON IMAX,CI,VXI,VYI,HI

MOFND = MOFND
 IF (NPRNT.NE.0) GO TO 20

PROGRAM
 NAMPOE
 PAGE
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C PRINT HEADING FOR PHI1 AND PHI2 PROFILE DATA TO BE PRINTED BY NAMPDE
WRITE (6,19)
19 FORMAT (1H1,41X,26HPHI1 AND PHI2 PROFILE DATA ///63H01AP1MX = NO.
10F LAYER FOR WHICH ABS(PHI1(IAP1MX)) IS A MAXIMUM/63H IAP2MX = NO.
2 OF LAYER FOR WHICH ABS(PHI2(IAP2MX)) IS A MAXIMUM/42H R1 = PH
311(IAP1MX) / ABS(PHI2(IAP2MX)) / 42H R2 = PHI2(IAP2MX) / ABS(PH
412(IAP2MX)) / 31H R3 = PHI2(1) / ABS(PHI2(IAP2MX)) / 40H NZC1
5= NO. OF TIMES PHI1 CHANGES SIGN /40H NZC2 = NO. OF TIMES PHI2 C
6HANGES SIGN)
20 CONTINUE
C DO LOOP TO COMPUTE AMP(J)
DO 25 I1=1,MCFND
IF (NPRNT.LT.0) GO TO 23
WRITE (6,21) I1
22 FORMAT (1H /// 1H ,51X,5HMODE ,12 /// 1H ,7X,5HOMEGA,7X,5HVPHSF
1,6X,6HIAP1MX,10X,2HR1,8X,4HNZC1,6X,6HIAP2MX,10X,2HR2,8X,4HNZC2,10X
2,2HR3 //)
23 J1=KST(I1)
J2=KFIN(I1)
DO 25 J=J1,J2
K = J
OMEGA = OMMOD(K)
VPHSF = VPMOD(K)
CALL NAMPDE(ZSCRCE,ZOBS,OMEGA,VPHSE,THFTK,X,NPRNT)
AMP(K) = X
25 CONTINUE
C END OF DO LOOP
C
C COMPUTATION OF AMBIENT PRESSURES
CALL AMBNT(ZSCRCE,PSCRCE,ISCR)
CALL AMBNT(ZOBS,POBS,IOBS)
C
C COMPUTATION OF SQRT(DENSITY RATIO)
UED = (CI(IISCR)/CI(IOBS)) * SQRT(POBS/PSCRCE)
C
C COMPUTATION OF ALAM AND FACT
ALAM=(1.E6/PSCRCE)**(0.333333)*(CI(1)/CI(IISCR))
C NOTE THAT CI(1) IS SOUND SPEED AT THE GROUND
CONST = 4.0/SQRT(2.0*3.141593)
FACT = CONST*(CI(1)*UED*(PSCRCE/1.E6)**(0.333333))
IF(NPRNT.NE.1) RETURN
WRITE (6,31) ZSCRCE,ZOBS,FACT,ALAM
31 FORMAT(1H1, 20X, 36HTABULATION OF SOURCE FREE AMPLITUDES,
1 23H FROM SUBROUTINE PAMPDE //21X, 19HHEIGHT OF BURST =,
1 F8.3, 3H KM / 21X, 19HHEIGHT OF CHSERVER=, F8.3, 3H KM /
1 21X, 4HFACT, 14X, 1H=, F8.3, 7H KM/SEC/ 21X,4HALAM,14X, 1H=,
1 F8.3)
DO 50 I1 =1,MCFND
WRITE (6,41) I1
41 FORMAT(1H /// 1H , 5HMODE , 13/ 1H , 20X,5HOMEGA,
1 15X, 5HVPHSF, 17X, 3HAMP)
K1=KST(I1)
K2=KFIN(I1)
DO 50 J=K1,K2
50 WRITE (6,51) OMMOD(J),VPMOD(J),AMP(J)
51 FORMAT(1H ,4X,F20.5,F20.5,F20.8)
RETURN
END

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PAMP 141
PAMP 142
PAMP 143
PAMP 144
PAMP 145
PAMP 146
PAMP 147
PAMP 148
PAMP 149
PAMP 150
PAMP 151
PAMP 152
PAMP 153
PAMP 154
PAMP 155
PAMP 156
PAMP 157
PAMP 158
PAMP 159
PAMP 160
PAMP 161
PAMP 162
PAMP 163
PAMP 164
PAMP 165
PAMP 166
PAMP 167
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PAMP 184
PAMP 185
PAMP 186
PAMP 187
PAMP 188
PAMP 189
PAMP 190
PAMP 191
PAMP 192
PAMP 193
PAMP 194
PAMP 195
PAMP 196
PAMP 197
PAMP 198

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PAMPDE

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C	PHASE (SUBROUTINE)	8/15/68	PHAS	1
C			PHAS	2
C			PHAS	3
C	----ABSTRACT----		PHAS	4
C			PHAS	5
C	TITLE - PHASE		PHAS	6
C	CONVERSION OF A COMPLEX NUMBER FROM RECTANGULAR FORM TO POLAR		PHAS	7
C	FORM		PHAS	8
C			PHAS	9
C	GIVEN TWO REAL NUMBERS RR AND RI, A MAGNITUDE R AND AN		PHAS	10
C	ANGLE PHI ARE COMPUTED SUCH THAT		PHAS	11
C			PHAS	12
C	$RR + I*RI = R * [EXP(I*PHI)]$		PHAS	13
C			PHAS	14
C	WHERE $I = (-1)**0.5$		PHAS	15
C			PHAS	16
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		PHAS	17
C			PHAS	18
C	AUTHORS - A.C.PIERCE AND J. POSEY, M.I.T., AUGUST, 1968		PHAS	19
C			PHAS	20
C	----USAGE----		PHAS	21
C			PHAS	22
C	NO SUBROUTINES ARE CALLED		PHAS	23
C			PHAS	24
C	FORTRAN USAGE		PHAS	25
C			PHAS	26
C	CALL PHASE(RR,RI,R,PHI)		PHAS	27
C			PHAS	28
C	INPUTS		PHAS	29
C			PHAS	30
C	RR REAL PART OF THE COMPLEX NUMBER BEING CONVERTED		PHAS	31
C	R#4		PHAS	32
C			PHAS	33
C	RI IMAGINARY PART OF COMPLEX NUMBER BEING CONVERTED		PHAS	34
C	R#4		PHAS	35
C			PHAS	36
C	OUTPUTS		PHAS	37
C			PHAS	38
C	R MAGNITUDE OF THE COMPLEX NUMBER		PHAS	39
C	R#4		PHAS	40
C			PHAS	41
C	PHI PHASE OF THE COMPLEX NUMBER (RADIAN) (-PI.LE.PHI.LE.PI)		PHAS	42
C	R#4		PHAS	43
C			PHAS	44
C			PHAS	45
C	----EXAMPLES----		PHAS	46
C			PHAS	47
C	CALL PHASE(0.0,1.0,R,PHI)		PHAS	48
C			PHAS	49
C	R = 1.0 AND PHI = 1.570796 ARE RETURNED		PHAS	50
C			PHAS	51
C	CALL PHASE(1.0,-1.0,R,PHI)		PHAS	52
C			PHAS	53
C	R = 1.414214 AND PHI = -0.785398 ARE RETURNED		PHAS	54
C			PHAS	55
C			PHAS	56
C	----PROGRAM FOLLOWS BELOW----		PHAS	57
C			PHAS	58
C			PHAS	59
C			PHAS	60
C	SUBROUTINE PHASE(RR,RI,R,PHI)		PHAS	61
C			PHAS	62
C	Q=ABS(RR)+ABS(RI)		PHAS	63
C	IF (Q-1.E-25) 1,1,30		PHAS	64
C	1 R=Q		PHAS	65
C	PHI=0.		PHAS	66
C	RETURN		PHAS	67
C	30 AR=RR/Q		PHAS	68
C	AI=RI/Q		PHAS	69
C	A=SQRT(AR**2+AI**2)		PHAS	70

PROGRAM
PHASE
PAGE
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R=Q*A
PHI=ARSIN(ABS(A11)/A)
IF(RR) 50,60,60
50 IF(RI) 300,300,200
60 IF(RI) 400,400,100
100 PHI=PHI
RETURN
200 PHI=3.1415927-PHI
RETURN
300 PHI=PHI-3.1415927
RETURN
400 PHI=-PHI
RETURN
END

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PHAS 71
PHAS 72
PHAS 73
PHAS 74
PHAS 75
PHAS 76
PHAS 77
PHAS 78
PHAS 79
PHAS 80
PHAS 81
PHAS 82
PHAS 83
PHAS 84

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PROGRAM
PHASF

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C	PPAMP (SUBROUTINE)	7/30/68	PPAM	1
C			PPAM	2
C	-----ABSTRACT-----		PPAM	3
C			PPAM	4
C	TITLE - PPAMP		PPAM	5
C	PROGRAM TO COMPUTE AND STORE AMPLITUDE ARRAY AMPLTD AND PHASE		PPAM	6
C	ARRAY PHASQ FOR GUIDED WAVES EXCITED BY A POINT ENERGY SOURCE		PPAM	7
C	WITH TIME DEPENDENCE CORRESPONDING TO A NUCLEAR EXPLOSION OF		PPAM	8
C	ENERGY DENOTED BY YIELD IN KT. THE VALUES FCND ARE TO BE		PPAM	9
C	SUBSEQUENTLY USED BY TMPT ACCORDING TO THE RELATION		PPAM	10
C			PPAM	11
C	(PRESSURE IN DYNES/CM**2 FOR A GIVEN MODE)*SQRT(R)		PPAM	12
C			PPAM	13
C	= INTEGRAL OVER OMEGA OF AMPLTD*COS(OMEGA*(T-R/VP)+PHASQ)		PPAM	14
C			PPAM	15
C	THE QUANTITIES AMPLTD AND PHASQ ARE BOTH DEPENDENT ON ANGULAR		PPAM	16
C	FREQUENCY AND ARE DIFFERENT FOR DIFFERENT MODES.		PPAM	17
C			PPAM	18
C	PROGRAM NOTES		PPAM	19
C			PPAM	20
C	IN THE FORMULATION FOR A POINT ENERGY SOURCE, THE ENERGY		PPAM	21
C	EQUATION IS WRITTEN		PPAM	22
C			PPAM	23
C	DP/DT -(C**2/D(RHO)/DT = 4*PI*C**2*F(T)*(DELTA FCYN)		PPAM	24
C	AN EXPRESSION FOR F(T) IS		PPAM	25
C			PPAM	26
C	F(T) = ((L**2)/CS)*POS*(INTEGRAL OVER X FROM 0 TO CS*T/L		PPAM	27
C	OF UNIVERSAL FUNCTION FUNIV(X))		PPAM	28
C			PPAM	29
C	WITH L=(ENERGY/POS)**(1/3) AND POS,CS REPRESENTING PRESSURE		PPAM	30
C	AND SOUND SPEED AT THE SOURCE. IF F1KT(T) IS THE PRESSURE		PPAM	31
C	AT A DISTANCE OF 1 KM FROM A 1 KT EXPLOSION AT SEA LEVEL		PPAM	32
C	AND WITH TIME ORIGIN CORRESPONDING TO BLAST WAVE ONSET,		PPAM	33
C	THEN		PPAM	34
C			PPAM	35
C	FUNIV(X)=((L1*P01)**(-1))*F1KT(L1=X/C1)		PPAM	36
C			PPAM	37
C	THE FOURIER TRANSFORM OF F(T) IS ACCORDINGLY FOUND TO BE		PPAM	38
C			PPAM	39
C	G(OMEGA)= (1/(2*PI))*(Y**(2/3))*(C1/CS)*(POS/P01)**(1/3)		PPAM	40
C			PPAM	41
C	*(1/(-1*OMEGA))*FTMAG(OMERAT)*EXP(I*FTPHSE(OMERAT))		PPAM	42
C			PPAM	43
C	WHERE Y IS YIELD IN KT, I=SQRT(-1), AND OMERAT=ALAM*		PPAM	44
C	OMEGA*Y**(1/3). THE FUNCTIONS FTMAG AND FTPHSE ARE AS		PPAM	45
C	COMPUTED BY SUBROUTINE SOURCE. THE QUANTITY ALAM IS		PPAM	46
C	(C1/CS)*(P01/POS)**1/3 AS COMPUTED BY SUBROUTINE		PPAM	47
C	PAMPDE.		PPAM	48
C	A LENGTHY DERIVATION NOT GIVEN HERE INDICATES THAT		PPAM	49
C			PPAM	50
C	AMPLTD*EXP(-I*PHASQ)		PPAM	51
C			PPAM	52
C	= -4*SQRT(K)*G(OMEGA)*C5*UED*SQRT(2*PI)*AMP		PPAM	53
C	*EXP(-I*PI/4)		PPAM	54
C			PPAM	55
C	WHERE AMP IS THE SAME AS THE AMPLTD COMPUTED BY PAMPDE AN		PPAM	56
C	WHERE UED IS THE DENSITY FACTOR (CS/C0BS)*SQRT(PSCRCE/POB		PPAM	57
C	COMPUTED IN SUBROUTINE PAMPDE. INSERTING G(OMEGA) INTO		PPAM	58
C	THE ABOVE, WE IDENTIFY		PPAM	59
C			PPAM	60
C	PHASQ =(3/4)*PI - FTPHSE(OMERAT)		PPAM	61
C			PPAM	62
C	AMPLTD*FACT*AMP*(Y**(2/3))*FTMAG(OMERAT)*SQRT(K)/OMEGA		PPAM	63
C			PPAM	64
C	WHERE FACT IS 4/SQRT(2*PI)*C1*UED*(PS/P1)**(1/3) AND IS		PPAM	65
C	COMPUTED BY SUBROUTINE PAMPDE.		PPAM	66
C			PPAM	67
C	THE QUANTITIES FACT, ALAM, AND AMP ARE IN THE INPUT LIST		PPAM	68
C	OF THE SUBROUTINE. NOTE THAT THESE ARE YIELD INDEPENDENT		PPAM	69
C			PPAM	70

PROGRAM
PPAMP
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C      THE SCHEME OF STORAGE FOR AMPLTD(J), AND PHASQ(J) IS THE SAME AS FOR QMMOD(J) AND VPMOD(J). SEE SUBROUTINE ALLMOD.
C
C LANGUAGE - FORTRAN IV (1960, REFERENCE MANUAL C28-6515-4)
C
C AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., JULY, 1968.
C
C      -----CALLING SEQUENCE-----
C
C SEE THE MAIN PROGRAM
C      DIMENSION KST(1), KFIN(1), QMMOD(1), VPMOD(1), AMP(1)
C      DIMENSION AMPLTD(1), PHASQ(1)
C THESE QUANTITIES MUST BE DIMENSIONED. THE PROGRAM USES VARIABLE
C DIMENSIONING. FOR ACTUAL DIMENSIONS ASSIGNED, SEE THE MAIN PROGRAM.
C      CALL PPAMP(YIELD, MDEFND, KST, KFIN, QMMOD, VPMOD, AMP, ALAM, FACT,
C      1 AMPLTD, PHASQ)
C
C      -----EXTERNAL SUBROUTINES REQUIRED-----
C
C SOURCE, PHASE (PHASE IS CALLED BY SOURCE)
C
C      -----ARGUMENT LIST-----
C
C      YIELD      R*4      NO      INP
C      MDEFND     I*4      NO      INP
C      KST        I*4      VAR      INP
C      KFIN       I*4      VAR      INP
C      QMMOD      R*4      VAR      INP
C      VPMOD      R*4      VAR      INP
C      AMP        R*4      VAR      INP
C      ALAM       R*4      NO      INP
C      FACT       R*4      NO      INP
C      AMPLTD     R*4      VAR      OUT
C      PHASQ      R*4      VAR      OUT
C
C NO COMMON STORAGE IS USED.
C
C      -----INPUTS-----
C
C      YIELD      =ENERGY RELEASE OF EXPLOSION IN EQUIVALENT KILOTONS OF
C                  TNT, 1 KT = 4.2E19 ERGS.
C
C      MDEFND     =NUMBER OF MODES FOUND IN PREVIOUS TABULATION OF
C                  DISPERSION CURVES.
C      KST(N)      =INDEX OF FIRST TABULATED POINT IN N-TH MODE.
C      KFIN(N)     =INDEX OF LAST TABULATED POINT IN N-TH MODE. IN
C                  GENERAL, KFIN(N)=KST(N+1)-1.
C      QMMOD(N)    =ARRAY STORING ANGULAR FREQUENCY ORDINATE OF POINTS
C                  ON DISPERSION CURVES. THE NMODE MODE IS STORED FOR
C                  N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      VPMOD(N)    =ARRAY STORING PHASE VELOCITY ORDINATE OF POINTS ON
C                  DISPERSION CURVES. THE NMODE MODE IS STORED FOR
C                  N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      AMP(N)      =AMPLITUDE FACTOR INDEPENDENT OF YIELD COMPUTED BY
C                  SUBROUTINE PAMPDF CORRESPONDING TO ANGULAR FREQUENCY
C                  QMMOD(N) AND PHASE VELOCITY VPMOD(N).
C      ALAM       =A SCALING FACTOR DEPENDENT ON HEIGHT OF BURST, EQUAL
C                  TO CUBE ROOT OF (PRESSURE AT GROUND)/(PRESSURE AT
C                  BURST HEIGHT) TIMES (SOUND SPEED AT GROUND)/SOUND
C                  SPEED AT BURST HEIGHT).
C      FACT       =A GENERAL AMPLITUDE FACTOR DEPENDENT ON BURST HEIGHT
C                  AND OBSERVER HEIGHT. A PRECISE DEFINITION IS GIVEN
C                  IN THE LISTING OF SUBROUTINE PAMPDF.
C
C      -----OUTPUTS-----
C
C      AMPLTD(N)  =AMPLITUDE FACTOR REPRESENTING TOTAL MAGNITUDE OF
C                  FOURIER TRANSFORM OF THE CONTRIBUTION TO THE WAVEFORM
C                  OF A SINGLE GUIDED MODE AT FREQUENCY QMMOD(N). IT
C                  REPRESENTS THE AMPLITUDE OF THE NMODE-TH MODE IF N IS

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PPAM 71
PPAM 72
PPAM 73
PPAM 74
PPAM 75
PPAM 76
PPAM 77
PPAM 78
PPAM 79
PPAM 80
PPAM 81
PPAM 82
PPAM 83
PPAM 84
PPAM 85
PPAM 86
PPAM 87
PPAM 88
PPAM 89
PPAM 90
PPAM 91
PPAM 92
PPAM 93
PPAM 94
PPAM 95
PPAM 96
PPAM 97
PPAM 98
PPAM 99
PPAM 100
PPAM 101
PPAM 102
PPAM 103
PPAM 104
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PPAM 121
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PPAM 124
PPAM 125
PPAM 126
PPAM 127
PPAM 128
PPAM 129
PPAM 130
PPAM 131
PPAM 132
PPAM 133
PPAM 134
PPAM 135
PPAM 136
PPAM 137
PPAM 138
PPAM 139
PPAM 140

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PPAMP
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C	BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE. THE	PPAM 141
C	PRECISE DEFINITION IS GIVEN IN THE ABSTRACT.	PPAM 142
C	PHASQ(N) = PHASE LAG AT FREQUENCY CMMD(N) FOR NMODE-TH MODE WHEN	PPAM 143
C	N IS BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE.	PPAM 144
C	THE INTEGRAND IS UNDERSTOOD TO HAVE THE FORM	PPAM 145
C	AMPLTD*CMMD*(TIME-DISTANCE/VPMD)+PHASQ).	PPAM 146
C		PPAM 147
C	-----PROGRAM FOLLOWS BELOW-----	PPAM 148
C	SUBROUTINE PPAMP(YIELD,MDFND,KST,KFIN,CMMD,VPMD,AMP,ALAM,FACT,	PPAM 149
C	AMPLTD,PHASQ)	PPAM 150
C	DIMENSION STATEMENTS USING VARIABLE DIMENSIONING	PPAM 151
C	DIMENSION KST(1),KFIN(1),CMMD(1),VPMD(1),AMP(1)	PPAM 152
C	DIMENSION AMPLTD(1),PHASQ(1)	PPAM 153
C		PPAM 154
C	Q=(YIELD)**10.333333	PPAM 155
C	ALAMP=Q*ALAM	PPAM 156
C		PPAM 157
C	START OF DO-LOOP: II IS MODE NUMBER	PPAM 158
C	DO 20 II=1,MDFND	PPAM 159
C	K1=KST(II)	PPAM 160
C	K2=KFIN(II)	PPAM 161
C		PPAM 162
C	DO 20 J=K1,K2	PPAM 163
C	COMPUTATION OF SCALED FREQUENCY UMERAT	PPAM 164
C	UMERAT=CMMD(J)*ALAMP	PPAM 165
C	COMPUTATION OF SORT(K)	PPAM 166
C	AKAY=SQRT(CMMD(J)/VPMD(J))	PPAM 167
C		PPAM 168
C	CALL SOURCE(UMERAT,FTMAG,FTPHSE,DMAG,DPHSE)	PPAM 169
C	AMPLTD(J)=(Q**2)*FACT*FTMAG*AMP(J)*AKAY/CMMD(J)	PPAM 170
C		PPAM 171
C	20 PHASQ(J)=.75*3.14159-FTPHSE	PPAM 172
C	END OF DO-LOOP	PPAM 173
C		PPAM 174
C	RETURN	PPAM 175
C	END	PPAM 176
		PPAM 177

PROGRAM
PPAMP

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C PRATMO (SUBROUTINE) 8/1/68
C
C ----ABSTRACT----
C
C TITLE - PRATMO
C PROGRAM TO PRINT OUT PARAMETERS DEFINING THE MODEL MULTILAYER
C ATMOSPHERE. A LISTING IS PRINTED OF LAYER NUMBER, HEIGHT OF
C LAYER BOTTOM, HEIGHT OF LAYER TOP, LAYER THICKNESS, SOUND SPEED,
C AND OF X AND Y COMPONENTS OF WIND VELOCITY.
C
C LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C2A-6515-4)
C
C AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., AUGUST, 1963
C
C ----CALLING SEQUENCE----
C
C SEE THE MAIN PROGRAM
C DIMENSION CI(100), VXI(100), VYI(100), HI(100)
C COMMON IMAX, CI, VXI, VYI, HI (THESE MUST BE IN COMMON)
C CALL PRATMO)
C
C ----EXTERNAL SUBROUTINES REQUIRED----
C
C NO EXTERNAL SUBROUTINES ARE REQUIRED.
C
C ----ARGUMENT LIST----
C
C COMMON STORAGE USED
C COMMON IMAX, CI, VXI, VYI, HI
C
C IMAX      I*4      NO      INP
C CI        R*4      100     INP
C VXI       R*4      100     INP
C VYI       R*4      100     INP
C HI        R*4      100     INP
C
C ----INPUTS----
C
C IMAX      =NUMBER OF LAYERS OF FINITE THICKNESS
C CI(1)     =SOUND SPEED IN KM/SEC IN I-TH LAYER
C VXI(1)    =X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C VYI(1)    =Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)
C HI(1)     =THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS.
C
C ----OUTPUTS----
C
C THE ONLY OUTPUT IS A PRINTOUT.
C
C ----EXAMPLE----
C
C MODEL ATMOSPHERE OF 10 LAYERS (TOP OF NEW PAGE)
C (IMAX = 9)
C
C LAYER    ZH      ZT      H      C      VX
C 10      22.50   INFINITE INFINITE 0.2972  0.0082
C 9        20.00   22.50   2.50   0.2958  0.0093
C 8        17.50   20.00   2.50   0.2938  0.0118
C 7        15.00   17.50   2.50   0.2931  0.0144
C 6        12.50   15.00   2.50   0.2931  0.0165
C 5        10.00   12.50   2.50   0.2951  0.0160
C 4         7.50   10.00   2.50   0.3012  0.0144
C 3         5.00   7.50    2.50   0.3117  0.0114
C 2         2.50   5.00    2.50   0.3264  0.0084
C 1         0.     2.50    2.50   0.3394  0.0057
C
C ZH=HEIGHT OF LAYER BOTTOM IN KM
C ZT=HEIGHT OF LAYER TOP IN KM
C H=WIDTH OF LAYER IN KM
C C=SOUND SPEED IN KM/SEC
C VX=X COMP. OF WIND VEL. IN KM/SEC
C
C (THE VY COLUMN IS NOT SHOWN BECAUSE OF LACK OF SPACE. IT DOES APPEAR ON

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PRTM 1
PRTM 2
PRTM 3
PRTM 4
PRTM 5
PRTM 6
PRTM 7
PRTM 8
PRTM 9
PRTM 10
PRTM 11
PRTM 12
PRTM 13
PRTM 14
PRTM 15
PRTM 16
PRTM 17
PRTM 18
PRTM 19
PRTM 20
PRTM 21
PRTM 22
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PRTM 57
PRTM 58
PRTM 59
PRTM 60
PRTM 61
PRTM 62
PRTM 63
PRTM 64
PRTM 65
PRTM 66
PRTM 67
PRTM 68
PRTM 69
PRTM 70

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PROGRAM
PRATMO
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C	VY=Y COMP. OF WIND VEL. IN KM/SEC	PRINTOUT.)	PRTM	71
C			PRTM	72
C	----	PROGRAM FOLLOWS BELOW----	PRTM	73
C			PRTM	74
C	SUBROUTINE PRATMO.		PRTM	75
C			PRTM	76
C	DIMENSION AND COMMON STATEMENTS LOCATING INPUT		PRTM	77
	DIMENSION CI(100),VXI(100),VYI(100),HI(100),ZI(100)		PRTM	78
	COMMON IMAX,CI,VXI,VYI,HI		PRTM	79
C			PRTM	80
C	LET JET DENOTE THE INDEX OF THE UPPER HALFSpace		PRTM	81
	JET=IMAX+1		PRTM	82
C			PRTM	83
C	PRINTING OF HEADING		PRTM	84
	WRITE (6,1) JET		PRTM	85
	1) FORMAT(1H,14X,19HMODEL ATMOSPHERE OF,14,7H LAYERS//)		PRTM	86
	WRITE (6,2)		PRTM	87
	2) FORMAT(1H,2X,5H LAYER,7X,2HZH,10X,2HZT,11X,1HH,11X,1HC,11X,2HVX,		PRTM	88
	110X,2HVV)		PRTM	89
C			PRTM	90
	IF(IMAX.EQ.0) GO TO 33		PRTM	91
C			PRTM	92
C	ZI(I) DENOTES THE HEIGHT OF TOP OF I-TH LAYER OF FINITE THICKNESS		PRTM	93
	ZI(I)=HI(I)		PRTM	94
	IF(IMAX.EQ.1) GO TO 31		PRTM	95
	DO 30 I=2,IMAX		PRTM	96
	30 ZI(I)=ZI(I-1)+HI(I)		PRTM	97
	31 CONTINUE		PRTM	98
C			PRTM	99
C	PRINTOUT FOR UPPER HALFSpace		PRTM	100
	XUV=ZI(IMAX)		PRTM	101
	33 IF(IMAX.EQ.0) XUV=0.0		PRTM	102
	C=CI(JET)		PRTM	103
	VX=VXI(JET)		PRTM	104
	VY=VYI(JET)		PRTM	105
	WRITE (6,4) JET,XUV,C,VX,VY		PRTM	106
	4) FORMAT(1H,17,3F12.2,4X,8HINFINITE,4X,8HINFINITE,3F12.4)		PRTM	107
C			PRTM	108
	IF(IMAX.EQ.0) GO TO 60		PRTM	109
	IF(IMAX.EQ.1) GO TO 52		PRTM	110
C			PRTM	111
C	TABULATION FOR LAYERS 2 THROUGH IMAX		PRTM	112
	DO 50 J=2,IMAX		PRTM	113
	I=IMAX+2-J		PRTM	114
	IL=I-1		PRTM	115
	50 WRITE (6,5) I,ZI(IL),ZI(I),HI(I),CI(I),VXI(I),VYI(I)		PRTM	116
	5) FORMAT(1H,17,3F12.2,3F12.4)		PRTM	117
C			PRTM	118
C	TABULATION FOR LAYER 1		PRTM	119
	52 I=1		PRTM	120
	USTED=0.0		PRTM	121
	WRITE (6,5) I,USTED,ZI(I),HI(I),CI(I),VXI(I),VYI(I)		PRTM	122
C			PRTM	123
C	PRINTOUT OF EXPLANATIONS		PRTM	124
	60 WRITE (6,6)		PRTM	125
	6) FORMAT(1H0,15X,31HZH=HEIGHT OF LAYER BOTTOM IN KM/ 1H,15X,2HZT=H		PRTM	126
	HEIGHT OF LAYER TOP IN KM/1H,15X,23HH=WIDTH OF LAYER IN KM/1H,		PRTM	127
	215X,24HC=SCUND SPEED IN KM/SEC/1H,15X,33HVX=X COMP. OF WIND VEL.		PRTM	128
	3 IN KM/SEC/1H,15X,33HVV=Y COMP. OF WIND VEL. IN KM/SEC)		PRTM	129
C			PRTM	130
	RETURN		PRTM	131
	END		PRTM	132

PROGRAM
PRATMO

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C      RRRR (SUBROUTINE)                      8/1/68
C
C      ----ABSTRACT----
C
C  TITLE - RRRR
C  THIS SUBROUTINE COMPUTES A 2-BY-2 TRANSFER MATRIX WHICH CONNECT
C  SOLUTIONS OF THE RESIDUAL EQUATIONS AT THE BOTTOM OF THE UPPER
C  HALFS-SPACE TO SOLUTIONS AT THE GROUND BY THE RELATIONS
C
C      PHI1(GROUND)= RPP(1,1)*PHI1(ZT(IMAX))+RPP(1,2)*PHI2(ZT(IMA
C      PHI2(GROUND)= RPP(2,1)*PHI1(ZT(IMAX))+RPP(2,2)*PHI2(ZT(IMA
C
C  WHERE ZT(IMAX) IS THE HEIGHT OF THE TOP OF THE IMAX LAYER AND
C  CONSEQUENTLY THE HEIGHT OF THE BOTTOM OF THE UPPER HALFS-SPACE.
C  THE FUNCTIONS PHI1(Z) AND PHI2(Z) SATISFY THE RESIDUAL EQUATIONS
C
C      D(PHI1)/DZ = A(1,1)*PHI1(Z) + A(1,2)*PHI2(Z)
C      D(PHI2)/DZ = A(2,1)*PHI1(Z) + A(2,2)*PHI2(Z)
C
C  WHERE THE A(I,J) ARE FUNCTIONS OF ALTITUDE BUT CONSTANT IN EACH
C  LAYER.
C
C  IF WE LET EM(I) BE THE EM MATRIX (COMPUTED BY SUBROUTINE MMMM)
C  FOR THE I-TH LAYER, THEN (IN MATRIX NOTATION)
C
C      RPP = EM(1)*EM(2)*.....*EM(IMAX-1)*EM(IMAX)
C
C  THE ABOVE FORMULA IS USED TO COMPUTE THE RPP(I,J).
C
C  THE PARAMETERS DEFINING THE MULTILAYER ATMOSPHERE ARE PRESUMED
C  TO BE STORED IN COMMON.
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C  AUTHOR - A.D. PIERCE, M.I.T., AUGUST, 1968
C
C      ----CALLING SEQUENCE----
C
C  SEE SUBROUTINE NMODFN
C  DIMENSION CI(100),VXI(100),VYI(100),HI(100)
C  COMMON IMAX,CI,VXI,VYI,HI (THESE MUST BE STORED IN COMMON)
C  DIMENSION RPP(2,2)
C  CALL RRRR(OMEGA,AKX,AKY,RPP,K)
C
C      ----EXTERNAL SUBROUTINES REQUIRED----
C
C  MMMM,AAAA,CAI,SAI
C
C      ----ARGUMENT LIST----
C
C  OMEGA      R*4      NO      INP
C  AKX        R*4      NO      INP
C  AKY        R*4      NO      INP
C  RPP        R*4      2-BY-2 OUT
C  X          I*4      NO      OUT (ALWAYS OUTPUT AS K=0)
C
C  COMMON STORAGE USED
C  COMMON IMAX,CI,VXI,VYI,HI
C
C  IMAX      I*4      NO      INP
C  CI        R*4      100     INP
C  VXI       R*4      100     INP
C  VYI       R*4      100     INP
C  HI        R*4      100     INP
C
C      ----INPUTS----
C
C  OMEGA      = ANGULAR FREQUENCY IN RAD/SEC

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RRRR 1
RRRR 2
RRRR 3
RRRR 4
RRRR 5
RRRR 6
RRRR 7
RRRR 8
RRRR 9
RRRR 10
RRRR 11
RRRR 12
RRRR 13
RRRR 14
RRRR 15
RRRR 16
RRRR 17
RRRR 18
RRRR 19
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PROGRAM
RRRR
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C	AKX	=X COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM	RRRR	71
C	AKY	=Y COMPONENT OF HORIZONTAL WAVE NUMBER VECTOR IN 1/KM	RRRR	72
C	IMAX	=NUMBER OF LAYERS OF FINITE THICKNESS	RRRR	73
C	CI(I)	=SOUND SPEED IN KM/SEC IN I-TH LAYER	RRRR	74
C	VXI(I)	=X COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)	RRRR	75
C	VYI(I)	=Y COMPONENT OF WIND VELOCITY IN I-TH LAYER (KM/SEC)	RRRR	76
C	HI(I)	=THICKNESS IN KM OF I-TH LAYER OF FINITE THICKNESS	RRRR	77
C			RRRR	78
C		----OUTPUTS----	RRRR	79
C			RRRR	80
C	RPP	=7-BY-2 TRANSFER MATRIX WHICH CONNECTS SOLUTIONS OF	RRRR	81
C		THE RESIDUAL EQUATIONS AT THE BOTTOM OF THE UPPER	RRRR	82
C		HALFSPACE TO SOLUTIONS AT THE GROUND.	RRRR	83
C	X	=DUMMY PARAMETER ALWAYS RETURNED AS 0.	RRRR	84
C			RRRR	85
C		----PROGRAM FOLLOWS BELOW----	RRRR	86
C			RRRR	87
C		SUBROUTINE RRRR(OMEGA,AKX,AKY,RPP,K)	RRRR	88
C			RRRR	89
C		DIMENSION AND COMMON STATEMENTS LOCATING PARAMETERS DEFINING THE MODEL	RRRR	90
C		MULTILAYER ATMOSPHERE	RRRR	91
C		DIMENSION CI(100),VXI(100),VYI(100),HI(100)	RRRR	92
C		COMMON IMAX,CI,VXI,VYI,HI	RRRR	93
C			RRRR	94
C		DIMENSION EM(2,2),AINT(2,2),RPP(2,2)	RRRR	95
C		K=0	RRRR	96
C			RRRR	97
C		RPP AT TOP OF IMAX LAYER IS THE IDENTITY MATRIX	RRRR	98
C		RPP(1,1)=1.0	RRRR	99
C		RPP(1,2)=0.0	RRRR	100
C		RPP(2,1)=0.0	RRRR	101
C		RPP(2,2)=1.0	RRRR	102
C			RRRR	103
C		START OF DO LOOP RUNNING THROUGH IMAX LAYERS IN DESCENDING ORDER	RRRR	104
C		DO 100 JASA=1,IMAX	RRRR	105
C		JASA=IMAX+1-JASA	RRRR	106
C		JASA IS THE INDEX OF THE LAYER CURRENTLY UNDER CONSIDERATION	RRRR	107
C			RRRR	108
C		COMPUTATION OF EM MATRIX FOR JASA LAYER	RRRR	109
C		C=CI(JASA)	RRRR	110
C		VX=VXI(JASA)	RRRR	111
C		VY=VYI(JASA)	RRRR	112
C		H=HI(JASA)	RRRR	113
C		CALL MMMH(OMEGA,AKX,AKY,C,VX,VY,H,EM)	RRRR	114
C			RRRR	115
C		MULTIPLICATION OF RPP AT TOP OF JASA LAYER BY EM FOR JASA LAYER	RRRR	116
C		DO 80 I=1,2	RRRR	117
C		DO 80 J=1,2	RRRR	118
C		80 AINT(I,J)=EM(I,1)*RPP(1,J)+EM(I,2)*RPP(2,J)	RRRR	119
C			RRRR	120
C		CURRENT AINT IS RPP AT BOTTOM OF JASA LAYER	RRRR	121
C		DO 85 I=1,2	RRRR	122
C		DO 85 J=1,2	RRRR	123
C		85 RPP(I,J)=AINT(I,J)	RRRR	124
C			RRRR	125
C		100 CONTINUE	RRRR	126
C		END OF OUTER DO LOOP	RRRR	127
C			RRRR	128
C		CURRENT RPP IS THAT AT BOTTOM OF FIRST LAYER	RRRR	129
C		RETURN	RRRR	130
C		END	RRRR	131

PROGRAM
RRRR

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C	SAI (FUNCTION)	7/25/68	SAI	1
C			SAI	2
C	----	ABSTRACT----	SAI	3
C			SAI	4
C	TITLE - SAI		SAI	5
C	PROGRAM TO EVALUATE FUNCTION SAI(X) FOR GIVEN VARIABLE X.		SAI	6
C	IF X IS NEGATIVE, SAI(X)=SINH(Y)/Y WITH Y=SORT(-X). IF X IS		SAI	7
C	POSITIVE, SAI(X)=SINH(Y)/Y WITH Y=SORT(X). THE FUNCTION IS		SAI	8
C	ALSO REPRESENTABLE BY THE POWER SERIES		SAI	9
C			SAI	10
C	SAI(X)= 1 + X/(3FACT) + X**2/(5FACT) + X**3/(7FACT) + ...		SAI	11
C			SAI	12
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		SAI	13
C			SAI	14
C	AUTHOR - A.D.PIERCE, M.I.T., JULY,1968		SAI	15
C			SAI	16
C	----	CALLING SEQUENCE----	SAI	17
C			SAI	18
C	SAI(ANY R*4 ARGUMENT) MAY BE USED IN ARITHMETIC EXPRESSIONS		SAI	19
C			SAI	20
C	----	EXTERNAL SUBROUTINES REQUIRED----	SAI	21
C			SAI	22
C	NO EXTERNAL SUBROUTINES ARE REQUIRED		SAI	23
C			SAI	24
C	----	ARGUMENT LIST----	SAI	25
C			SAI	26
C	X R*4 NO INP		SAI	27
C	SAI R*4 NO OUT		SAI	28
C			SAI	29
C	NO COMMON STORAGE IS USED		SAI	30
C			SAI	31
C	----	PROGRAM FOLLOWS BELOW----	SAI	32
C			SAI	33
C	FUNCTION SAI(X)		SAI	34
C			SAI	35
C	1 IF(ABS(X) .GT. 1.E-15) GO TO 9		SAI	36
C			SAI	37
C	ABS(X) IS SO SMALL THAT SAI IS VIRTUALLY 1.0		SAI	38
C	SAI=1.0		SAI	39
C	RETURN		SAI	40
C			SAI	41
C	CONTINUING FROM 1		SAI	42
C	9 Y=SORT(ABS(X))		SAI	43
C	IF(X) 10,10,11		SAI	44
C			SAI	45
C	X IS LESS THAN 0.		SAI	46
C	10 SAI=SINH(Y)/Y		SAI	47
C	RETURN		SAI	48
C			SAI	49
C	X IS POSITIVE. SAI= SINH(Y)/Y.		SAI	50
C	11 E=EXP(Y)		SAI	51
C	SAI=0.5*(E-1./E)/Y		SAI	52
C	RETURN		SAI	53
C	END		SAI	54

PROGRAM
SAI

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C	SOURCE (SUBROUTINE)	8/15/68	SRCE	1
C			SRCE	2
C			SRCE	3
C	-----ABSTRACT-----		SRCE	4
C			SRCE	5
C	TITLE - SOURCE		SRCE	6
C	EVALUATION OF FOURIER TRANSFORM OF NEAR FIELD ACOUSTIC RESPONSE		SRCE	7
C	TO EXPLOSIVE SOURCE		SRCE	8
C			SRCE	9
C	SOURCE COMPUTES THE FOURIER TRANSFORM OF THE NEAR FIELD		SRCE	10
C	PRESSURE AT 1 KM FROM A 1 KT EXPLOSION AT SEA LEVEL. THE		SRCE	11
C	AMBIENT PRESSURE IS ASSUMED TO BE 1.E6 DYNES/CM**2 AND		SRCE	12
C	THE TIME LAPSE FROM TIME ZERO IS NEGLECTED. AN EMPIRICAL		SRCE	13
C	FORMULA FOR THIS PRESSURE IS		SRCE	14
C			SRCE	15
C	$F(T) = P_{AS} * (1 - (T/T_{AS})) * \exp(-T/T_{AS}) \quad , \quad T \geq T_{AS}$		SRCE	16
C	$= 0 \quad , \quad T < T_{AS}$		SRCE	17
C			SRCE	18
C	WITH $P_{AS} = (34.45E+3) * (1.61)$ DYNES/CM**2		SRCE	19
C	AND $T_{AS} = 0.48$ SEC .		SRCE	20
C			SRCE	21
C	THEREFORE, ITS FOURIER TRANSFORM IS		SRCE	22
C			SRCE	23
C	$F(\Omega) = -I * \Omega * P_{AS} / (1/T_{AS} - I * \Omega)**2$		SRCE	24
C			SRCE	25
C	WHERE $I = (-1)**0.5$.		SRCE	26
C			SRCE	27
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		SRCE	28
C			SRCE	29
C	AUTHORS - A.D.PIERCE AND J.POSEY, M.I.T., AUGUST, 1968		SRCE	30
C			SRCE	31
C			SRCE	32
C	-----USAGE-----		SRCE	33
C			SRCE	34
C	SUBROUTINE PHASE IS CALLED		SRCE	35
C			SRCE	36
C	FORTRAN USAGE		SRCE	37
C			SRCE	38
C	CALL SOURCE(OMEGA,FTMAG,FTPHSE,DMAG,DPHSE)		SRCE	39
C			SRCE	40
C	INPUTS		SRCE	41
C			SRCE	42
C	OMEGA ANGULAR FREQUENCT (RADIAN/SEC)		SRCE	43
C	R*4		SRCE	44
C			SRCE	45
C	OUTPUTS		SRCE	46
C			SRCE	47
C	FTMAG MAGNITUDE OF F(OMEGA) DEFINED ABOVE IN SUBROUTINE ABSTRA		SRCE	48
C	R*4 ((DYNES/CM**2) / (RAD/SEC))		SRCE	49
C			SRCE	50
C	FTPHSE PHASE OF F(OMEGA) DEFINED ABOVE IN SUBROUTINE ABSTRACT		SRCE	51
C	R*4 (RADIAN)		SRCE	52
C			SRCE	53
C	DMAG DERIVATIVE OF FTMAG WITH RESPECT TO OMEGA ((DYNES/CM**2)		SRCE	54
C	R*4 / (RAD/SEC)**2)		SRCE	55
C			SRCE	56
C	DPHSE DERIVATIVE OF FTPHSE WITH RESPECT TO OMEGA (RAD / (RAD/		SRCE	57
C	R*4 SEC))		SRCE	58
C			SRCE	59
C			SRCE	60
C	-----PROGRAM FOLLOWS BELOW-----		SRCE	61
C			SRCE	62
C			SRCE	63
C			SRCE	64
C	SUBROUTINE SOURCE(OMEGA,FTMAG,FTPHSE,DMAG,DPHSE)		SRCE	65
C	WE ASSUME INVERSE R DEPENDENCE		SRCE	66
C	PAS=(34.45E+3/1.01)*(1.61)		SRCE	67
C	PAS IS IN DYNES/CM**2		SRCE	68
C	THIS IS THE PEAK OVERPRESSURE AT 1 KM		SRCE	69
C	TAS=0.48		SRCE	70

PROGRAM
SOURCE

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C	TAS IS THE LENGTH OF THE POSITIVE PHASE	SRCE	71
	OMO=1.0/TAS	SRCE	72
	DENOM=OMEGA**2+OMO**2	SRCE	73
	FTMAG=PAS*OMEGA/DENOM	SRCE	74
	DMAG=PAS/DENOM-2.0*PAS*OMEGA**2/DENOM**2	SRCE	75
	CALL PHASE(OMO,OMEGA,X,PHI)	SRCE	76
C	PHI IS THE ARCTAN OF OMEGA/OMO	SRCE	77
	FTPHSF=-3.1415927/2.0+2.0*PHI	SRCE	78
	OPHSE=2.0*OMO/DENOM	SRCE	79
C	THE DERIVATIVE OF THE ARCTAN IS 1./(1.+Y**2)	SRCE	80
	RETURN	SRCE	81
	END	SRCE	82

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C	SUSPCT (SUBROUTINE)	7/19/68	SPCT	1	
C			SPCT	2	
C			SPCT	3	
C	-----ABSTRACT-----		SPCT	4	
C			SPCT	5	
C	TITLE - SUSPCT		SPCT	6	
C	EVALUATION OF SUSPICION INDEX OF ELEMENT (N,M) OF MATRIX INMODE		SPCT	7	
C			SPCT	8	
C	SUSPCT EVALUATES THE SUSPICION INDEX, ISUS, OF THE ELEMEN		SPCT	9	
C	IN ROW N, COLUMN M OF THE MATRIX INMODE ((N,M) MUST BE		SPCT	10	
C	AN INTERIOR ELEMENT). THE NEIGHBORS OF (N,M) ARE DEFINED		SPCT	11	
C	TO BE THE EIGHT ELEMENTS WHICH FORM THE THREE BY THREE		SPCT	12	
C	ELEMENT SQUARE WHICH HAS (N,M) AT ITS CENTER. THEY ARE		SPCT	13	
C	NUMBERED FROM ONE TO NINE BEGINNING IN THE UPPER LEFT AND		SPCT	14	
C	PROCEEDING CLOCKWISE (NO. 1 AND NO. 9 ARE SAME ELEMENT).		SPCT	15	
C	EACH ELEMENT OF MATRIX INMODE MUST HAVE ONE OF THREE		SPCT	16	
C	VALUES, -1, 1, OR 5. (N,M) IS NOT SUSPICIOUS AND ISUS =		SPCT	17	
C	0 IF ANY ONE OF THE FOLLOWING CONDITIONS HOLDS.		SPCT	18	
C			SPCT	19	
C	1. ELEMENT (N,M) = 5		SPCT	20	
C	2. ANY OF ITS NEIGHBORS = 5		SPCT	21	
C	3. NOWHERE IN THE 3X3 ARRAY OF (N,M) AND ITS NEIGH-		SPCT	22	
C	BORS DOES THERE APPEAR TO BE A DISPERSION CURVE		SPCT	23	
C	WITH POSITIVE SLOPE		SPCT	24	
C			SPCT	25	
C	OTHERWISE ISUS IS SET EQUAL TO THE NUMBER OF THE QUADRANT		SPCT	26	
C	IN WHICH THE POSITIVE SLOPE APPEARS. THE QUADRANTS ARE		SPCT	27	
C	NUMBERED BEGINNING IN THE UPPER LEFT AND PROCEEDING CLOCK-		SPCT	28	
C	WISE.		SPCT	29	
C			SPCT	30	
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		SPCT	31	
C			SPCT	32	
C	AUTHORS - A.C. PIERCE AND J. POSEY, M.I.T., JUNE, 1968		SPCT	33	
C			SPCT	34	
C	-----USAGE-----		SPCT	35	
C			SPCT	36	
C	NO FORTRAN SUBROUTINES ARE CALLED		SPCT	37	
C			SPCT	38	
C	FORTRAN USAGE		SPCT	39	
C	CALL SUSPCT(N,M,NROW,INMODE,ISUS)		SPCT	40	
C			SPCT	41	
C	INPUTS		SPCT	42	
C			SPCT	43	
C	N ROW NUMBER OF ELEMENT UNDER CONSIDERATION (MAY NOT BE		SPCT	44	
C	I*4 FIRST OR LAST ROW)		SPCT	45	
C			SPCT	46	
C	M COLUMN NUMBER OF ELEMENT UNDER CONSIDERATION (MAY NOT BE		SPCT	47	
C	I*4 FIRST OR LAST COLUMN)		SPCT	48	
C			SPCT	49	
C	NROW TOTAL NUMBER OF ROWS IN INMODE		SPCT	50	
C	I*4		SPCT	51	
C			SPCT	52	
C	INMODE MATRIX UNDER CONSIDERATION STORED IN VECTOR FORM, COLUMN		SPCT	53	
C	I*4(D) AFTER COLUMN. EACH ELEMENT MUST BE -1, 1, OR 5.		SPCT	54	
C			SPCT	55	
C	OUTPUTS		SPCT	56	
C			SPCT	57	
C	ISUS SUSPICION INDEX OF ELEMENT (N,M). SEE ABSTRACT ABOVE FOR		SPCT	58	
C	I*4 DEFINITION.		SPCT	59	
C			SPCT	60	
C			SPCT	61	
C	-----EXAMPLES-----		SPCT	62	
C			SPCT	63	
C	CALLING PROGRAM		SPCT	64	
C			SPCT	65	
C	DIMENSION INMODE(9)		SPCT	66	PROGRAM
C	INMODE = -1, -1, 1, 1, -1, 1, 1, 1, -1		SPCT	67	SUSPCT
C	CALL SUSPCT(2,3,INMODE,ISUS)		SPCT	68	
C	WRITE (6,200) ISUS		SPCT	69	PAGE
C	200 FORMAT (10H EXAMPLE 1,6X, 6HISUS =,12)		SPCT	70	74

C	INMODE = -1, -1, 1, 1, -1, -1, 1, 1, 1	SPCT 71
C	CALL SUSPCT(2,2,3,INMODE,ISUS)	SPCT 72
C	WRITE (6,300) ISUS	SPCT 73
C	300 FORMAT (10H EXAMPLE 2,6X, 6HISUS =,12)	SPCT 74
C	END	SPCT 75
C		SPCT 76
C	TABLES OF INMODE	SPCT 77
C		SPCT 78
C	EXAMPLE 1 EXAMPLE 2	SPCT 79
C		SPCT 80
C	--+	SPCT 81
C	--+	SPCT 82
C	++-	SPCT 83
C		SPCT 84
C	PRINTOUT	SPCT 85
C		SPCT 86
C	EXAMPLE 1 ISUS = 3	SPCT 87
C	EXAMPLE 2 ISUS = 0	SPCT 88
C		SPCT 89
C		SPCT 90
C	-----PROGRAM FOLLOWS BELOW-----	SPCT 91
C		SPCT 92
C	SUBROUTINE SUSPCT(N,M,NROW,INMODE,ISUS)	SPCT 93
C		SPCT 94
C	VARIABLE DIMENSIONING OF INMODE	SPCT 95
C	DIMENSION IPP(9),IQUA(4),INMODE(1)	SPCT 96
C		SPCT 97
C	ELEMENT (N,M) OF INMODE IS ICEN	SPCT 98
C	ICEN= INMODE((M-1)*NROW+N)	SPCT 99
C	ISUS= 0	SPCT 100
C		SPCT 101
C	IF ICEN IS 4, IT IS NOT SUSPICIOUS AND ISUS = 0	SPCT 102
C	IF(ICEN .EQ. 5) RETURN	SPCT 103
C		SPCT 104
C	IPPIN) IS NEIGHBOR NO. N (SEE ABSTRACT ABOVE FOR NUMBERING SCHEME)	SPCT 105
C	IPP(1)= INMODE((M-2)*NROW+(N-1))	SPCT 106
C	IPP(2)= INMODE((M-1)*NROW+(N-1))	SPCT 107
C	IPP(3)= INMODE((M-0)*NROW+(N-1))	SPCT 108
C	IPP(4)= INMODE((M-0)*NROW+(N-0))	SPCT 109
C	IPP(5)= INMODE((M-0)*NROW+(N+1))	SPCT 110
C	IPP(6)= INMODE((M-1)*NROW+(N+1))	SPCT 111
C	IPP(7)= INMODE((M-2)*NROW+(N+1))	SPCT 112
C	IPP(8)= INMODE((M-2)*NROW+(N+0))	SPCT 113
C	IPP(9)= IPP(1)	SPCT 114
C	NX = 0	SPCT 115
C	DO 10 I=1,8	SPCT 116
C	IF(IPP(I) .EQ. 5) NX=NX+1	SPCT 117
C	10 CONTINUE	SPCT 118
C	NX IS THE NUMBER OF NEIGHBORS WHICH EQUAL +5	SPCT 119
C		SPCT 120
C	IF MORE THAN ONE NEIGHBOR IS EQUAL TO +5, THEN ISUS=0	SPCT 121
C	IF (NX .GT. 1) RETURN	SPCT 122
C		SPCT 123
C	IF NEIGHBOR 3 IS THE ONLY ONE EQUAL TO +5 AND EITHER NEIGHBOR 2 OR	SPCT 124
C	NEIGHBOR 4 DOES NOT AGREE WITH ICEN, THEN ISUS=2	SPCT 125
C	ISUM = IABS(ICEN + IPP(2) + IPP(4))	SPCT 126
C	IF (IPP(3).EQ.5 .AND. ISUM.NE.3) ISUS=2	SPCT 127
C	IF (NX.GT.0) RETURN	SPCT 128
C	30 DO 50 I=1,9	SPCT 129
C	50 IPP(I)=(IABS(IPP(I)+ICEN))/2	SPCT 130
C	IPP(I) IS 1 IF NEIGHBOR I AGREES WITH ICEN, IT IS 0 IF THEY DISAGREE	SPCT 131
C	(TO REACH THIS POINT, NEITHER ICEN NOR ANY OF ITS NEIGHBORS COULD BE 5	SPCT 132
C		SPCT 133
C		SPCT 134
C	ISUS = 1	SPCT 135
C	IF(IPP(1) .EQ. 0 .AND. IPP(7) .EQ. 1 .AND. IPP(8) .EQ. 1)	SPCT 136
C	1 RETURN	SPCT 137
C	IF(IPP(8) .EQ. 0 .AND. IPP(2) .EQ. 0) RETURN	SPCT 138
C	ISUS = 2	SPCT 139
C	IF(IPP(7) .EQ. 0 .AND. IPP(3) .EQ. 1) RETURN	SPCT 140

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IF( IPP(3) .EQ. 1 .AND. IPP(4) .EQ. 0) RETURN
ISUS = 3
IF( IPP(5) .EQ. 0 .AND. IPP(4) .EQ. 1 .AND. IPP(6) .EQ. 1)
1 RETURN
IF( IPP(4) .EQ. 0 .AND. IPP(6) .EQ. 0) RETURN
ISUS = 4
IF( IPP(6) .EQ. 0 .AND. IPP(7) .EQ. 1) RETURN
IF( IPP(7) .EQ. 1 .AND. IPP(8) .EQ. 0) RETURN
ISUS = 0
RETURN
END

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SPCT 141
SPCT 142
SPCT 143
SPCT 144
SPCT 145
SPCT 146
SPCT 147
SPCT 148
SPCT 149
SPCT 150
SPCT 151

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C	TABLE (SUBROUTINE)	7/19/68	TABL	1
C			TABL	2
C			TABL	3
C	-----ABSTRACT-----		TABL	4
C			TABL	5
C	TITLE - TABLE		TABL	6
C	GENERATION OF SUSPICIONLESS TABLE OF NORMAL MODE DISPERSION		TABL	7
C	FUNCTION SIGNS		TABL	8
C			TABL	9
C	TABLE CALLS SUBROUTINE MPOUT TO CONSTRUCT THE MATRIX OF		TABL	10
C	NORMAL MODE DISPERSION FUNCTION SIGNS INMODE (STORED IN		TABL	11
C	VECTOR FORM COLUMN AFTER COLUMN) FOR REGION IN FREQUENCY-		TABL	12
C	PHASE VELOCITY PLANE (OM1.LE.OMEGA.LE.OM2.AND.V1.LE.VP.LE		TABL	13
C	.V2). SUBROUTINE SUSPCT IS CALLED TO EVALUATE THE SUSPI-		TABL	14
C	TION INDEX ,ISUS, OF EACH INTERIOR ELEMENT IN THE MATRIX		TABL	15
C	SCANNING FROM LEFT TO RIGHT, TOP TO BOTTOM. IF ISUS .NE.		TABL	16
C	0 , INMODE IS ALTERED AS FOLLOWS.		TABL	17
C	ISUS=1 ROW ADDED ABOVE SUSPICIOUS ELEMENT AND COLUMN		TABL	18
C	ADDED TO ITS LEFT		TABL	19
C	=2 COLUMN ADDED TO RIGHT OF SUSPICIOUS ELEMENT		TABL	20
C	AND ROW ADDED ABOVE IT		TABL	21
C	=3 ROW ADDED BELOW SUSPICIOUS ELEMENT AND COLUMN		TABL	22
C	ADDED TO ITS RIGHT		TABL	23
C	=4 COLUMN ADDED TO LEFT OF SUSPICIOUS ELEMENT		TABL	24
C	AND ROW ADDED BELOW IT		TABL	25
C	HOWEVER, NEITHER THE NUMBER OF ROWS NVP NOR THE NUMBER OF		TABL	26
C	COLUMNS NOM WILL BE INCREASED BEYOND 100. IF ISUS CALLS		TABL	27
C	FOR AN ADDITIONAL ROW WHEN NVP = 100 , THE MESSAGE		TABL	28
C	(NVP = 100 N = XX M = XX) WILL BE PRINTED.		TABL	29
C	N IS ROW NO. OF SUSPICIOUS ELEMENT. M IS COLUMN NO. IF		TABL	30
C	ISUS CALLS FOR ADDITION OF A COLUMN WHEN NOM = 100, THE		TABL	31
C	MESSAGE (NOM = 100 N = XX M = XX) IS PRINTED		TABL	32
C	WHEN INMODE HAS BEEN EXPANDED SCANNING IS RESUMED AT THE		TABL	33
C	ELEMENT IN NEW MATRIX WITH SAME ROW AND COLUMN NOS. AS		TABL	34
C	THOSE OF SUSPICIOUS ELEMENT IN OLD MATRIX. IF NOPT IS		TABL	35
C	POSITIVE INMODE WILL BE PRINTED AS IT IS RETURNED FROM		TABL	36
C	MPOUT AND IN ITS FINAL FORM.		TABL	37
C			TABL	38
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL - C28-6515-4)		TABL	39
C			TABL	40
C	AUTHOR - J.W.POSEY, M.I.T., JUNE, 1968		TABL	41
C			TABL	42
C			TABL	43
C	-----USAGE-----		TABL	44
C			TABL	45
C	SUBROUTINES MPOUT,SUSPCT,LANGTHN,WIDEN,NMODN ARE CALLED IN TABLE.		TABL	46
C			TABL	47
C	FORTRAN USAGE		TABL	48
C	CALL TABLE(OM1,OM2,V1,V2,NOM,NVP,THETK,OM,V,INMODE,NOPT)		TABL	49
C			TABL	50
C	INPUTS		TABL	51
C			TABL	52
C	OM1 MINIMUM VALUE OF FREQUENCY TO BE CONSIDERED.		TABL	53
C	R*4		TABL	54
C	OM2 MAXIMUM VALUE OF FREQUENCY TO BE CONSIDERED		TABL	55
C	R*4		TABL	56
C	V1 MINIMUM VALUE OF PHASE VELOCITY TO BE CONSIDERED		TABL	57
C	R*4		TABL	58
C	V2 MAXIMUM VALUE OF PHASE VELOCITY TO BE CONSIDERED		TABL	59
C	R*4		TABL	60
C	NOM INITIAL NO. OF FREQUENCIES TO BE CONSIDERED		TABL	61
C	I*4		TABL	62
C	NVP INITIAL NO. OF PHASE VELOCITIES TO BE CONSIDERED		TABL	63
C	I*4		TABL	64
C	THETK PHASE VELOCITY DIRECTION (RADIAN)		TABL	65
C	R*4		TABL	66
C	NOPT PRINT OUT OPTION. IF NOPT = -1, NO PRINT. IF NOPT = 1,		TABL	67
C	I*4 INMODE IS PRINTED IN ITS INITIAL FORM (GENERATED BY MPOUT		TABL	68
C	AND IN ITS FINAL FORM.		TABL	69
C			TABL	70
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C	OUTPUTS	TABL	71
C		TABL	72
C	NOM TOTAL NO. OF FREQUENCIES CONSIDERED	TABL	73
C	I*4	TABL	74
C	NVP TOTAL NO. OF PHASE VELOCITIES CONSIDERED	TABL	75
C	I*4	TABL	76
C	OM VECTOR WHOSE ELEMENTS ARE THE VALUES OF ANGULAR FREQUENCY	TABL	77
C	R*4(D) CORRESPONDING TO THE COLUMNS OF THE INMODE MATRIX	TABL	78
C		TABL	79
C	V VECTOR WHOSE ELEMENTS ARE THE VALUES OF PHASE VELOCITY	TABL	80
C	R*4(D) CORRESPONDING TO THE ROWS OF THE INMODE MATRIX	TABL	81
C		TABL	82
C	INMODE EACH ELEMENT OF THIS MATRIX CORRESPONDS TO A POINT IN THE	TABL	83
C	I*4(D) FREQUENCY (OM) - PHASE VELOCITY (V) PLANE. IF THE NORMAL	TABL	84
C	MODE DISPERSION FUNCTION (FPP) IS POSITIVE AT THAT POINT,	TABL	85
C	THE ELEMENT IS +1. IF FPP IS NEGATIVE, THE ELEMENT IS -1.	TABL	86
C	IF FPP DOES NOT EXIST, THE ELEMENT IS 5. INMODE HAS NVP	TABL	87
C	ROWS AND NOM COLUMNS. MATRIX IS STORED AS A VECTOR,	TABL	88
C	COLUMN AFTER COLUMN.	TABL	89
C		TABL	90
C		TABL	91
C	-----EXAMPLE-----	TABL	92
C		TABL	93
C	LET INMODE = -1,5,5,5,1,-1,-1,-1,1,1,-1,-1,1,1,1	TABL	94
C	WITH NOM = NVP = 4	TABL	95
C	AND OM = 1.0,1.5,2.0,2.5 THETK = 3.14159	TABL	96
C	V = 1.0,2.0,3.0,4.0	TABL	97
C	(VALUES NOT CORRECT, FOR ILLUSTRATION ONLY)	TABL	98
C		TABL	99
C	THEN THE TABLE WILL BE PRINTED AS FOLLOWS.	TABL	100
C		TABL	101
C	VPHASE NORMAL MODE DISPERSION FUNCTION SIGN	TABL	102
C	1.00000 -+++	TABL	103
C	2.00000 X-++	TABL	104
C	3.00000 X--+	TABL	105
C	4.00000 X--+	TABL	106
C		TABL	107
C	OMEGA 1234	TABL	108
C	PHASE VELOCITY DIRECTION IS 90.000DEGREES	TABL	109
C		TABL	110
C	OMEGA =	TABL	111
C	0.10000E 01 0.15000E 01 0.20000E 01 0.25000E 01	TABL	112
C		TABL	113
C		TABL	114
C	-----PROGRAM FOLLOWS BELOW-----	TABL	115
C		TABL	116
C	SUBROUTINE TABLE(OM1,OM2,V1,V2,NOM,NVP,THETK,OM,V,INMODE,NOPT)	TABL	117
C		TABL	118
C	DIMENSION OM(100),V(100),INMODE(10000),DORN(100),KORN(100)	TABL	119
C	COMMON IMAX,CI(100),VXI(100),VYI(100),HI(100)	TABL	120
C		TABL	121
C	MPOUT IS CALLED TO PRODUCE INMODE MATRIX AND OM AND V VECTORS.	TABL	122
C	CALL MPOUT(OM1,OM2,V1,V2,NOM,NVP,INMODE,OM,V,THETK)	TABL	123
C		TABL	124
C	IFLAG = 1 INDICATES FIRST TIME THROUGH WRITE PROCEDURE	TABL	125
C	IFLAG = 1	TABL	126
C		TABL	127
C	INMODE IS PRINTED IF NOPT IS POSITIVE	TABL	128
C	IF (NOPT.GE.0) GO TO 123	TABL	129
C	5 IFLAG = 0	TABL	130
C	NOPER=0	TABL	131
C		TABL	132
C	NOPER IS THE NUMBER OF EXPANSION OPERATIONS PERFORMED IN THE PRESENT	TABL	133
C	SCAN OF THE MATRIX. THUS, NOPER IS THE NUMBER OF SUSPICIOUS POINTS	TABL	134
C	FOUND IN THE PRESENT SCAN.	TABL	135
C		TABL	136
C	BEGIN SCANNING OF INTERIOR ELEMENTS OF INMODE IN UPPER LEFT CORNER	TABL	137
C	N = 2	TABL	138
C	M = 2	TABL	139
C	10 CALL SUSPCT(N,M,NVP,INMODE,ISUS)	TABL	140

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C POINT (N,M) IS SUSPICIOUS IF ISUS.NE.0	TABL 141	
IF (ISUS.NE.0) GO TO 60	TABL 142	
C CHECK FOR END OF ROW	TABL 143	
20 IF (M.LT.(NOM-1)) GO TO 30	TABL 144	
C CHECK FOR LAST ROW	TABL 145	
IF (M.LT.(NVP-1)) GO TO 40	TABL 146	
GO TO 121	TABL 147	
C MOVE ONE COLUMN TO RIGHT	TABL 148	
30 M = M+1	TABL 149	
GO TO 10	TABL 150	
C ADVANCE ONE ROW AND START AT COLUMN TWO	TABL 151	
40 N = N+1	TABL 152	
M = 2	TABL 153	
GO TO 10	TABL 154	
C CHECK FOR MAXIMUM VALUE OF NVP	TABL 155	
60 IF (NVP.LT.100) GO TO 62	TABL 156	
61 FORMAT (24H NVP = 100	TABL 157	N = ,13,8H M = ,13)
WRITE (6,61) N,M	TABL 158	
GO TO 20	TABL 159	
62 IF (NOM.LT. 100) GO TO 70	TABL 160	
63 FORMAT (24H NOM = 100	TABL 161	N = ,13, 8H M = ,13)
64 WRITE (6,63) N,M	TABL 162	
GO TO 20	TABL 163	
70 IF (ISUS .NE. 1) GO TO 75	TABL 164	
C ADD ROW ABOVE SUSPICIOUS POINT	TABL 165	
N1=N-1	TABL 166	
C ADD A COLUMN TO LEFT OF SUSPICIOUS POINT	TABL 167	
M1=M-1	TABL 168	
GO TO 100	TABL 169	
75 IF (ISUS .NE. 2) GO TO 80	TABL 170	
C ADD A COLUMN TO RIGHT OF SUSPICIOUS POINT	TABL 171	
M1=M	TABL 172	
C ADD ROW ABOVE SUSPICIOUS POINT	TABL 173	
N1=N-1	TABL 174	
GO TO 100	TABL 175	
80 IF (ISUS .NE. 3) GO TO 85	TABL 176	
C ADD A COLUMN TO RIGHT OF SUSPICIOUS POINT	TABL 177	
M1=M	TABL 178	
C ADD ROW ABOVE SUSPICIOUS POINT	TABL 179	
N1=N-1	TABL 180	
GO TO 100	TABL 181	
85 IF (ISUS .NE. 4) GO TO 90	TABL 182	
C ADD A COLUMN TO RIGHT OF SUSPICIOUS POINT	TABL 183	
M1=M	TABL 184	
C ADD ROW BELOW SUSPICIOUS POINT	TABL 185	
N1=N	TABL 186	
GO TO 100	TABL 187	
C ADD ROW BELOW SUSPICIOUS POINT	TABL 188	
85 N1=N	TABL 189	
C ADD A COLUMN TO LEFT OF SUSPICIOUS POINT	TABL 190	
M1=M-1	TABL 191	
100 CONTINUE	TABL 192	
CALL LNGTHN(OM,V,INMODE,NOM,NVP,NVPP,M1,1,THETK)	TABL 193	
CALL WIDEN(OM,V,INMODE,NOM,NOMP,NVPP,M1,1,THETK)	TABL 194	
NVP=NVPP	TABL 195	
NOM=NOMP	TABL 196	
NOPER=NOPER+1	TABL 197	
GO TO 10	TABL 198	
121 CONTINUE	TABL 199	
IF (INOPER .NE. 0 .AND. NVP .LT. 100 .AND. NOM .LT. 100) GO TO 5	TABL 200	
C DO NOT PRINT INMODE IF NOPT IS NEGATIVE	TABL 201	
	TABL 202	
	TABL 203	
	TABL 204	
	TABL 205	
	TABL 206	
	TABL 207	
	TABL 208	
	TABL 209	
	TABL 210	

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IF(NOPT .LT. 0) RETURN	TABL 211
C LABELING	TABL 212
122 FORMAT (6H1VPHSF,6X,36HNORMAL MODE DISPERSION FUNCTION SIGN/)	TABL 213
123 WRITE (6,122)	TABL 214
DO 133 I=1,NVP	TABL 215
DO 128 J=1,NOM	TABL 216
IF (INMODE((J-1)*NVP+I)-1) 126,125,124	TABL 217
124 CONTINUE	TABL 218
C	TABL 219
C IF INMODE = 5, DORN = 1H+	TABL 220
DATA Q1/1H+	TABL 221
DORN(J) = Q1	TABL 222
GO TO 127	TABL 223
125 CONTINUE	TABL 224
C	TABL 225
C IF INMODE = 1, DORN = 1H+	TABL 226
DATA Q2/1H+	TABL 227
DORN(J) = Q2	TABL 228
GO TO 127	TABL 229
126 CONTINUE	TABL 230
C	TABL 231
C IF INMODE = -1, DORN = 1H-	TABL 232
DATA Q3/1H-	TABL 233
DORN(J) = Q3	TABL 234
127 CONTINUE	TABL 235
128 CONTINUE	TABL 236
C	TABL 237
C PRINT ROW I OF TABLE	TABL 238
WRITE (6,130)I,(DORN(J), J=1,NOM)	TABL 239
130 FORMAT(1H ,F9.5,3X,100A1)	TABL 240
133 CONTINUE	TABL 241
J10 = 10	TABL 242
DO 150 J=1,NOM	TABL 243
C	TABL 244
C NUMBER COLUMNS	TABL 245
150 KORN(J) = MOD(J,J10)	TABL 246
WRITE (6,213) (KORN(J), J=1,NOM)	TABL 247
213 FORMAT (6H00MEGA,6X,100I1)	TABL 248
C	TABL 249
C CONVERT THETA FROM RADIAN TO DEGREE	TABL 250
X = THETA*180/3.14159	TABL 251
WRITE (6,413) X	TABL 252
413 FORMAT (1H ,11X,27HPHASE VELOCITY DIRECTION IS,F9.3,	TABL 253
1 AHDEGREES)	TABL 254
WRITE (6,513)	TABL 255
513 FORMAT (AH00MEGA =)	TABL 256
C	TABL 257
C LIST VALUES OF OMEGA WHICH CORRESPOND TO COLUMNS OF TABLE	TABL 258
WRITE (6,613) (OM(I),I=1,NOM)	TABL 259
613 FORMAT (1H ,5E14.5)	TABL 260
C	TABL 261
C IF SUSPICION ELIMINATION HAS NOT BEEN PERFORMED, BEGIN IT AT THIS TIME	TABL 262
IF(IFLAG.FQ.1) GO TO 5	TABL 263
RETURN	TABL 264
END	TABL 265
	TABL 266

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C      TABPRT (SUBROUTINE)              7/31/68
C
C      -----ABSTRACT-----
C
C      TITLE - TABPRT
C      PROGRAM TO PRINT OUT LISTS OF FREQUENCY, PHASE VELOCITY,
C      AMPLITUDE, AND PHASE FOR EACH GUIDED MODE EXCITED BY A NUCLEAR
C      EXPLOSION OF GIVEN YIELD. THE SIMULTANEOUS LISTING OF FREQUENCY
C      AND PHASE VELOCITY REPRESENTS THE DISPERSION CURVE FOR THE
C      GUIDED MODE. THE QUANTITIES AMPLTD AND PHASE DEPEND ON SOURCE
C      AND OBSERVER HEIGHTS AS WELL AS THE MODEL ATMOSPHERE. HOWEVER,
C      THE LATTER INFORMATION IS NOT LISTED BY TABPRT AND IS PRESUMED
C      TO BE LISTED BY ANOTHER SUBROUTINE. THE SUBROUTINE TABPRT
C      SHOULD NOT BE CALLED UNTIL ALL THE QUANTITIES TO BE LISTED
C      HAVE BEEN COMPUTED AND STORED IN THE MACHINE. NORMALLY,
C      ATMOS, TABLE, ALLMOD, PAMPDE, AND PPAMP WOULD BE CALLED BEFORE
C      TABPRT.
C
C      LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C      AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., JULY, 1968
C
C      -----CALLING SEQUENCE-----
C
C      DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMPLTD(1),PHASQ(1)
C      THE SUBROUTINE USES VARIABLE DIMENSIONING. THE TRUE DIMENSIONS MUST
C      BE GIVEN IN THE PROGRAM WHICH DEFINES THESE QUANTITIES. SEE THE
C      DIMENSION STATEMENTS IN THE MAIN PROGRAM.
C      CALL TABPRT(YIELD,MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASQ)
C
C      NO EXTERNAL SUBROUTINES ARE REQUIRED
C
C      -----ARGUMENT LIST-----
C
C      YIELD      R*4      ND      INP
C      MDFND      I*4      ND      INP
C      KST        I*4      VAR      INP
C      KFIN       I*4      VAR      INP
C      OMMOD      R*4      VAR      INP
C      VPMOD      R*4      VAR      INP
C      AMPLTD     R*4      VAR      INP
C      PHASQ      R*4      VAR      INP
C
C      NO COMMON STORAGE USED
C
C      -----INPUTS-----
C
C      YIELD      =ENERGY YIELD OF EXPLOSION IN EQUIVALENT KILOTONS (KT)
C                  OF TNT. 1 KT = 4.2X(10)**19 ERGS.
C      MDFND      =NUMBER OF NORMAL MODES FOUND
C      KST(N)     =INDEX OF FIRST TABULATED POINT IN N-TH MODE
C      KFIN(N)    =INDEX OF LAST TABULATED POINT IN N-TH MODE. IN
C                  GENERAL, KFIN(N)=KST(N+1)-1.
C      OMMOD(N)   =ARRAY STORING ANGULAR FREQUENCY ORDINATE (RAD/SEC) OF
C                  POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORED
C                  FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      VPMOD(N)   =ARRAY STORING PHASE VELOCITY ORDINATE (KM/SEC) OF
C                  POINTS ON DISPERSION CURVES. THE NMODE MODE IS STORED
C                  FOR N BETWEEN KST(NMODE) AND KFIN(NMODE).
C      AMPLTD(N)  =AMPLITUDE FACTOR REPRESENTING TOTAL MAGNITUDE OF
C                  FOURIER TRANSFORM OF THE CONTRIBUTION TO THE WAVEFORM
C                  FROM A SINGLE GUIDED MODE AT FREQUENCY OMMOD(N).
C                  ITS UNITS SHOULD BE (DYNES/CM**2)*(KM**((1/2)))*SEC.
C                  IT REPRESENTS THE AMPLITUDE OF NMODE-TH MODE IF N IS
C                  BETWEEN KST(NMODE) AND KFIN(NMODE), INCLUSIVE. FOR
C                  PRECISE DEFINITION, SEE SUBROUTINE PPAMP.
C      PHASQ(N)   =PHASE LAG IN RADIAN AT FREQUENCY OMMOD(N) FOR THE
C                  NMODE-TH MODE WHEN N IS BETWEEN KST(NMODE) AND
C                  KFIN(NMODE), INCLUSIVE. THE INTEGRAND IS UNDERSTOOD
C                  TO HAVE THE FORM AMPLTD*COS(OMMOD*(TIME-DISTANCE/VPMO
C                  +PHASQ). FOR A PRECISE DEFINITION, SEE SUBROUTINE

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C          PPAMP.
C          ----OUTPUTS----
C PRINTOUT. THE ONLY FUNCTION OF TABPRT IS TO PRINT OUT RESULTS.
C          ----EXAMPLE----
C THE OUTPUT FORMAT IS ILLUSTRATED BELOW.
C          MODE TABULATION FOR Y= 100.00 KILOTONS
C
C          MODE 1
C          OMEGA      VPHSE      AMPLTD      PHASE
C          .00100      0.33426  -7.01342E 20  -3.72139
C          .00200      0.24372  -8.02394E 20  -4.56028
C
C          MODE 2
C          OMEGA      VPHSE      AMPLTD      PHASE
C          .00100      0.55298  -7.95321E 10  -2.40798
C          .00200      0.48321  -1.23108E 11  -2.30524
C
C ETC.
C          ----PROGRAM FOLLOWS BELOW----
C
C          SUBROUTINE TABPRT(YFLD,MDFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASQ)
C          VARIABLE DIMENSIONING IS USED
C          DIMENSION KST(1),KFIN(1),OMMOD(1),VPMOD(1),AMPLTD(1),PHASQ(1)
C
C          WRITE (6,11) YIELD
C          11 FORMAT( 1H1 ,1H ,25X,22HMODE TABULATION FOR Y=,F9.2,9H KILOTONS)
C
C          START OF OUTER DO LOOP
C          DO 50 I1=1,MDFND
C
C          WRITE (6,21) I1
C          21 FORMAT(1H ///,1H ,4X, 5HMODE ,I3//, 1H ,9X,5HOMEGA,9X,5HVPHSE,9X,
C          1 6HAMPLTD,9X,5HPHASE/ )
C
C          K1=KST(I1)
C          K2=KFIN(I1)
C
C          START OF INNER DO LOOP
C          DO 50 J=K1,K2
C
C          50 WRITE (6,51) OMMOD(J),VPMOD(J),AMPLTD(J),PHASQ(J)
C          51 FORMAT( 1H ,4X,F14.5,F14.5,1P614.5,0PF14.5)
C          END OF LOOPS
C
C          RETURN
C          END

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TABPR 72
TABPR 73
TABPR 74
TABPR 75
TABPR 76
TABPR 77
TABPR 78
TABPR 79
TABPR 80
TABPR 81
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TABPR 124
TABPR 125
TABPR 126
TABPR 127
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TABPR 129
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TABPR 131

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C	TMPT - SUBROUTINE	7/19/68	TMPT	1
C			TMPT	2
C			TMPT	3
C	-----ABSTRACT-----		TMPT	4
C			TMPT	5
C	TITLE - TMPT		TMPT	6
C	CALCULATION AND PLOTTING OF FAR-FIELD TRANSIENT RESPONSE TO A		TMPT	7
C	PRESSURE SOURCE IN THE ATMOSPHERE		TMPT	8
C			TMPT	9
C	THE RESPONSE OF MODE N IS FOUND BY INTEGRATING (AMPLD * COS(OMEGA * (T - R/VP) + PHASQ) OVER OMEGA FROM OMMOD		TMPT	10
C	(KST(N)) TO OMMOD(KFIN(N)) AND DIVIDING BY SORT(R). VP,		TMPT	11
C	PHASQ, AND AMPLD ARE FUNCTIONS OF BOTH N AND OMEGA. THE		TMPT	12
C	TOTAL RESPONSE IS THE SUM OF THE MODAL RESPONSES. THE		TMPT	13
C	RESPONSE IS CALCULATED FOR TIME TFIRST AND AT INTERVALS		TMPT	14
C	OF DELTT THEREAFTER UNTIL TFND IS REACHED. THE VALUE OF		TMPT	15
C	IOPT DETERMINES WHAT WILL BE CALCULATED, PRINTED AND		TMPT	16
C	PLOTTED. (SEE INPUT LIST FOR POSSIBLE IOPT VALUES.) THE		TMPT	17
C	RESULTS ARE TABULATED IN THE PRINTOUT AND GRAPHED BY THE		TMPT	18
C	CALCOMP PLOTTER.		TMPT	19
C			TMPT	20
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		TMPT	21
C	AUTHOR - J.W. POSEY, M.I.T., JUNE, 1968		TMPT	22
C			TMPT	23
C			TMPT	24
C	-----USAGE-----		TMPT	25
C			TMPT	26
C	FORTRAN SUBROUTINE AKI IS CALLED		TMPT	27
C			TMPT	28
C	CALCOMP PLOTTER SUBROUTINES PLOT1, AXIS1, NUMB1, SYMBL5, AND		TMPT	29
C	SCLGPH ARE CALLED TO WRITE THE CALCOMP TAPE. SUBROUTINE NEWP1T		TMPT	30
C	MUST HAVE BEEN CALLED PRIOR TO CALLING TMPT, AND ENDP1T MUST BE		TMPT	31
C	CALLED AFTER RETURNING FROM TMPT. (SEE MAIN PROGRAM)		TMPT	32
C			TMPT	33
C	FORTRAN USAGE		TMPT	34
C	CALL TMPT(TFIRST,TFND,DELTT,ROSS,MDFND,KST,KFIN,OMMOD,VPMOD,AMPLD)		TMPT	35
C	1 ,PHASQ,IOPT)		TMPT	36
C			TMPT	37
C	INPUTS		TMPT	38
C			TMPT	39
C	TFIRST TIME AT WHICH TABULATION AND PLOTTING OF RESPONSE IS TO		TMPT	40
C	R*4 BEGIN (SEC)		TMPT	41
C			TMPT	42
C	TFND TIME AT WHICH TABULATION AND PLOTTING OF RESPONSE IS TO		TMPT	43
C	R*4 END (.LE.(TFIRST+5400.)) (SEC)		TMPT	44
C			TMPT	45
C	DELTT TIME INTERVAL BETWEEN SUCCESSIVE CALCULATIONS OF THE		TMPT	46
C	R*4 RESPONSE (.GE.(TFND-TFIRST)/1000)) (SEC)		TMPT	47
C			TMPT	48
C	ROSS DISTANCE OF THE OBSERVER FROM THE SOURCE OF THE DISTUR-		TMPT	49
C	R*4 BANCE (KM)		TMPT	50
C			TMPT	51
C	MDFND NUMBER OF MODES FOUND (.LE.10)		TMPT	52
C	I*4		TMPT	53
C			TMPT	54
C	KST ELEMENT N OF THIS VECTOR IS NUMBER OF OMMOD ELEMENT WHICH		TMPT	55
C	I*4(N) IS FIRST FREQUENCY CONSIDERED FOR MODE N		TMPT	56
C			TMPT	57
C	KFIN ELEMENT N OF THIS VECTOR IS NUMBER OF OMMOD ELEMENT WHICH		TMPT	58
C	I*4(N) IS LAST FREQUENCY CONSIDERED FOR MODE N		TMPT	59
C			TMPT	60
C	OMMOD ELEMENTS OF THIS VECTOR NUMBERED KST(N) THROUGH KFIN(N)		TMPT	61
C	R*4(N) ARE THE VALUES OF FREQUENCY (IN INCREASING ORDER) FOR		TMPT	62
C	WHICH THE CORRESPONDING MODE N PHASE VELOCITIES HAVE BEEN		TMPT	63
C	DETERMINED		TMPT	64
C			TMPT	65
C	VPMOD VECTOR OF PHASE VELOCITIES WHICH CORRESPOND TO THE FRE-		TMPT	66
C	R*4(N) QUENCIES OF VECTOR OMMOD		TMPT	67
C			TMPT	68
C	AMPLD VALUES OF AMPLITUDE FUNCTION IN AKI INTEGRAL ELEMENTS		TMPT	69
C			TMPT	70

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C      R*4(D) CORRESPOND DIRECTLY TO ELEMENTS OF OMMOD (DYNES/CM**2)      TMPT 71
C      PHASQ TERM IN ARGUMENT OF COS IN AKI INTEGRAL WHICH IS INDEPEN-      TMPT 72
C      R*4(D) DENT OF TIME AND DISTANCE (ROBS)                             TMPT 73
C      TMPT COMPUTATION AND PRINT OPTION INDICATOR                         TMPT 74
C      I*4 = 1,2,....,10 CALCULATE, PRINT AND PLOT MODE NO. TOPT ONL      TMPT 75
C      = 11 CALCULATE, PRINT AND PLOT ALL MODES AS WELL AS THE           TMPT 76
C      TOTAL RESPONSE                                                       TMPT 77
C      = 12 CALCULATE ALL MODES, PRINT AND PLOT TOTAL RESPONSE           TMPT 78
C      ONLY                                                                  TMPT 79
C      TMPT 80
C      TMPT 81
C      TMPT 82
C      TMPT 83
C      TMPT 84
C      THE ONLY OUTPUTS ARE THE PRINTOUTS AND PLOTS CALLED FOR BY IOPT.    TMPT 85
C      ALL GRAPHS ARE DRAWN TO THE SAME SCALE. THE PRESSURE SCALE IS      TMPT 86
C      DETERMINED BY THE MAXIMUM AMPLITUDE OF THE TOTAL PRESSURE, AND THE  TMPT 87
C      TIME SCALE IS 600 SEC PER INCH. PRESSURE IS EXPRESSED IN DYNES/CM*  TMPT 88
C      TMPT 89
C      TMPT 90
C      TMPT 91
C      TMPT 92
C      TMPT 93
C      SUBROUTINE TMPT(TFIRST,TEND,DELT,ROBS,                               TMPT 94
C      1 MOFND,KST,KFIN,OMMOD,VPMOD,AMPLTD,PHASQ,IOPT)                   TMPT 95
C      TMPT 96
C      DIMENSION KST(10),KFIN(10),OMMOD(1000),VPMOD(1000),AMPLTD(1000),  TMPT 97
C      1 PHASQ(1000),T(1001),TOTINT(1001),TNINT(10,1001),Y(1001)      TMPT 98
C      TMPT 99
C      C YAX IS VECTOR OF LITERAL CONSTANTS. ELEMENT N IS THE EIGHT SPACE LABE  TMPT 100
C      FOR THE PRESSURE AXIS ON THE GRAPH OF THE MODE N RESPONSE.          TMPT 101
C      DOUBLE PRECISION YAX(10)                                           TMPT 102
C      DATA YAX/AM MODE 1 ,8H MODE 2 ,8H MODE 3 ,8H MODE 4 ,8H MODE 5 ,  TMPT 103
C      1 8H MODE 6 ,8H MODE 7 ,8H MODE 8 ,8H MODE 9 ,8H MODE 10/         TMPT 104
C      IF(IOPT.NE.11) GO TO 4                                              TMPT 105
C      WRITE(6,7)                                                         TMPT 106
C      2 FORMAT (1H1,40X,23HTABULATION OF RESPONSES//)                  TMPT 107
C      WRITE(6,7)                                                         TMPT 108
C      3 FORMAT (1H ,20X,4HTIME,12X,5HTOTAL,11X,6HMODE 1,10X,6HMODE 2,10X,  TMPT 109
C      1 6HMODE 3,10X,6HMODE 4,10X,6HMODE 5//)                          TMPT 110
C      4 IF(IOPT.EQ.12) WRITE(6,753)                                       TMPT 111
C      753 FORMAT (1H1,45X,40HTABULATION OF ACOUSTIC PRESSURE RESPONSE///1H , TMPT 112
C      1 48X,10HTIME (SEC),9X,15HP (DYNES/CM**2)///)                   TMPT 113
C      TMPT 114
C      C L IS NUMBER OF TIMES AT WHICH RESPONSE IS TO BE CALCULATED      TMPT 115
C      L = (TEND - TFIRST) / DELT + 1                                     TMPT 116
C      TMPT 117
C      C SIZE IS THE LENGTH OF THE TIME AXIS IN INCHES                    TMPT 118
C      SIZE = ( TEND - TFIRST ) / 600.0                                  TMPT 119
C      TMPT 120
C      C PRESET ALL RESPONSE VALUES TO 0.0                               TMPT 121
C      5 DO 7 K=1,L                                                       TMPT 122
C      TOTINT(K) = 0.0                                                    TMPT 123
C      DO 7 N=1,10                                                         TMPT 124
C      7 TNINT(N,K) = 0.0                                                 TMPT 125
C      TMPT 126
C      C SET UP TABLE OF TIMES BEGINNING AT TFIRST AND TAKING VALUES OF TIME AT  TMPT 127
C      INTERVALS OF DELT UNTIL TEND IS REACHED                           TMPT 128
C      9 DO 10 IT=1,L                                                     TMPT 129
C      10 T(IT) = TFIRST + (IT-1)*DELT                                     TMPT 130
C      TMPT 131
C      C BEGIN SET UP TO CALCULATE MODE 1 RESPONSE                       TMPT 132
C      N = 1                                                                TMPT 133
C      TMPT 134
C      C IF TOPT.LE.10 CALCULATE ONLY MODE IOPT RESPONSE                 TMPT 135
C      IF (TOPT.LE.10) N = IOPT                                           TMPT 136
C      11 NOST = KST(N) + 1                                               TMPT 137
C      NOFN = KFIN(N)                                                    TMPT 138
C      TMPT 139
C      C THE MODE N RESPONSE IS FOUND FOR ALL VALUES OF T BEFORE NEXT MODE IS  TMPT 140

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C CONSIDERED	TMPT 141
DO 51 IT=1,L	TMPT 142
C	TMPT 143
C SET A2,PH2 EQUAL TO VALUES FOR A1,PH1 IN FIRST INTEGRATION INTERVAL	TMPT 144
A2 = AMPLTD(KST(N))	TMPT 145
S2=OMMOD(KST(N))/VPMOD(KST(N))-PHASQ(KST(N))/ROBS	TMPT 146
SLOW=(IT)/ROBS	TMPT 147
DIDDLE=SLOW-1.0/VPMOD(KST(N))	TMPT 148
PH2=ROBS*(OMMOD(KST(N))*DIDDLE+PHASQ(KST(N))/ROBS)	TMPT 149
CTRIG2=COS(PH2)	TMPT 150
STRIG2=SIN(PH2)	TMPT 151
C	TMPT 152
C THE INTEGRAL OF (AMPLTD * COS(OMEGA * (T - ROBS/VP) + PHASQ)) OVER THE	TMPT 153
C INTERVAL FROM OMMOD(KST(N)) TO OMMOD(KFIN(N)) IS FOUND BY SUMMING THE	TMPT 154
C INTEGRALS FROM OMMOD(NOM-1) TO OMMOD(NOM) FOR NOM FROM KST(N)+1 TO	TMPT 155
C KFIN(N)	TMPT 156
DO 50 NOM = MOST,NOM	TMPT 157
A1 = A2	TMPT 158
PH1 = PH2	TMPT 159
CTRIG1=CTRIG2	TMPT 160
STRIG1=STRIG2	TMPT 161
S1=S2	TMPT 162
A2 = AMPLTD(NOM)	TMPT 163
S2=OMMOD(NOM)/VPMOD(NOM)-PHASQ(NOM)/ROBS	TMPT 164
DIDDLE=SLOW-1.0/VPMOD(NOM)	TMPT 165
PH2=ROBS*(OMMOD(NOM)*DIDDLE+PHASQ(NOM)/ROBS)	TMPT 166
OMEG1=OMMOD(NOM-1)	TMPT 167
OMEG2=OMMOD(NOM)	TMPT 168
DELPH = ROBS * (SLOW * (OMEG2 - OMEG1) - (S2 - S1))	TMPT 169
CALL AKI(OMEG1,OMEG2,A1,A2,CTRIG1,STRIG1,CTRIG2,STRIG2,	TMPT 170
1 DELPH,AKIINT)	TMPT 171
50 TNINT(N,IT) = TNINT(N,IT) + AKIINT	TMPT 172
C	TMPT 173
C PRESSURE IS FOUND TO (1 / SORT(ROBS)) * (VALUE OF OMEGA INTEGRAL)	TMPT 174
51 TNINT(N,IT) = (1/SORT(ROBS)) * TNINT(N,IT)	TMPT 175
C	TMPT 176
C IF IOPT.LE.10 ALL THAT IS REQUESTED IS THE MODE IOPT RESPONSE, WHICH	TMPT 177
C HAS JUST BEEN CALCULATED	TMPT 178
IF (IOPT.LE.10) GO TO 101	TMPT 179
C	TMPT 180
C INCREASE MODE NUMBER BY ONE	TMPT 181
N = N + 1	TMPT 182
C	TMPT 183
C IF N IS GREATER THAN MODND, ALL MODAL RESPONSES HAVE BEEN DETERMINED	TMPT 184
IF (N.LE.MODND) GO TO 11	TMPT 185
C	TMPT 186
C FOR EACH TIME IN T SET TOTAL PRESSURE EQUAL TO SUM OF MODAL PRESURES	TMPT 187
DO 60 IT=1,L	TMPT 188
DO 53 N = 1,MODND	TMPT 189
53 TOTINT(IT) = TOTINT(IT) + TNINT(N,IT)	TMPT 190
IF (IOPT.EQ. 11) GO TO 55	TMPT 191
C	TMPT 192
C WRITE TIME AND CORRESPONDING TOTAL ACOUSTIC RESPONSE (DYNES/CM**2)	TMPT 193
WRITE (6,54) T(IT),TOTINT(IT)	TMPT 194
54 FORMAT (1H ,49X,F9.1,10X,F12.2)	TMPT 195
C	TMPT 196
C WHEN IOPT.EQ.12 ONLY TOTAL RESPONSE IS PRINTED	TMPT 197
IF (IOPT.EQ.12) GO TO 59	TMPT 198
C	TMPT 199
C WHEN IOPT.EQ.11 ALL MODAL RESPONSES ARE ALSO PRINTED	TMPT 200
55 MM = MINO(MODND,5)	TMPT 201
WRITE (6,57) T(IT),TOTINT(IT), (TNINT(N,IT),N=1,MM)	TMPT 202
57 FORMAT (1H ,3X,14,10X,F9.1,5X,F12.2,4X,F12.2,4X,F12.2,4X,F12.2,	TMPT 203
1 4X,F12.2,4X,F12.2)	TMPT 204
59 CONTINUE	TMPT 205
60 CONTINUE	TMPT 206
IF (MODND .EQ. 5 .OR. IOPT .NE. 11) GO TO 65	TMPT 207
WRITE (6,61)	TMPT 208
61 FORMAT (1H0,20X,4HTIME,12X,5HTOTAL,11X,6HMODE 6,10X,6HMODE 7,10X,	TMPT 209
1 6HMODE 8,10X,6HMODE 9,10X,7HMODE 10/)	TMPT 210

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ON 63 IT=1.L	TMPT 211
63 WRITE (6,57) IT,T(I),TOTINT(I),(TNINT(N,IT),N=6,MDFND)	TMPT 212
C	TMPT 213
65 CONTINUE	TMPT 214
66 CALL PLOT1(2,..3,-3)	TMPT 215
SIZE = (T(I)-T(1))/600.	TMPT 216
IF (IOPT.LE.10) GO TO 107	TMPT 217
CALL DXDY1(TOTINT,L,3.0,UMIN,DY,ND,K)	TMPT 218
UMIN = AINT(UMIN/25) * 25.0	TMPT 219
UMIN = AMIN1(UMIN,-25.)	TMPT 220
DY = ABS(UMIN)	TMPT 221
C	TMPT 222
C IF IOPT.EQ.12 PLOT ONLY THE TOTAL ACOUSTIC RESPONSE	TMPT 223
IF (IOPT.EQ.12) GO TO 70	TMPT 224
C	TMPT 225
C DRAW PRESSURE AXIS	TMPT 226
CALL PLOT1(0..0..3)	TMPT 227
ABC = MDFND	TMPT 228
CALL PLOT1(ARC ,0..2)	TMPT 229
ON 69 N=1,MDFND	TMPT 230
ON 67 J=1.L	TMPT 231
67 V(J) = -1 * TNINT(N,J)	TMPT 232
68 CALL PLOT1(1..0..-3)	TMPT 233
C	TMPT 234
C PLOT ACOUSTIC RESPONSE (DYNES/CM**2) OF MODE N VERSUS TIME (SEC)	TMPT 235
69 CALL SCLGPH(V,T,L,0..1,0..DY,T(1),600.)	TMPT 236
C	TMPT 237
70 ON 73 J=1.L	TMPT 238
73 V(J) = (-1) * TOTINT(J)	TMPT 239
C	TMPT 240
C DRAW PRESSURE AXIS	TMPT 241
75 CALL PLOT1(0..0..3)	TMPT 242
CALL PLOT1(3..0..2)	TMPT 243
CALL PLOT1(1.5,0..-3)	TMPT 244
CALL NUMBRI(4,-.15,.15,DY,180..0)	TMPT 245
CALL SYMBL(5(4,-.15,.15,'MICROBARS PER INCH',180..18)	TMPT 246
CALL AXISI(1.5,0..' '.1,SIZE,90..T(1),1800..0,0,3.)	TMPT 247
CALL SYMBL(5(1.8,2..15,'TIME (SEC)',90..10)	TMPT 248
CALL SCLGPH(V,T,L,0..1,0..DY,T(1),600.)	TMPT 249
CALL PLOT1(8..-3,-3)	TMPT 250
GO TO 200	TMPT 251
C	TMPT 252
C PRINT HISTORY OF MODE IOPT ONLY	TMPT 253
101 WRITE (6,102) IOPT	TMPT 254
102 FORMAT (1H1,49X,19HTABULATION OF MODE ,12, 9H RESPONSE///1H ,48X,	TMPT 255
1 10HTIME (SEC),9X,15HP (DYNES/CM**2)///)	TMPT 256
ON 103 IT=1.L	TMPT 257
103 WRITE (6,104) T(IT),TNINT(IOPT,IT)	TMPT 258
104 FORMAT (1H ,49X,F9.1,10X,F12.2)	TMPT 259
GO TO 66	TMPT 260
C	TMPT 261
C IF IOPT.LT.11 PLOT ONLY ACOUSTIC RESPONSE OF MODE IOPT	TMPT 262
107 ON 108 J=1.L	TMPT 263
108 V(J) = (-1) * TNINT(IOPT,J)	TMPT 264
C	TMPT 265
C DETERMINE SCALE FOR PRESSURE AXIS WHEN IOPT.LT.11	TMPT 266
111 CALL DXDY1(V,L,2..UMIN,DY,ND,K)	TMPT 267
UMIN = AINT(UMIN/25) * 25.0	TMPT 268
UMIN = AMIN1(UMIN,-25.)	TMPT 269
DY = ABS(UMIN)	TMPT 270
GO TO 75	TMPT 271
C	TMPT 272
200 RETURN	TMPT 273
END	TMPT 274

PROGRAM
TMPT

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C	TOTINT (SUBROUTINE)	7/27/68	TOTI	1
C			TOTI	2
C	----		TOTI	3
C	-----ABSTRACT-----		TOTI	4
C	TITLE - TOTINT		TOTI	5
C	THIS SUBROUTINE COMPUTES THE TOTAL INTEGRAL		TOTI	6
C			TOTI	7
C	XINT = INTEGRAL OVER Z FROM 0 TO INFINITY OF		TOTI	8
C			TOTI	9
C	$A_3(Z) * (A_1(Z) * F_1(Z) + A_2(Z) * F_2(Z)) * Z$	(1)	TOTI	10
C			TOTI	11
C	THE ATMOSPHERE IS ASSUMED TO BE REPRESENTED IN A MULTILAYER FOR		TOTI	12
C	WITH A1, A2, AND A3 CONSTANT IN EACH LAYER. THE INTEGRAL IS		TOTI	13
C	EVALUATED AS A SUM OF INTEGRALS OVER INDIVIDUAL LAYERS.		TOTI	14
C			TOTI	15
C	THE FUNCTIONS F1(Z) AND F2(Z) ARE CONTINUOUS ACROSS LAYER		TOTI	16
C	BOUNDARIES AND SATISFY THE RESIDUAL EQUATIONS		TOTI	17
C			TOTI	18
C	$DF_1(Z)/DZ = A(1,1) * F_1(Z) + A(1,2) * F_2(Z)$	(2A)	TOTI	19
C	$DF_2(Z)/DZ = A(2,1) * F_1(Z) + A(2,2) * F_2(Z)$	(2B)	TOTI	20
C			TOTI	21
C	WHERE THE ELEMENTS OF THE MATRIX A (COMPUTED BY SUBROUTINE AAAA		TOTI	22
C	ARE CONSTANT IN EACH LAYER.		TOTI	23
C			TOTI	24
C	THE FUNCTIONS F1(Z) AND F2(Z) ARE ASSUMED TO SATISFY THE UPPER		TOTI	25
C	BOUNDARY CONDITION THAT BOTH DECREASE EXPONENTIALLY WITH		TOTI	26
C	INCREASING HEIGHT IN THE UPPER HALFSpace. THE NORMALIZATION		TOTI	27
C	OF THE FUNCTIONS IS SUCH THAT AT THE LOWER BOUNDARY ZO OF THE		TOTI	28
C	UPPER HALFSpace		TOTI	29
C			TOTI	30
C	$F_1(Z_0) = -\text{SORT}(G) * A(1,2)$	(3A)	TOTI	31
C	$F_2(Z_0) = \text{SORT}(G) * (G + A(1,1))$	(3B)	TOTI	32
C			TOTI	33
C	WITH		TOTI	34
C			TOTI	35
C	$G = \text{SORT}(A(1,1)**2 + A(1,2)*A(2,1))$	(4)	TOTI	36
C			TOTI	37
C	THE ELEMENTS A(I,J) IN EONS. (3) AND (4) ARE THOSE APPROPRIATE		TOTI	38
C	TO THE UPPER HALFSpace. IF G**2 IS NEGATIVE, THE PROGRAM		TOTI	39
C	RETURNS L=-1. OTHERWISE IT RETURNS L=1.		TOTI	40
C			TOTI	41
C	PROGRAM NOTES		TOTI	42
C			TOTI	43
C	THE INTEGRATION OVER THE UPPER HALFSpace IS PERFORMED BY		TOTI	44
C	CALLING UPINT. THE INTEGRATIONS OVER THE LAYERS OF FINITE		TOTI	45
C	THICKNESS ARE PERFORMED BY CALLING ELINT.		TOTI	46
C			TOTI	47
C	THE PARAMETERS A1, A2, A3 DEPEND IN GENERAL ON ANGULAR		TOTI	48
C	FREQUENCY OMEGA, HORIZONTAL WAVENUMBER COMPONENTS AKX		TOTI	49
C	AND AKY, SOUND SPEED C, AND WIND VELOCITY COMPONENTS VX		TOTI	50
C	AND VY. THE FORMULAS USED ARE CONTROLLED BY THE INPUT		TOTI	51
C	PARAMETER IT WHICH IS TRANSMITTED TO SUBROUTINE USEAS.		TOTI	52
C			TOTI	53
C	THE PARAMETERS DEFINING THE MULTILAYER ATMOSPHERE ARE		TOTI	54
C	PRESUMED STORED IN COMMON		TOTI	55
C			TOTI	56
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)		TOTI	57
C			TOTI	58
C	AUTHOR - A.D. PIERCE, M.I.T., JULY, 1968		TOTI	59
C			TOTI	60
C	-----CALLING SEQUENCE-----		TOTI	61
C			TOTI	62
C	SEE SUBROUTINE NAMPDE		TOTI	63
C	DIMENSION C(100), VX(100), VY(100), H(100), PHI1(100), PHI2(100)		TOTI	64
C	COMMON IMAX, CI, VXI, VYI, HI (THESE MUST BE IN COMMON)		TOTI	65
C	CALL TOTINT(OMEGA, AKX, AKY, IT, L, XINT, PHI1, PHI2)		TOTI	66
C			TOTI	67
C	-----EXTERNAL SUBROUTINES REQUIRED-----		TOTI	68
C			TOTI	69
C	AAAA, MMMM, CAT, SAI, USEAS, UPINT, ELINT, RRRR		TOTI	70

PROGRAM
TOTINT

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C	AAAA AND BBBB ARE CALLED BY ELINT.	TOTI	71
C	CAT AND SAT ARE CALLED BY BBBB.	TOTI	72
C		TOTI	73
C	----ARGUMENT LIST----	TOTI	74
C		TOTI	75
C	OMEGA R#4 NO INP	TOTI	76
C	AKX R#4 NO INP	TOTI	77
C	AKY R#4 NO INP	TOTI	78
C	IT I#4 NO INP	TOTI	79
C	L I#4 NO OUT	TOTI	80
C	XINT I#4 NO OUT	TOTI	81
C	PHI1 R#4 100 INP	TOTI	82
C	PHI2 R#4 100 INP	TOTI	83
C		TOTI	84
C	COMMON STORAGE USED	TOTI	85
C	COMMON IMAX,CI,VXI,VYI,HI	TOTI	86
C		TOTI	87
C	IMAX I#4 NO INP	TOTI	88
C	CI R#4 100 INP	TOTI	89
C	VXI R#4 100 INP	TOTI	90
C	VYI R#4 100 INP	TOTI	91
C	HI R#4 100 INP	TOTI	92
C		TOTI	93
C	----INPUTS----	TOTI	94
C		TOTI	95
C	OMEGA =ANGULAR FREQUENCY IN RAD/SEC	TOTI	96
C	AKX =X COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)	TOTI	97
C	AKY =Y COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)	TOTI	98
C	IT =PARAMETER TRANSMITTED TO USEAS DEFINING FUNCTIONAL	TOTI	99
C	DEPENDENCE OF A1,A2,A3 COMPUTED BY USEAS.	TOTI	100
C	PHI1(I) =VALUE OF F1 AT BOTTOM OF LAYER I	TOTI	101
C	PHI2(I) =VALUE OF F2 AT BOTTOM OF LAYER I	TOTI	102
C	IMAX =NUMBER OF ATMOSPHERIC LAYERS WITH FINITE THICKNESS	TOTI	103
C	CI(I) =SOUND SPEED (KM/SEC) IN I-TH LAYER	TOTI	104
C	VXI(I) =X COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER	TOTI	105
C	VYI(I) =Y COMPONENT OF WIND VELOCITY (KM/SEC) IN I-TH LAYER	TOTI	106
C	HI(I) =THICKNESS IN KM OF I-TH LAYER	TOTI	107
C		TOTI	108
C	----OUTPUTS----	TOTI	109
C		TOTI	110
C	L =1 OR -1 DEPENDING ON WHETHER UPPER BOUNDARY CONDITION	TOTI	111
C	CAN OR CANNOT BE SATISFIED. SEE SUBROUTINE UPINT	TOTI	112
C	XINT =INTEGRAL OVER Z FROM 0 TO INFINITY AS DEFINED IN THE	TOTI	113
C	ABSTRACT.	TOTI	114
C		TOTI	115
C		TOTI	116
C		TOTI	117
C	----PROGRAM FOLLOWS BELOW----	TOTI	118
C		TOTI	119
C	SUBROUTINE TOTINT(OMEGA,AKX,AKY,IT,L,XINT,PHI1,PHI2)	TOTI	120
C		TOTI	121
C	DIMENSION AND COMMON STATEMENTS	TOTI	122
C	DIMENSION CI(100),VXI(100),VYI(100),HI(100),EM(2,2)	TOTI	123
C	DIMENSION PHI1(100),PHI2(100)	TOTI	124
C	COMMON IMAX,CI,VXI,VYI,HI	TOTI	125
C		TOTI	126
C	COMPUTATION OF CONTRIBUTION FROM UPPER HALFSPACE	TOTI	127
C	J=IMAX+1	TOTI	128
C	C=CI(J)	TOTI	129
C	VX=VXI(J)	TOTI	130
C	VY=VYI(J)	TOTI	131
C	CALL USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)	TOTI	132
C	CALL UPINT(OMEGA,AKX,AKY,C,VX,VY,A1,A2,L,F1,F2,UINT)	TOTI	133
C		TOTI	134
C		TOTI	135
C	CHECK IF L NEGATIVE	TOTI	136
C	IF(L .LT. 0) RETURN	TOTI	137
C		TOTI	138
C	WE DEFINE THE CONTRIBUTION AS UINT BY XINT. AS THE COMPUTATION CON-	TOTI	139
C	TINUES, XINT WILL SUCCESSIVELY REPRESENT THE VARIOUS SUBTOTALS UNTIL	TOTI	140

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C CONTRIBUTIONS FROM ALL THE LAYERS HAVE BEEN ADDED IN.	TOTI 141
XINT=A3*UINT	TOTI 142
C	TOTI 143
C START OF DO LOOP	TOTI 144
DO 90 I=1,IMAX	TOTI 145
J=IMAX+1-I	TOTI 146
C	TOTI 147
C COMPUTATION OF CONTRIBUTION FROM J-TH LAYER OF FINITE THICKNESS.	TOTI 148
C THE CURRENT VALUES F1 AND F2 REPRESENT F1(Z) AND F2(Z) AT TOP OF	TOTI 149
C J-TH LAYER.	TOTI 150
C=C1(IJ)	TOTI 151
VX=VXI(IJ)	TOTI 152
VY=VYI(IJ)	TOTI 153
H=HI(IJ)	TOTI 154
CALL USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)	TOTI 155
CALL ELINT(OMEGA,AKX,AKY,C,VX,VY,H,F1,F2,A1,A2,AINT)	TOTI 156
XINT=XINT+AINI*A3	TOTI 157
C	TOTI 158
C COMPUTATION OF F1 AND F2 APPROPRIATE TO TOP OF (J-1)-TH LAYER	TOTI 159
F1 = PHI1(IJ)	TOTI 160
90 F2 = PHI2(IJ)	TOTI 161
C END OF DO LOOP	TOTI 162
C	TOTI 163
RETURN	TOTI 164
END	TOTI 165

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TOTINT

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C      UPINT (SUBROUTINE)              7/25/68
C
C      ----ABSTRACT----
C
C  TITLE - UPINT
C      THIS SUBROUTINE COMPUTES AN INTEGRAL OF THE FORM
C
C      UINT = INTEGRAL OVER Z FROM Z0 TO INFINITY OF
C
C      (A1*F1(Z) + A2*F2(Z))**2
C
C      (1)
C
C      THE FUNCTIONS F1(Z) AND F2(Z) ARE THE SOLUTIONS OF THE COUPLED
C      ORDINARY DIFFERENTIAL EQUATIONS
C
C      DF1/DZ = A11*F1 + A12*F2
C      DF2/DZ = A21*F1 + A22*F2
C
C      (2A)
C      (2B)
C
C      WHERE THE ELEMENTS OF THE MATRIX A ARE INDEPENDENT OF Z. THE
C      FUNCTIONS F1(Z) AND F2(Z) ARE SUBJECT TO THE UPPER BOUNDARY
C      CONDITION THAT BOTH DECREASE EXPONENTIALLY WITH INCREASING
C      ALTITUDE. SINCE THE MATRIX A IS COMPUTED BY AAAA, INSURING
C      A(2,2)=-A(1,1), BOTH SHOULD VARY WITH HEIGHT AS EXP(-G*(Z-Z0))
C      WHERE
C
C      G = SORT(A(1,1)**2+A(1,2)*A(2,1))
C
C      (3)
C
C      IT IS ASSUMED G**2 IS POSITIVE. OTHERWISE L=-1 IS RETURNED.
C
C      THE EXPLICIT FORMS ADOPTED FOR F1 AND F2 WHICH SATISFY (2) ARE
C
C      F1 = -SORT(G)*A(1,2)*EXP(-G*(Z-Z0))
C      F2 = SORT(G)*(G+A(1,1))*EXP(-G*(Z-Z0))
C
C      (4A)
C      (4B)
C
C      THUS UINT IS GIVEN BY
C
C      UINT = ((-A1*A(1,2)+A2*(G+A(1,1))**2)/2.0
C
C      (5)
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C
C  AUTHOR - A.D.PIERCE, N.I.T., JULY, 1968
C
C      ----CALLING SEQUENCE----
C
C  SEE SUBROUTINE TOTINT
C  NO DIMENSION STATEMENTS REQUIRED
C  CALL UPINT(OMEGA,AKX,AKY,C,VX,VY,A1,A2,L,F1,F2,UINT)
C
C      ----EXTERNAL SUBROUTINES REQUIRED----
C
C  AAAA
C
C      ----ARGUMENT LIST----
C
C  OMEGA      R*4      ND      INP
C  AKX        R*4      ND      INP
C  AKY        R*4      ND      INP
C  C          R*4      ND      INP
C  VX         R*4      ND      INP
C  VY         R*4      ND      INP
C  A1         R*4      ND      INP
C  A2         R*4      ND      INP
C  L          I*4      ND      OUT
C  F1         R*4      ND      OUT
C  F2         R*4      ND      OUT
C  UINT       R*4      ND      OUT
C
C  NO COMMON STORAGE USED
C
C      ----INPUTS----

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UPIN 1
UPIN 2
UPIN 3
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UPINT
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C	OMEGA	=ANGULAR FREQUENCY IN RAD/SEC	UPIN 71
C	AKX	=X COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)	UPIN 72
C	AKY	=Y COMPONENT OF WAVE NUMBER VECTOR IN KM**(-1)	UPIN 73
C	C	=SOUND SPEED IN KM/SEC	UPIN 74
C	VX	=X COMPONENT OF WIND VELOCITY IN KM/SEC	UPIN 75
C	VY	=Y COMPONENT OF WIND VELOCITY IN KM/SEC	UPIN 76
C	A1	=COEFFICIENT OF F1(Z) IN INTEGRAND	UPIN 77
C	A2	=COEFFICIENT OF F2(Z) IN INTEGRAND	UPIN 78
C		---- <td>UPIN 79</td>	UPIN 79
C	L	=1 OR -1 DEPENDING ON WHETHER UPPER BOUNDARY CONDITION	UPIN 80
C		OF F1(Z), F2(Z) DECREASING EXPONENTIALLY WITH INCREASE	UPIN 81
C		HEIGHT CAN OR CANNOT BE SATISFIED. IT REPRESENTS THE	UPIN 82
C		SIGN OF G**2 WHERE G IS DEFINED IN THE ABSTRACT.	UPIN 83
C	F1	=VALUE OF F1(Z) AT BOTTOM OF HALFSpace, DEFINED AS	UPIN 84
C		-SORT(G)*A(1,2) FROM EQN. (4A).	UPIN 85
C	F2	=VALUE OF F2(Z) AT BOTTOM OF HALFSpace, DEFINED AS	UPIN 86
C		SORT(G)*(G+A(1,1)) FROM EQN. (4B)	UPIN 87
C	UINT	=THE INTEGRAL DEFINED BY EQNS. (1) AND (5) IN THE	UPIN 88
C		ABSTRACT	UPIN 89
C			UPIN 90
C			UPIN 91
C			UPIN 92
C		-----PROGRAM FOLLOWS BELOW-----	UPIN 93
C			UPIN 94
C			UPIN 95
C			UPIN 96
C		SUBROUTINE UPINT (OMEGA, AKX, AKY, C, VX, VY, A1, A2, L, F1, F2, UINT)	UPIN 97
C		DIMENSION A(2,2)	UPIN 98
C			UPIN 99
C		COMPUTATION OF A MATRIX AND OF X=G**2	UPIN 100
C		CALL AAAA(OMEGA, AKX, AKY, C, VX, VY, A)	UPIN 101
C		X=A(1,1)**2+A(1,2)*A(2,1)	UPIN 102
C		CHECK ON SIGN OF X	UPIN 103
C		2 IF(X .GT. 0.0) GO TO 3	UPIN 104
C			UPIN 105
C		X IS NEGATIVE	UPIN 106
C		L=-1	UPIN 107
C		RETURN	UPIN 108
C		CONTINUING FROM 2 WITH X POSITIVE	UPIN 109
C		3 L=1	UPIN 110
C		G=SQRT(X)	UPIN 111
C		GRT=SQRT(G)	UPIN 112
C		F1=-GRT*A(1,2)	UPIN 113
C		F2=GRT*(G+A(1,1))	UPIN 114
C		COMPUTATION OF UINT	UPIN 115
C		UINT=(-A1*A(1,2)+A2*(G+A(1,1))**2/2.0	UPIN 116
C		RETURN	UPIN 117
C		END	UPIN 118

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UPINT

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C      USEAS (SUBROUTINE)              7/25/68
C
C      ----ABSTRACT----
C
C  TITLE - USEAS
C      THE PURPOSE OF THIS SUBROUTINE IS TO COMPUTE THE NUMBERS A1, A2
C      AND A3 WHICH DEPEND ON ANGULAR FREQUENCY OMEGA, HORIZONTAL WAVE
C      NUMBER COMPONENTS AKX AND AKY, THE SOUND SPEED C, AND THE WIND
C      SPEED COMPONENTS VX AND VY. THE INTEGER IT DETERMINES WHICH
C      FORMULAS ARE USED FOR A1, A2, AND A3 ACCORDING TO THE FOLLOWING
C      TABLE
C
C      (IT)      (A1)      (A2)      (A3)
C      -----
C      1          1          0          1
C      2          0          1          1
C      3          1          0      BOM*(KDOTV)/(C**2*K)
C      4          1          0      BOM/C**2
C      5          1          0      VX*BOM/C**2
C      6          1          0      VY*BOM/C**2
C      7          G/C        -C      K*OMEGA/BOM**3
C      8          G/C        -C      1.0/BOM**2
C      9          G/C        -C      K**2/BOM**3
C     10          G/C        -C      VX*K**2/BOM**3
C     11          G/C        -C      VY*K**2/BOM**3
C
C      HERE BOM=OMEGA-KDOTV IS THE DOPPLER SHIFTED ANGULAR FREQUENCY,
C      KDOTV=AKX*VX+AKY*VY IS THE DOT PRODUCT OF WAVE NUMBER WITH
C      THE WIND VELOCITY, AND K=SQRT(AKX**2+AKY**2) IS THE MAGNITUDE
C      OF THE WAVE NUMBER VECTOR. THE ACCELERATION OF GRAVITY G IS
C      TAKEN AS .0098 KM/SEC**2 IN THE COMPUTATION. COMPUTED VALUES
C      SHOULD BE IN KM/SEC SYSTEM OF UNITS.
C
C  LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C28-6515-4)
C  AUTHOR   - A.D.PIERCE, M.I.T., JUNE,1968
C
C      ----CALLING SEQUENCE----
C
C  SEE SUBROUTINE TOTINT
C  NO DIMENSION STATEMENTS ARE REQUIRED
C  IT=
C  CALL USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)
C  A1,A2,A3 ARE NOW AVAILABLE FOR FUTURE COMPUTATIONS
C
C  NO EXTERNAL LIBRARY SUBROUTINES ARE REQUIRED
C
C      ----ARGUMENT LIST----
C
C      OMEGA      R04      NO      INP
C      AKX        R04      NO      INP
C      AKY        R04      NO      INP
C      C          R04      NO      INP
C      VX         R04      NO      INP
C      VY         R04      NO      INP
C      IT         I04      NO      INP
C      A1         R04      NO      OUT
C      A2         R04      NO      OUT
C      A3         R04      NO      OUT
C
C  NO COMMON STORAGE USED
C
C      ----INPUTS----
C
C      OMEGA      =ANGULAR FREQUENCY IN RAD/SEC
C      AKX        =X COMPONENT OF WAVE NUMBER VECTOR IN KM*(-1)
C      AKY        =Y COMPONENT OF WAVE NUMBER VECTOR IN KM*(-1)
C      C          =SOUND SPEED IN KM/SEC
C      VX         =X COMPONENT OF WIND VELOCITY IN KM/SEC
C      VY         =Y COMPONENT OF WIND VELOCITY IN KM/SEC
C      IT         =CONTROL PARAMETER FOR SELECTION OF FORMULAS (SEE

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PROGRAM
USEAS
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C	ABSTRACT).	USEA	71	
C		USEA	72	
C	----OUTPUTS----	USEA	73	
C		USEA	74	
C	A1 =PARAMETER DEFINED BY FORMULAS IN ABSTRACT	USEA	75	
C	A2 =PARAMETER DEFINED BY FORMULAS IN ABSTRACT	USEA	76	
C	A3 =PARAMETER DEFINED BY FORMULAS IN ABSTRACT	USEA	77	
C		USEA	78	
C		USEA	79	
C		USEA	80	
C	----PROGRAM FOLLOWS BELOW----	USEA	81	
C		USEA	82	
C	SUBROUTINE USEAS(OMEGA,AKX,AKY,C,VX,VY,IT,A1,A2,A3)	USEA	83	
C		USEA	84	
C	WE ASSIGN VALUES TO A1,A2,A3 WHICH WILL NOT NECESSARILY BE THEIR EXIT	USEA	85	
C	VALUES.	USFA	86	
	A1=1.0	USEA	87	
	A2=0.0	USEA	88	
	A3=1.0	USEA	89	
C	IF IT IS 1. THESE ARE CORRECT, HOWEVER.	USEA	90	
	IF(IT.EQ. 1) RETURN	USEA	91	
	IF(IT.GT. 1) GO TO 200	USEA	92	
C		USEA	93	
C	IT IS 2. THE CURRENT VALUES ARE 1.0,1. WE CHANGE THE FIRST TWO.	USEA	94	
	A1=0.0	USEA	95	
	A2=1.0	USEA	96	
	RETURN	USEA	97	
C		USEA	98	
C	IT IS .GT. 2. WE COMPUTE SOME QUANTITIES FOR FUTURE REFERENCE	USEA	99	
	200 AKV=AKX*VX+AKY*VY	USEA	100	
	AKSQ=AKX**2+AKY**2	USEA	101	
	BOM=OMEGA*AKV	USEA	102	
	AK=SQRT(AKSQ)	USEA	103	
C	THE CURRENT VALUES OF A1,A2,A3 ARE STILL 1.0,1.	USEA	104	
	IF(IT.GT. 3) GO TO 300	USEA	105	
C		USEA	106	
C	IT IS EQUAL TO 3. ONLY A3 NEED BE CHANGED.	USEA	107	
	A3=BOM*AKV/(C**2*AK)	USEA	108	
	RETURN	USEA	109	
C		USEA	110	
C	IT IS 4 OR GREATER. WE SET A3 TO VALUE APPROPRIATE FOR IT=4.	USEA	111	
	300 A3=BOM/C**2	USEA	112	
C	THE CURRENT VALUES OF A1 AND A2 ARE 1 AND 0	USEA	113	
	IF(IT.EQ. 4) RETURN	USEA	114	
	IF(IT.EQ. 5) A3=VX*A3	USEA	115	
	IF(IT.EQ. 6) A3=VY*A3	USEA	116	
	IF(IT.EQ. 5 .OR. IT.EQ. 6) RETURN	USEA	117	
C		USEA	118	
C	IT IS 7 OR LARGER	USEA	119	
	A1=.0098/C	USEA	120	
	A2=-C	USEA	121	
C		USEA	122	
C	THE ONLY QUANTITY WE NEED DETERMINING IS A3	USEA	123	
C		USEA	124	
	IF(IT.GT. 7) GO TO 700	USEA	125	
C	IT=7	USEA	126	
	A3=AK*OMEGA/BOM**3	USEA	127	
	RETURN	USEA	128	
C		USEA	129	
	700 IF(IT.GT.8) GO TO 800	USFA	130	
C	IT=8	USEA	131	
	A3=1.0/BOM**2	USEA	132	
	RETURN	USEA	133	
C		USEA	134	
C	FOR IT=9,10,11 WE NEED THE FACTOR AKSQ/BOM**3	USEA	135	
	800 A3=AKSQ/BOM**3	USEA	136	PROGRAM
	IF(IT.EQ. 9) RETURN	USEA	137	USFAS
	IF(IT.GT. 10) GO TO 1000	USEA	138	
C	IT=10	USEA	139	PAGE
	A3=VX*A3	USEA	140	93

RETURN
C IT=11 (YOU SHOULDN'T INPUT IT FOR VALUES OUTSIDE RANGE OF 1 TO 11.)
1000 A3=VY*A3
RETURN
END

USEA 141
USEA 142
USEA 143
USEA 144
USEA 145
USEA 146

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USEAS

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C	WIDEN (SUBROUTINE)	6/19/68	WIDE	1
C			WIDE	2
C			WIDE	3
C	-----ABSTRACT-----		WIDE	4
C			WIDE	5
C	TITLE - WIDEN		WIDE	6
C	WIDEN MATRIX INMODE BY ADDING KW COLUMNS BETWEEN COLUMNS N1 AND		WIDE	7
C	N1+1		WIDE	8
C			WIDE	9
C	WIDEN ADDS KW ELEMENTS TO THE VECTOR OF ANGULAR FREQUENCY		WIDE	10
C	OM, DIVIDING THE INTERVAL BETWEEN OM(N1) AND OM(N1+1) IN		WIDE	11
C	KW+1 EQUAL PARTS. FOR EACH NEW ANGULAR FREQUENCY, A NEW		WIDE	12
C	COLUMN IS ADDED TO THE INMODE MATRIX (DEFINED IN SUBROUTINE		WIDE	13
C	TIME MPUT). INMODE IS STORED IN VECTOR FORM, COLUMN AFTER		WIDE	14
C	COLUMN.		WIDE	15
C			WIDE	16
C	LANGUAGE - FORTRAN IV (360, REFERENCE MANUAL C2A-6515-4)		WIDE	17
C			WIDE	18
C	AUTHORS - A.D. PIERCE AND J. POSEY, M.I.T., JUNE, 1968		WIDE	19
C			WIDE	20
C			WIDE	21
C	-----USAGE-----		WIDE	22
C			WIDE	23
C	OM, V, INMODE MUST BE DIMENSIONED IN CALLING PROGRAM		WIDE	24
C	THE ONLY SUBROUTINE CALLED IS NMODF, DESCRIBED ELSEWHERE IN THIS		WIDE	25
C	SERIES		WIDE	26
C			WIDE	27
C	FORTRAN USAGE		WIDE	28
C	CALL WIDEN(OM, V, INMODE, NOM, NOMP, NVP, N1, KW, THETK)		WIDE	29
C			WIDE	30
C	INPUTS		WIDE	31
C			WIDE	32
C	OM VECTOR WHOSE ELEMENTS ARE THE VALUES OF ANGULAR FREQUENCY		WIDE	33
C	R*4(D) CORRESPONDING TO THE COLUMNS OF MATRIX INMODE. (RAD/SEC)		WIDE	34
C			WIDE	35
C	V VECTOR WHOSE ELEMENTS ARE THE VALUES OF PHASE VELOCITY		WIDE	36
C	R*4(D) CORRESPONDING TO THE ROWS OF MATRIX INMODE. (KM/SEC)		WIDE	37
C			WIDE	38
C	INMODE EACH ELEMENT OF THIS MATRIX CORRESPONDS TO A POINT IN THE		WIDE	39
C	I*4(D) FREQUENCY (OM) - PHASE VELOCITY (V) PLANE. IF THE NORMAL		WIDE	40
C	MODE DISPERSION FUNCTION (FPP, FOUND BY CALLING SUBROUTINE		WIDE	41
C	NMODF) IS POSITIVE AT THAT POINT, THE ELEMENT IS +1, IF		WIDE	42
C	FPP IS NEGATIVE, THE ELEMENT IS -1, IF FPP DOES NOT EXIST		WIDE	43
C	THE ELEMENT IS 5. INMODE IS STORED IN VECTOR FORM, COLUMN		WIDE	44
C	AFTER COLUMN.		WIDE	45
C			WIDE	46
C	NOM NUMBER OF ELEMENTS IN OM (AND NO. OF COLUMNS IN INMODE)		WIDE	47
C	I*4 WHEN WIDEN IS CALLED.		WIDE	48
C	NVP NUMBER OF ELEMENTS IN V (AND NO. OF ROWS IN INMODE).		WIDE	49
C	I*4		WIDE	50
C			WIDE	51
C	N1 NUMBER OF INMODE COLUMN IMMEDIATELY LEFT OF SPACE IN WHICH		WIDE	52
C	I*4 NEW COLUMNS ARE TO BE ADDED.		WIDE	53
C			WIDE	54
C	KW NUMBER OF COLUMNS TO BE ADDED TO INMODE.		WIDE	55
C	I*4		WIDE	56
C			WIDE	57
C	THETK PHASE VELOCITY DIRECTION MEASURED COUNTER-CLOCKWISE FROM		WIDE	58
C	R*4 X-AXIS (RADIAN).		WIDE	59
C			WIDE	60
C	OUTPUTS		WIDE	61
C			WIDE	62
C	THE OUTPUTS ARE NOMP (= NOM + KW = THE NEW NUMBER OF ELEMENTS IN OM		WIDE	63
C	AND THE NEW NUMBER OF COLUMNS IN INMODE) AND REVISED VERSIONS OF OM		WIDE	64
C	AND INMODE.		WIDE	65
C			WIDE	66
C			WIDE	67
C	-----EXAMPLE-----		WIDE	68
C			WIDE	69
C	SUPPOSE OM = 1.0, 2.0, 3.0 AND WIDEN IS CALLED WITH KW = 3, AND N1 =		WIDE	70

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C	2. THEN UPON RETURN TO CALLING PROGRAM, OM = 1.0,2.0,2.25,2.5,2.75,	WIDE	71
C	3.0, NOMP = 6, AND INMODE WILL HAVE THREE NEW ROWS CORRESPONDING TO	WIDE	72
C	THE NEW ELEMENTS OF OM.	WIDE	73
C		WIDE	74
C		WIDE	75
C	-----PROGRAM FOLLOWS BELOW-----	WIDE	76
C		WIDE	77
C		WIDE	78
C	SUBROUTINE WIDEN(OM,V,INMODE,NOM,NOMP,NVP,N1,KW,THETK)	WIDE	79
C		WIDE	80
C	VARIABLE DIMENSIONING	WIDE	81
C	DIMENSION OM(1),V(1),INMODE(1)	WIDE	82
C	COMMON /MAX,C(100),VXI(100),VVI(100),NI(100)	WIDE	83
C		WIDE	84
C	INTERVAL AT WHICH NEW VALUES OF OM ARE RE PLACED BETWEEN OM(N1) AND	WIDE	85
C	OM(N1+1) IS DETERMINED	WIDE	86
C	DELOM=(OM(N1+1)-OM(N1))/(KW+1)	WIDE	87
C		WIDE	88
C	NOMP IS NUMBER OF ELEMENT IN EXPANDED OM	WIDE	89
C	NOMP=NOM*KW	WIDE	90
C		WIDE	91
C	NSTART IS THE NUMBER OF THE ELEMENT IN THE NEW OM WHICH CORRESPONDS TO	WIDE	92
C	ELEMENT N1+1 IN THE OLD OM VECTOR	WIDE	93
C	NSTART=N1+1+KW	WIDE	94
C		WIDE	95
C	MOVE ALL ELEMENTS OF OM BEYOND ELEMENT N1 TO THEIR NEW POSITIONS, BEGI	WIDE	96
C	NING WITH THE LAST ELEMENT	WIDE	97
C	DO 90 NJ=NSTART,NOMP	WIDE	98
C	J=NOMP-(NJ-NSTART)	WIDE	99
C	JOLD=J-KW	WIDE	100
C		WIDE	101
C	MOVE COLUMN JOLD OF INMODE INTO POSITION FOR COLUMN J	WIDE	102
C	OM(J)=OM(JOLD)	WIDE	103
C	DO 90 IP=1,NVP	WIDE	104
C	IJ=(J-1)*NVP+(NVP-IP) + 1	WIDE	105
C	JOLD=(JOLD-1)*NVP+(NVP-IP) + 1	WIDE	106
C	INMODE(IJ)=INMODE(IJOLD)	WIDE	107
C	90 CONTINUE	WIDE	108
C		WIDE	109
C	NSTART IS NUMBER OF FIRST NEW COLUMN	WIDE	110
C	NSTART=N1+1	WIDE	111
C		WIDE	112
C	NEND IS NUMBER OF LAST NEW COLUMN	WIDE	113
C	NEND=N1+KW	WIDE	114
C		WIDE	115
C	NEW VALUES OF OM ARE ESTABLISHED	WIDE	116
C	OMEGA=OM(N1)	WIDE	117
C	DO 190 J=NSTART,NEND	WIDE	118
C	OM(J)=OMEGA + DELOM	WIDE	119
C	OMEGA = OM(J)	WIDE	120
C	90 190 I=1,NVP	WIDE	121
C		WIDE	122
C	IJ IS NUMBER OF ELEMENT IN VECTOR REPRESENTATION OF INMODE WHICH IS	WIDE	123
C	ELEMENT J IN ROW I OF INMODE	WIDE	124
C	IJ=(J-1)*NVP+I	WIDE	125
C	VPHSE=V(I)	WIDE	126
C		WIDE	127
C	CALL NMODN TO EVALUATE THE NORMAL MODE DISPERSION FUNCTION (FPP)	WIDE	128
C	CALL NMODN(OMEGA,VPHSE,THETK,L,FPP,K)	WIDE	129
C		WIDE	130
C	IF FPP DOES NOT EXIST L = -1	WIDE	131
C	IF(L .EQ. -1) GO TO 150	WIDE	132
C		WIDE	133
C	IF FPP DOES EXIST L = 1 AND INMODE(IJ) = (FPP/ABS(FPP))	WIDE	134
C	INMODE(IJ) = 1	WIDE	135
C	IF (FPP.LE.0.0) INMODE(IJ) = -1	WIDE	136
C	GO TO 190	WIDE	137
C	150 INMODE(IJ)=5	WIDE	138
C	190 CONTINUE	WIDE	139
C	190 CONTINUE	WIDE	140

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RETURN
END

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13. ABSTRACT <p>A computer program is described which enables one to compute the pressure wave-form at a distant point following the detonation of a nuclear explosion in the atmosphere. The theoretical basis of the program and the numerical methods used in its formulation are explained; a deck listing and instructions for the program's operation are included. The primary limitation on the program's applicability to realistic situations is that the atmosphere is assumed to be perfectly stratified. However, the temperature and wind profiles may be arbitrarily specified. Numerical studies carried out by the program show some discrepancies with previous computations by Harkrider for the case of an atmosphere without winds. These discrepancies are analyzed and shown to be due to different formulations of the source model for a nuclear explosion. Other numerical studies explore the effects of various atmospheric parameters on the waveforms. In the remainder of the report, two alternate theoretical formulations of the problem are described. The first of these is based on the neglect of the vertical acceleration term in the equations of hydrodynamics and allows a solution by Cagniard's integral transform technique. The second is based on the hypothesis of propagation in a single guided mode and permits a study of the effects of departures from stratification on the waveforms.</p>		

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